Interactions between optical bullets with different velocities in the three–dimensional cubic–quintic complex Ginzburg–Landau equation*

Zheng Lang(郑 浪) and Tang Yi(唐 翌)†

Department of Physics and Institute of Modern Physics, Xiangtan University, Xiangtan 411105, *China*

(Received 14 April 2009; revised manuscript received 8 July 2009)

By using the three-dimensional complex Ginzburg–Landau equation with cubic–quintic nonlinearity, this paper numerically investigates the interactions between optical bullets with different velocities in a dissipative system. The results reveal an abundance of interesting behaviours relating to the velocities of bullets: merging of the optical bullets into a single one at small velocities; periodic collisions at large velocities and disappearance of two bullets after several collisions in an intermediate region of velocity. Finally, it also reports that an extra bullet derives from the collision of optical bullets when optical bullets are at small velocities but with high energies.

Keywords: optical bullet, dissipative soliton, complex Ginzburg–Landau equation

PACC: 4265S, 4265T, 4265J

1. Introduction

A dissipative system, a term coined by Nikolis and Prigogine,[1] is an open system that is kept in contact with an external source and exchanges energy and matter with the source. The complex Ginzburg–Landau equation (CGLE) is one of the notable models for description of dissipative systems and it models a vast variety of nonlinear phenomena in physics. To name but a few, we mention pulse generation by passively mode-locked soliton lasers, signal transmission in all-optical communication lines, superconductivity, superfluidity and Bose–Einstein condensation.[2−5] Many basic properties of the equation and its solutions are reviewed in Refs. [6] and [7] in great detail. An important class of patterns, properly reproduced by the CGLE, is in forms of localized pulses, which are called dissipative solitons (DSs). These DSs include plain solitons, pulsating solitons, creeping solitons, erupting solitons,[8] optical bullets and double optical bullets,[9] occurring especially in dissipative optical systems. The DSs are qualitatively different from solitons in Hamiltonian systems. In Hamiltonian systems, soliton solutions appear as the result of balance between diffraction (dispersion) and nonlinearity while in dissipative systems the soliton occurs mainly as a balance between nonlinear gain and loss. When the system parameters are chosen in special regions, DSs can propagate stably.

Owing to a variety of DSs being found, more and more attention is being paid to their interactions. Interactions of DSs in one- and two-dimensional (1D and 2D) spaces have been systemically studied.[10−14] In recent years, analysis of interactions of dissipative optical bullets (spatiotemporal solitons) in three-dimensional (3D) space has gradually attracted more and more attention because of the abundant and interesting behaviours occurring in interactions of optical bullets. Soto-Crespo *et al.*[15] made use of the interaction plane to numerically analyse interactions between dissipative optical bullets in normal and anomalous dispersion regions. D. Mihalache *et al.*[16] studied the collisions between coaxial vortex solitons in the 3D cubic–quintic CGLE. Outcomes observed in those simulations included fusion of the bullets into a single one, quasi-elastic interaction, formation of a "bullet molecule", or creation of an extra soliton.

A dissipative optical bullet can propagate with variable velocities in space, whose velocities play a relevant role in collision. Evidence can be found from two cases of bullet interactions.[15] In one case, the interaction between two bullets with zero velocity displays fusion of the bullets into a single one, in the other case with nonzero velocity, two colliding bullets show

*Project supported by the Key Project of the Educational Department of Hunan Province of China (Grant No. 04A058), and the General Project of the Educational Department of Hunan Province of China (Grant No. 07C754).

†Corresponding author. E-mail: tangyi@xtu.edu.cn

© 2010 Chinese Physical Society and IOP Publishing Ltd

quasi–elastic collision. Obviously, the behaviours of the two cases are very different because of their different velocities. However, up to now, a detailed study of dynamical features of the interaction of bullets with different velocities is still missing. In this work, we investigate interactions of light bullets with the same phase and different velocities in the space domain and an abundance of interesting behaviours are observed. The rest of the paper is organized as follows. In Section 2, we present the master equation which we are solving and provide a stationary optical bullet as a localized initial condition. The interactions of light bullets with different velocities in the space domain are investigated in Section 3. In the last section, we summarize our main conclusions.

