Decoherence-immune generation of highly entangled states for two atoms*

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This paper proposes a decoherence-immune scheme for generating highly entangled states for two atoms trapped in a cavity. The scheme is based on two resonant atom-cavity interactions. Conditional upon the detection of no photon, the two atoms may exchange an excitation via the first resonant interaction, which leads to entanglement. Due to the loss of the excitation, the two atoms are in a mixed entangled state. With the help of an auxiliary ground state not coupled to the cavity mode, the state related to the excitation loss is eliminated by the detection of a photon resulting from the second resonant interaction. Thus, the fidelity of entanglement is almost not affected by the decoherence.

Keywords: entangled state, resonant interaction, auxiliary ground state

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Entanglement is not only an essential ingredient for the test of quantum mechanics against local hidden variable theories,[1] but also a key resource for implementation of tasks of quantum information, such as quantum cryptography[2] and teleportation.[3] One of the main obstacles for the generation of entangled states and realization of quantum information processors is decoherence, which arises from the coupling between quantum systems and the environment. In recent years, various physical systems have been suggested as candidates for quantum entanglement engineering and quantum information processing. Of special interest is the cavity quantum electrodynamics (QED) system. The atoms are ideal for storing quantum information, while the photons are ideal for transmitting information. Schemes have been proposed for producing entangled states of atoms in cavity QED.[4−6] Entangled states for two or three particles have been demonstrated with Rydberg atoms passing through a superconducting microwave cavity.[7,8] However, the fidelity of entanglement needs to be significantly improved in order to be useful for a test of Bell's inequalities and quantum information processing.

The main sources of decoherence in cavity QED are cavity loss and atomic spontaneous emission. In the above-mentioned schemes the fidelity of entanglement is sensitive to cavity loss and/or atomic spontaneous emission. Plenio *et al.* have suggested a scheme for the generation of entangled states of two two-level atoms via cavity loss.[9] The success of the scheme depends upon the detection of no photon leaking out of the cavity. Due to the imperfection of the photodetector, there is a probability that a photon has been emitted from the cavity but the photodetector fails to detect it. Thus the generated state is a mixed state and the fidelity of entanglement depends upon the detection efficiency. Hong and Lee presented a scheme to generate a maximally entangled state of two three-level atoms trapped in a cavity by detecting the polarization of the photon leaking out of the cavity.[10] The scheme requires a photon to be initially injected into the cavity, which is a difficult task with presently available experimental techniques. In this paper we propose a decoherence-immune scheme for generating highly entangled states of two atoms in a cavity. Our scheme involves two resonant atom-cavity interactions. Under the condition that no photon is detected, the two atoms are entangled after the first resonant interaction. However, due to the imperfection of the photodetector a photon may have escaped from the cavity but it is not detected, so the two atoms are not in a pure entangled state. With the help of an auxiliary ground state not coupled to the cavity mode, the state associated with the loss of the photon is eliminated by the detection of a photon during the second interaction. Thus, the fidelity of entanglement is almost not affected by the cavity decay, atomic

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spontaneous emission, and imperfection of the photodetector. In contrast with the scheme of Ref. [10], our scheme does not require the source of a single polarized photon. Based on cavity QED techniques, our scheme is realizable.

Assume that two atoms are trapped in a single-mode optical cavity. The atoms have one excited state $|e\rangle$ and two ground states $|f\rangle$ and $|g\rangle$. The transition $|g\rangle \rightarrow |e\rangle$ is resonantly coupled to the cavity mode. The ground state $|f\rangle$ is not affected during the atom-cavity interaction. The interaction is described by the Hamiltonian

$$H = \sum_{j=1}^{2} g(aS_j^+ + a^\dagger S_j^-), \tag{1}$$

where $S_j^+ = |e_j\rangle\langle g_j|$ and $S_j^- = |g_j\rangle\langle e_j|$ are the atomic flipping operators, $a^\dagger$ and a are the creation and annihilation operators for the cavity mode, and g is the atom–cavity coupling strength. We now consider the atomic spontaneous emission and cavity decay. Under the condition that no photon is observed either by the spontaneous emission or by the leakage through the cavity mirrors, the evolution of the system is governed by the conditional Hamiltonian

$$H_{\rm con} = H - \mathrm{i}\frac{\kappa}{2}a^\dagger a - \mathrm{i}\frac{\Gamma}{2}\sum_{j=1}^{2}|e_j\rangle\langle e_j|, \tag{2}$$

where κ is the cavity decay rate and Γ is the atomic spontaneous emission rate. Suppose that the two atoms are initially in the state $|e_1\rangle|g_2\rangle$ and the cavity mode is initially in the vacuum state $|0\rangle$. After an interaction time t_1 the evolution of the system is