2. The cubic–quintic Ginzburg–Landau model and stable light bullet

Our studies are based on an extended CGLE model which includes cubic and quintic nonlinear terms. The normalized propagation equation reads as[9]

$$\begin{aligned} &\mathrm{i}\frac{\partial U}{\partial z}+\frac{D}{2}\frac{\partial^2 U}{\partial t^2}+\frac{1}{2}\frac{\partial^2 U}{\partial x^2} \\ &+\frac{1}{2}\frac{\partial^2 U}{\partial y^2}+|U|^2 U+\kappa|U|^4 U \\ &=\mathrm{i}\delta U+\mathrm{i}\varepsilon|U|^2 U+\mathrm{i}\beta\frac{\partial^2 U}{\partial t^2}+\mathrm{i}\mu|U|^4 U, \end{aligned} \quad (1)$$

where D, κ, δ, ε, β, μ are real constants (we do not require them to be small) and U is a complex function. In the present work, the equation is written in the "optical" notation, assuming evolution along the propagation coordinate z of a beam with a 3D cross section in the plane t, x, y. Physically, $U(t, x, y, z)$ stands for the local amplitude of the normalized envelope of the field. The left-hand side of Eq. (1) contains the conservative terms, where D represents the coefficient for group velocity dispersion and κ is the saturation coefficient of the Kerr nonlinearity. The right-hand side of Eq. (1) includes all dissipative terms: δ, ε, β and μ are the coefficients for linear loss (if negative), nonlinear gain (if positive), spectral filtering,[17,18] and saturation of the nonlinear gain (if negative), respectively.

We solve Eq. (1) by using a split-step Fourier method[19,20] and periodic boundary conditions are applied in x, y and t. Most simulations presented in the paper are carried out on a $256 \times 256 \times 256$ (or $512 \times 256 \times 256$ when necessary) mesh. We have used various values of step sizes along the spatial and temporal domains to verify that the results do not depend on the mesh intervals.

In order to investigate the interactions of dissipative optical bullets, we firstly look for stationary dissipative optical bullet solutions for Eq. (1). We use a symmetric initial condition in the following form:

$$\begin{aligned} &U(t, x, y, z) \\ &=3.0\exp\left[-\left(\frac{x}{1.2}\right)^2-\left(\frac{y}{1.2}\right)^2-\left(\frac{z}{1.2}\right)^2\right]. \end{aligned} \quad (2)$$

If we choose the parameters: $D=-0.1$, $\kappa=-0.01$, $\delta=-0.4$, $\varepsilon=0.95$, $\beta=0.1$, $\mu=-0.1$, equation (1) with these parameters is found to admit a stable optical bullet defined by Eq. (2).[15] One of the major characteristics of the optical bullet is the total amount of energy Q defined by

$$Q(z)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|U(x, y, z, t)|^2\,\mathrm{d}x\mathrm{d}y\mathrm{d}t. \quad (3)$$

However, the energy is not a conserved quantity for a dissipative system and it evolves in accordance with the so-called balance equation.[21] If the solution stays localized, the energy evolves but remains finite. When a stationary optical bullet is reached, the total energy Q converges to a constant value.

3. Interactions of dissipative optical bullets with velocities

To study collisions of two dissipative optical bullets with velocities, we must introduce the initial velocity by multiplying the individual input profiles by $\exp(\mathrm{i}v_1 x)$ and $\exp(\mathrm{i}v_2 x)$,[15,22] respectively. Thus the initial condition for two moving optical bullets is defined as

$$\begin{aligned} U(t, x, y, 0) &= U_0\left(t, x-\frac{R}{2}, y\right)\exp(-\mathrm{i}v_1 x) \\ &+U_0\left(t, x+\frac{R}{2}, y\right)\exp(\mathrm{i}v_2 x), \end{aligned} \quad (4)$$

where U_0 is defined by Eq. (2) and R stands for the initial separation between two bullets. The v_1 and v_2, which are called initial velocities of two optical bullets in the present work, represent the amount of spatial chirp related to the angle of collision.[15] The phase difference between them in our simulations is set to be zero.

Firstly, several cases of collisions of two dissipative optical bullets with small velocities are given in

Fig. 1. In these cases, the initial separation is chosen to be $R = 5$ in Eq. (4) and the velocities v are chosen to be: (a) $v_1 = 0.3$, $v_2 = 0.5$, (b) $v_1 = 1$, $v_2 = 1$ and (c) $v_1 = 2$, $v_2 = 2$, respectively. Figures 1(a)–1(c) are the contour plots of $|U(t = 0, x, y = 0, z)|$ in the x–z plane displaying the evolution of the amplitude of two optical bullets versus z. Two optical bullets are initially moving in opposite directions. Then they collide and merge into a single stationary one after a finite number of oscillations. The stationary bullet created by the in phase bullets is a consequence of the small velocity of the bullet (the shallow angle of collision). The angle of collision of the soliton plays an important role in interaction of the soliton in a variety of nonintegrable systems.[23,24] When a soliton is viewed as a guided mode of its own self-induced waveguide,[25] v_c is the (complementary) critical angle for total internal reflection in the waveguide,[26] as is known from waveguide theory. When the collision angle is less than the critical angle v_c, solitons can couple light into each other's waveguides; hence the collisions are inelastic, which leads to fusion. When the collision angle is larger than the critical angle v_c, solitons pass through each other.[27,28] In Fig. 1, all the angles of collision are less than the critical angle v_c, therefore the two bullets merge into one in the three cases. Meanwhile, we show the evolution of the energy Q versus z for all the above-mentioned cases; it can be seen that the total energies of bullets converge to a constant value finally. This is because the stationary solution is completely fixed by the parameters of Eq. (1) in the dissipative system.[15] The energy of a stationary bullet is equal to that for the bullet at the chosen set of parameters. When the states of constant energy have been reached, two colliding bullets actually merge into a single stationary bullet, which occurs in all three cases. Such a phenomenon has also been observed in the case of interaction between bullets with zero velocities.[15]