$$\begin{aligned}|\psi_1\rangle = {} & \frac{1}{2}\left\{\mathrm{e}^{-(\kappa+\Gamma)t_1/4}\left[\cos(\alpha t_1)+\frac{\kappa-\Gamma}{4\alpha}\sin(\alpha t_1)\right]+\mathrm{e}^{-\Gamma t_1/2}\right\}|e_1\rangle|g_2\rangle|0\rangle \\ & +\frac{1}{2}\left\{\mathrm{e}^{-(\kappa+\Gamma)t_1/4}\left[\cos(\alpha t_1)+\frac{\kappa-\Gamma}{4\alpha}\sin(\alpha t_1)\right]-\mathrm{e}^{-\Gamma t_1/2}\right\}|g_1\rangle|e_2\rangle|0\rangle\} \\ & -\mathrm{i}\frac{g}{\alpha}\mathrm{e}^{-(\kappa+\Gamma)t_1/4}\sin(\alpha t_1)|g_1\rangle|g_2\rangle|1\rangle,\end{aligned} \tag{3}$$

where

$$\alpha = \sqrt{2g^2-(\kappa-\Gamma)^2/16}. \tag{4}$$

Set $t_1 = \pi/2\alpha$. Then we have

$$\begin{aligned}|\psi_1\rangle = {} & \frac{1}{2}\left[\frac{\kappa-\Gamma}{4\alpha}\mathrm{e}^{-\pi(\kappa+\Gamma)/8\alpha}+\mathrm{e}^{-\pi\Gamma/4\alpha}\right]|e_1\rangle|g_2\rangle|0\rangle \\ & +\frac{1}{2}\left[\frac{\kappa-\Gamma}{4\alpha}\mathrm{e}^{-\pi(\kappa+\Gamma)/8\alpha}-\mathrm{e}^{-\pi\Gamma/4\alpha}\right]|g_1\rangle|e_2\rangle|0\rangle-\mathrm{i}\frac{g}{\alpha}\mathrm{e}^{-\pi(\kappa+\Gamma)/8\alpha}|g_1\rangle|g_2\rangle|1\rangle.\end{aligned} \tag{5}$$

Now we sequentially perform the transformations $|g_j\rangle \rightarrow |f_j\rangle$ and $|e_j\rangle \rightarrow |g_j\rangle$. This leads to

$$\begin{aligned}|\psi_2\rangle = {} & \frac{1}{2}\left[\frac{\kappa-\Gamma}{4\alpha}\mathrm{e}^{-\pi(\kappa+\Gamma)/8\alpha}+\mathrm{e}^{-\pi\Gamma/4\alpha}\right]|g_1\rangle|f_2\rangle|0\rangle \\ & +\frac{1}{2}\left[\frac{\kappa-\Gamma}{4\alpha}\mathrm{e}^{-\pi(\kappa+\Gamma)/8\alpha}-\mathrm{e}^{-\pi\Gamma/4\alpha}\right]|f_1\rangle|g_2\rangle|0\rangle-\mathrm{i}\frac{g}{\alpha}\mathrm{e}^{-\pi(\kappa+\Gamma)/8\alpha}|f_1\rangle|f_2\rangle|1\rangle.\end{aligned} \tag{6}$$

After the transformation, the atomic system is decoupled with the cavity mode since $H|\psi_2\rangle = 0$. Furthermore, the atomic spontaneous emission does not occur as the atoms are in ground states. Then we wait for a time τ. Suppose that no photon is detected during this period. Due to the cavity decay the system evolves to

$$\begin{aligned}|\psi_3\rangle = {} & \frac{1}{2}\left[\frac{\kappa-\Gamma}{4\alpha}\mathrm{e}^{-\pi(\kappa+\Gamma)/8\alpha}+\mathrm{e}^{-\pi\Gamma/4\alpha}\right]|g_1\rangle|f_2\rangle|0\rangle \\ & +\frac{1}{2}\left[\frac{\kappa-\Gamma}{4\alpha}\mathrm{e}^{-\pi(\kappa+\Gamma)/8\alpha}-\mathrm{e}^{-\pi\Gamma/4\alpha}\right]|f_1\rangle|g_2\rangle|0\rangle-\mathrm{i}\frac{g}{\alpha}\mathrm{e}^{-\pi(\kappa+\Gamma)/8\alpha}\mathrm{e}^{-\kappa\tau/2}|f_1\rangle|f_2\rangle|1\rangle.\end{aligned} \tag{7}$$