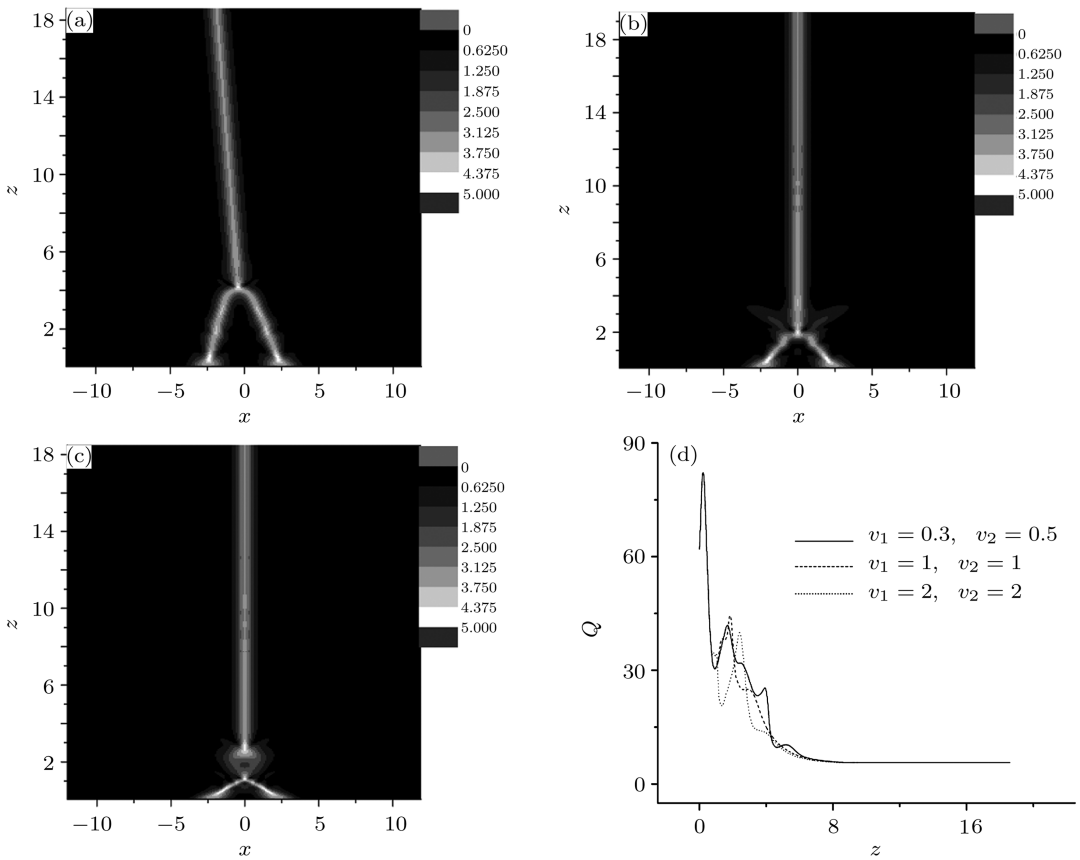


Fig. 1. The contour plot of the interaction between two neighbouring optical bullets for the same separation $R = 5$ and different initial velocities: (a) $v_1 = 0.3$, $v_2 = 0.5$; (b) $v_1 = 1$, $v_2 = 1$; (c) $v_1 = 2$, $v_2 = 2$. Figure (d) shows the total energy Q of the optical bullets vs. propagation distance z in the three cases.

Secondly, we investigate interaction of two dissipative optical bullets with intermediate velocities. Scenarios are illustrated in Fig. 2 by pictures of the evolution of the field in the x–z plane. Two bullets initially move

head on, because the collision angle is larger than the critical angle v_c, after the first collision they pass through each other. In the intermediate region of velocity, the collisions break the balance of gain and loss of bullets. The colliding bullets fail to adjust them to the system defined by the parameters of Eq. (1). After several collisions, they both disappear gradually owing to complete symmetry of the bullets. The evolution of the total energy is also shown in Fig. 2(d). Because of the disappearance of the two bullets, the total energy becomes zero eventually.

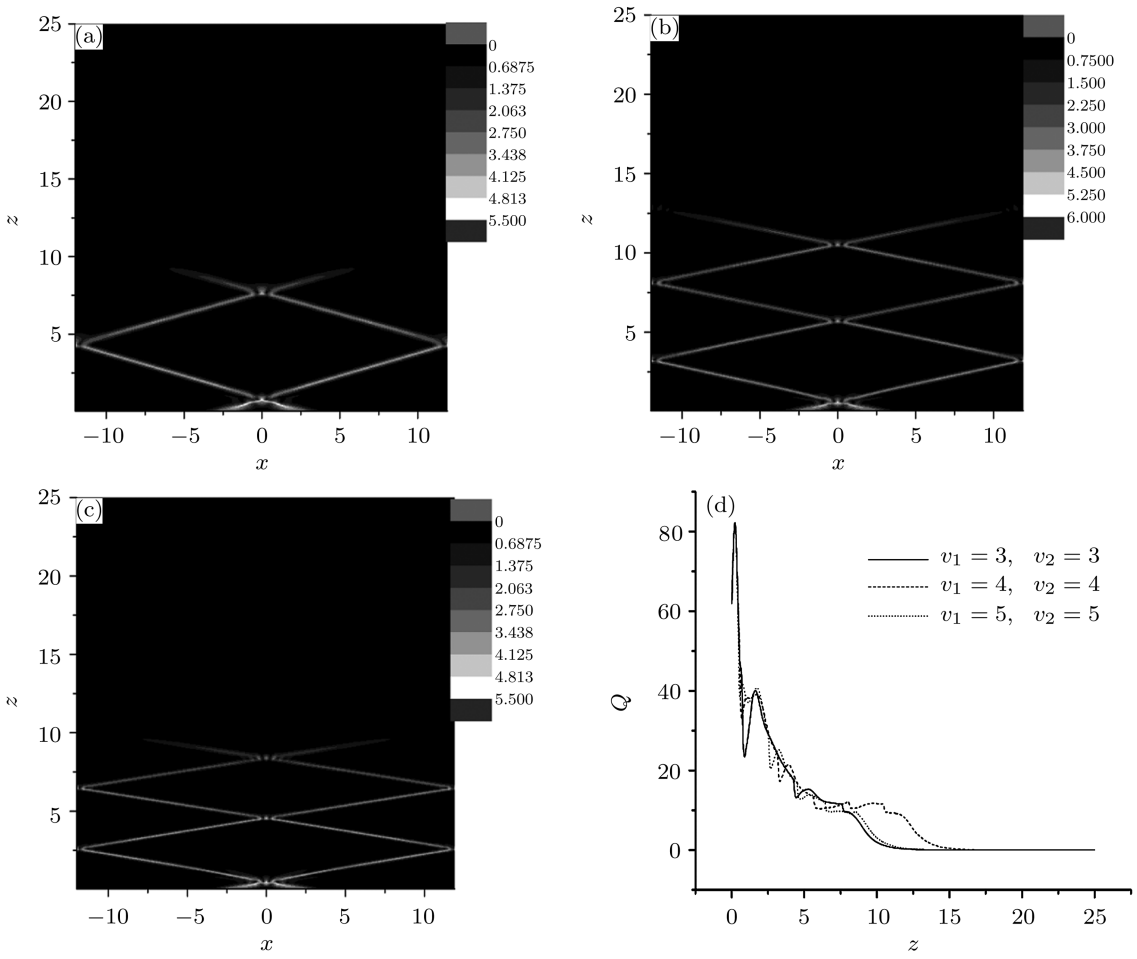


Fig. 2. Contour plot of the interactions between two neighbouring optical bullets for the same separation $R = 5$ and different initial velocities. Here, (a) $v_1 = 3$, $v_2 = 3$; (b) $v_1 = 4$, $v_2 = 4$; (c) $v_1 = 5$, $v_2 = 5$. Figure (d) shows the total energy Q of the optical bullets vs. propagation distance z in three cases.

Thirdly, we study the interactions of dissipative optical bullets with large velocities. The x–z plane plots of the bullets with large velocities and their corresponding evolutions of the total energy are shown in Fig. 3. From Figs. 3(a)–3(c), it can be seen that the bullets collide quickly for the first time because of the large velocities. After the collision, they move away from each other. Owing to the periodic boundary condition, the bullets will collide with each other on the boundary, and then they come into the computation window from the other boundary of space x and start the subsequent collision. The energy of the initial condition is higher than that for the optical bullet at the chosen set of parameters.[9] Therefore the whole energy Q decreases rapidly at first. When the energy is equal to that for the bullet at the chosen set of parameters, the whole energy Q reaches a stable state, in this state the energy Q of the bullets slightly fluctuates in a simple periodic form. The evolutions of energy Q are displayed in Figs. 3(d)–3(f) clearly. In the process of collisions, the frequency of collision depending on the bullet velocity and the energy bullets kept making the bullets adaptable to the dissipative system exactly. The bullets can survive and form stable collisions in the perturbation caused by collision.