If τ is long enough so that $\mathrm{e}^{-\kappa\tau/2} \ll 1$ the last term of $|\psi_2\rangle$ can be discarded, then we have

$$\begin{aligned}|\psi_3\rangle = \frac{1}{2}\Bigg\{ & \left[\frac{\kappa-\Gamma}{4\alpha}\mathrm{e}^{-\pi(\kappa+\Gamma)/8\alpha}+\mathrm{e}^{-\pi\Gamma/4\alpha}\right]|g_1\rangle|f_2\rangle|0\rangle \\ & +\left[\frac{\kappa-\Gamma}{4\alpha}\mathrm{e}^{-\pi(\kappa+\Gamma)/8\alpha}-\mathrm{e}^{-\pi\Gamma/4\alpha}\right]|f_1\rangle|g_2\rangle|0\rangle\Bigg\}.\end{aligned} \tag{8}$$

The two atoms are now prepared in an entangled state with the cavity mode left in the vacuum state.

We now take into account the nonunit detection efficiency. There is a probability that a photon has escaped from the cavity but the photodetector fails to detect it. This leads to the state $|f_1\rangle\,|f_2\rangle\,|0\rangle$. In order to eliminate this unwanted state we perform the transformations $|g_j\rangle \to |e_j\rangle$. Then the state $|\psi_3\rangle$ is transformed into

$$\begin{aligned}|\psi_4\rangle = \frac{1}{2}\Bigg\{&\left[\frac{\kappa-\Gamma}{4\alpha}\mathrm{e}^{-\pi(\kappa+\Gamma)/8\alpha}+\mathrm{e}^{-\pi\Gamma/4\alpha}\right]|e_1\rangle\,|f_2\rangle\,|0\rangle\\&+\left[\frac{\kappa-\Gamma}{4\alpha}\mathrm{e}^{-\pi(\kappa+\Gamma)/8\alpha}-\mathrm{e}^{-\pi\Gamma/4\alpha}\right]|f_1\rangle\,|e_2\rangle\,|0\rangle\Bigg\}.\end{aligned} \tag{9}$$

After the transformations the atom–cavity coupling restarts. Supposing that no photon is detected during the interaction time t_2, the state $|\psi_4\rangle$ evolves to

$$\begin{aligned}|\psi_5\rangle = &\frac{1}{2}\mathrm{e}^{-(\kappa+\Gamma)t_2/4}\left[\frac{\kappa-\Gamma}{4\alpha}\mathrm{e}^{-\pi(\kappa+\Gamma)/8\alpha}+\mathrm{e}^{-\pi\Gamma/4\alpha}\right]\\&\times\left\{\left[\cos(\beta t_2)+\frac{\kappa-\Gamma}{4\beta}\sin(\beta t_2)\right]|e_1\rangle\,|0\rangle-\mathrm{i}\,\frac{g}{\beta}\sin(\beta t_2)\,|g_1\rangle\,|0\rangle\right\}|f_2\rangle\\&+\frac{1}{2}\mathrm{e}^{-(\kappa+\Gamma)t_2/4}\left[\frac{\kappa-\Gamma}{4\alpha}\mathrm{e}^{-\pi(\kappa+\Gamma)/8\alpha}-\mathrm{e}^{-\pi\Gamma/4\alpha}\right]\\&\times|f_1\rangle\left\{\left[\cos(\beta t_2)+\frac{\kappa-\Gamma}{4\beta}\sin(\beta t_2)\right]|e_2\rangle\,|0\rangle-\mathrm{i}\,\frac{g}{\beta}\sin(\beta t_2)\,|g_2\rangle\,|0\rangle\right\},\end{aligned} \tag{10}$$

where

$$\beta=\sqrt{g^2-(\kappa-\Gamma)^2/16}. \tag{11}$$

The detection of a photon at some moment projects the two atoms to the entangled state

$$|\psi_5\rangle=\mathcal{N}\left\{\left[\frac{\kappa-\Gamma}{4\alpha}\mathrm{e}^{-\pi(\kappa+\Gamma)/8\alpha}+\mathrm{e}^{-\pi\Gamma/4\alpha}\right]|g_1\rangle\,|f_2\rangle+\left[\frac{\kappa-\Gamma}{4\alpha}\mathrm{e}^{-\pi(\kappa+\Gamma)/8\alpha}-\mathrm{e}^{-\pi\Gamma/4\alpha}\right]|f_1\rangle\,|g_2\rangle\right\}, \tag{12}$$

where

$$\mathcal{N}=2\alpha\sqrt{\frac{2}{(\kappa-\Gamma)^2\mathrm{e}^{-\pi(\kappa+\Gamma)/4\alpha}+16\alpha^2\mathrm{e}^{-\pi\Gamma/2\alpha}}}. \tag{13}$$