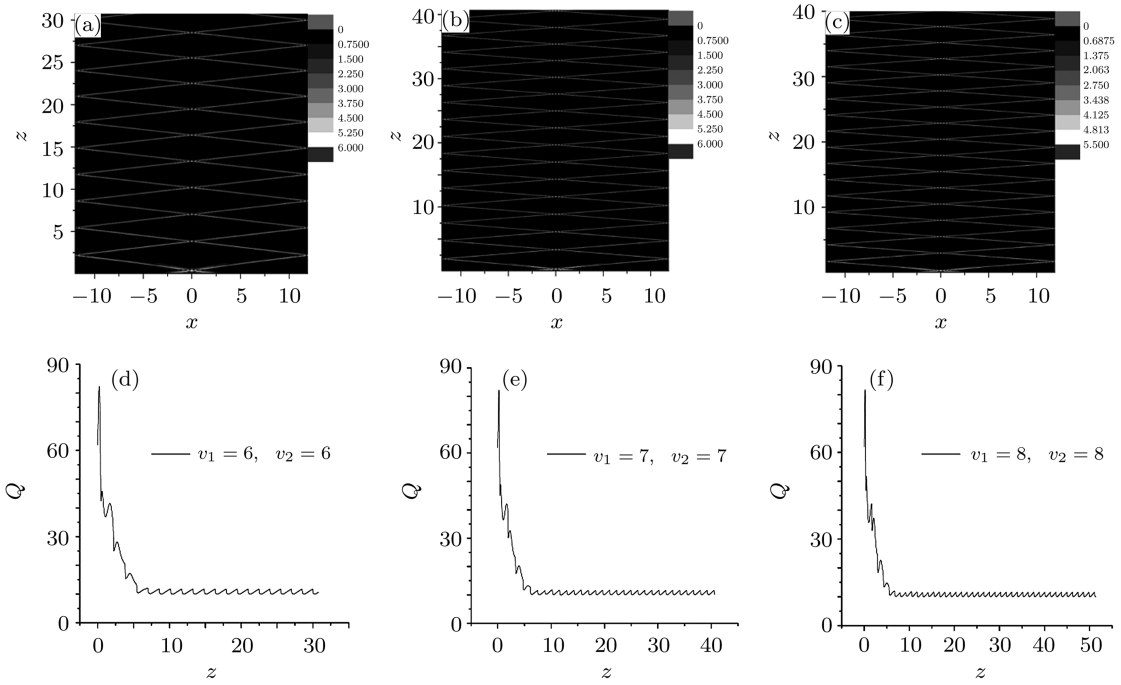


Fig. 3. The contour plot of the interaction between two neighbouring optical bullets for the same separation $R = 5$ and different initial velocities: (a) $v_1 = 6$, $v_2 = 6$; (b) $v_1 = 7$, $v_2 = 7$; (c) $v_1 = 8$, $v_2 = 8$. Figures (d), (e), (f) show the total energies Q of the optical bullets vs. propagation distance z in three cases.

Finally, we present an interesting phenomenon: when we vastly increase the amplitudes of the colliding bullets and keep their velocities small, an extra bullet will form. An example of the collision is given in Fig. 4. In this simulation, the initial amplitude of bullet increases to 6 and the other parameters of Eq. (4) are the same as those in Fig. 1(b). The two initial bullets with high energy evolve with z. Similar to the case of Fig. 1(b), but with high initial energy much more than that of creating a new bullet, there is a difference: the remaining energy can form two bullets propagating in the previous directions of the waveguides in addition to some loss of energy. The new created bullet with no velocity can propagate stably. After the first collision, the initial two bullets collide and merge into a single one on the boundary, which disappears immediately. Finally, the surviving bullet becomes stationary and its energy is up to 5.6897, which is equal to the value in the case of Fig. 1(b). This is because the stationary solution is completely fixed by the parameters of Eq. (1) in the dissipative system.[15] Note that a similar effect of "soliton birth" was observed experimentally in the case of collinear collisions of two one-dimensional dissipative spatial solitons in periodically patterned semiconductor amplifiers.[29]

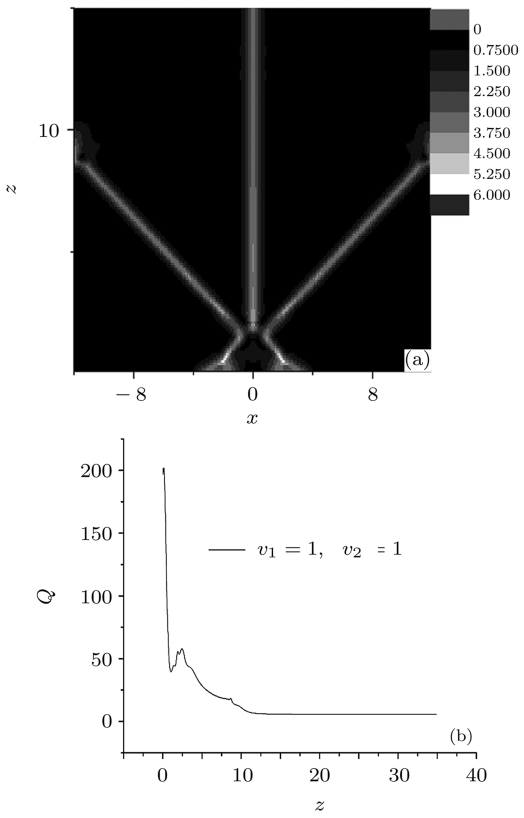


Fig. 4. (a) The contour plot of the interaction between two neighbouring optical bullets for the amplitude being 6, $R = 5$ and the initial velocities $v_1 = 1$, $v_2 = 1$. (b) Total energy Q of the optical bullets vs. propagation distance z in the same case.

4. Conclusion

In conclusion, based on the 3D CGLE with cubic–quintic nonlinearity, we have systematically studied the properties of interactions between dissipative optical bullets with different velocities in the space domain. Collisions of dissipative optical bullets with different velocities exhibit a wealth of phenomena, including merging, creation of an extra bullet, disappearance of two bullets and periodic oscillations. It is worthwhile to note that we observe the process that the colliding bullets keep their shapes and energies approximately stable except for periodic slight oscillations. Obviously, this is very useful in applications for transmission of information, all-optical logic, switching devices, and so on. Numerical simulations for dissipative optical bullets are tedious and time-consuming. It is impossible to perform a complete study of the interaction of dissipative optical bullets as the CGLE has six independent parameters to vary. Here, we have just studied one aspect of dissipative optical bullets, but we believe that our present numerical results are not only helpful for a full grasp of the properties of colliding optical bullets but also instructive for future applications of dissipative optical bullets.

References

[1] Nicolis G and Prigogine I 1977 *Self Organization in Nonequilibrium Systems—From Dissipative Structures to Order through Fluctuations* (New York: Academic) p. 76
[2] Aronson I S and Kramer L 2002 *Rev. Mod. Phys.* **74** 99
[3] Ippen E P 1994 *Appl. Phys.* B **58** 159
[4] Bakonyi Z, Michaelis D, Peschel U, Onishchukov G and Lederer F 2002 *J. Opt. Soc. Am.* B **19** 487
[5] Song L J, Shi X J, Li L and Zhou G S 2005 *Chin. Phys.* **14** 1174
[6] Aranson I S and Kramer L 2002 *Rev. Mod. Phys.* **74** 99
[7] Bowman C and Newell A C 1998 *Rev. Mod. Phys.* **70** 289
[8] Soto-Crespo J M, Akhmediev N N and Ankiewicz A 2000 *Phys. Rev. Lett.* **85** 2937
[9] Soto-Crespo J M, Akhmediev N N and Grelu P 2006 *Phys. Rev.* E **74** 046612
[10] Afanasjev V V, Akhmediev N N and Soto-Crespo J M 1996 *Phys. Rev.* E **53** 1931
[11] Akhmediev N N and Ankiewicz A 2005 *Dissipative Solitons* (New York: Academic) p. 216
[12] Quiroga-Teixeiro M and Michinel H 1997 *J. Opt. Soc. Am.* B **14** 2004
[13] Quiroga-Teixeiro M, Berntson A and Michinel H 1999 *J. Opt. Soc. Am.* B **16** 1697
[14] Song L J, Li L and Zhou G S 2007 *Chin. Phys.* **16** 148
[15] Soto-Crespo J M, Grelu P H and Akhmediev N N 2006 *Opt. Express* **14** 4013
[16] Mihalache D, Mazilu D, Lederer F, Leblond H and Malomed B A 2008 *Phys. Rev.* A **77** 033817
[17] Agrawal G P 1991 *Phys. Rev.* A **44** 7493
[18] Kivshar Y S and Agrawal G P 2003 *Optical Solitons* (London: Academic) p. 173
[19] Peng J Z, Yang H and Tang Y 2009 *Chin. Phys.* B **18** 2364
[20] Zheng H J, Liu S L and Xu J P 2007 *Chin. Phys.* **16** 2023
[21] Akhmediev N N and Ankiewicz A 1997 *Solitons, Nonlinear Pulses and Beams* (London: Academic) p. 227
[22] Edmundson D E and Enns R H 1993 *Opt. Lett.* **18** 1609
[23] Makhankov V G 1990 *Soliton Phenomenology* (London: Academic) p. 115
[24] Jakubowski M H and Steiglitz K 1997 *Phys. Rev.* E **56** 7267
[25] Snyder A W, Mitchell D J, Polodian L and Ladouceur F 1991 *Opt. Lett.* **16** 21
[26] Snyder A W and Sheppard A P 1993 *Opt. Lett.* **18** 482
[27] Pigier C, Uzdin R, Carmon T, Segev M, Nepomnyaschchy A and Musslimani Z H 2001 *Opt. Lett.* **26** 1577
[28] Stegeman G I and Segev M 1999 *Science* **286** 1518
[29] Ultanir E A, Stegeman G I, Lange C H and Lederer F 2004 *Opt. Lett.* **29** 283