We here have used the normalized state. Suppose that one wishes to wait a time t_d during the second stage, the probability of success is

$$P=\left[\frac{(\kappa-\Gamma)^2}{32\alpha^2}\mathrm{e}^{-\pi(\kappa+\Gamma)/4\alpha}+\frac{1}{2}\mathrm{e}^{-\pi\Gamma/2\alpha}\right]\left\{1-\mathrm{e}^{-\kappa t_\mathrm{d}}\left[1+\frac{\kappa-\Gamma}{4\beta}\sin(2\beta t_\mathrm{d})\right]\right\}. \tag{14}$$

If t_d is long enough so that $\mathrm{e}^{-\kappa t_\mathrm{d}}\ll 1$ the success probability is

$$P=\frac{(\kappa-\Gamma)^2}{32\alpha^2}\mathrm{e}^{-\pi(\kappa+\Gamma)/4\alpha}+\frac{1}{2}\mathrm{e}^{-\pi\Gamma/2\alpha}. \tag{15}$$

The success probablity increases as the decay rates κ and Γ decrease, approaching its maximum value of 1/2 in the strong coupling regime: $g\gg\kappa,\Gamma$. It should be noted that the second procedure is performed if and only if no photon is detected during the first interaction. If a photon is detected during the first interaction, the two atoms are in the ground state $|g_1\rangle\,|g_2\rangle$ and the cavity mode is in the vacuum state. Then the experiment can be simply repeated by exciting the first atom to the excited state $|e_1\rangle$.

If the system is in the state $|f_1\rangle\,|f_2\rangle\,|0\rangle$ after the first atom–cavity interaction and subsequent transformations no atoms can emit a photon during the second atom–cavity interaction. As the scheme is conditional upon the detection of a photon during the second stage, the state $|f_1\rangle\,|f_2\rangle\,|0\rangle$ is discarded and the fidelity of entanglement is not affected by the imperfection of the photodetector. The corresponding success probability is given by $P'=\eta P$, with η being the detection efficiency. Remarkably, even if the atomic spontaneous emission rate and the cavity decay rate cannot be neglected and the photodetector is imperfect one can still produce a pure entangled state, which is in contrast with previous work.[9]

The degree of entanglement between two qubits can be quantified by concurrence,[11] which is defined as

$$C = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \tag{16}$$

where λ_i's are the square roots in decreasing order of the eigenvalues of the operator $\rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$, with ρ being the density operator and σ_y the Pauli operator. The concurrence $C = 0$ corresponds to an unentangled state, and $C = 1$ corresponds to a maximally entangled state. For the pure two-qubit state $|\psi_5\rangle$ in Eq. (12), the concurrence is reduced to

$$\begin{aligned} C &= |\langle\psi_5| \sigma_y \otimes \sigma_y |\psi_5^*\rangle| \\ &= \frac{16\alpha^2}{(\kappa-\Gamma)^2 \mathrm{e}^{-\pi(\kappa+\Gamma)/4\alpha} + 16\alpha^2 \mathrm{e}^{-\pi\Gamma/2\alpha}} \left| \frac{(\kappa-\Gamma)^2}{16\alpha^2} \mathrm{e}^{-\pi(\kappa+\Gamma)/4\alpha} - \mathrm{e}^{-\pi\Gamma/2\alpha} \right|. \end{aligned} \tag{17}$$

The concurrence increases as the difference between the two decay rates κ and Γ decreases. If $\kappa = \Gamma$ then we have $C = 1$, i.e., the two atoms are in a maximally entangled state.

Due to the imperfect timing of the interaction time t_1, the condition $t_1 = \pi/2\alpha$ cannot be strictly satisfied. Thus, the two atoms are finally in the state

$$\begin{aligned} |\psi_5\rangle = {} & \mathcal{N}\left\{ \mathrm{e}^{-(\kappa+\Gamma)t_1/4} \left[\cos(\alpha t_1) + \frac{\kappa-\Gamma}{4\alpha} \sin(\alpha t_1) \right] + \mathrm{e}^{-\Gamma t_1/2} \right\} |g_1\rangle\,|f_2\rangle \\ & + \mathcal{N}\left\{ \mathrm{e}^{-(\kappa+\Gamma)t_1/4} \left[\cos(\alpha t_1) + \frac{\kappa-\Gamma}{4\alpha} \sin(\alpha t_1) \right] - \mathrm{e}^{-\Gamma t_1/2} \right\} |f_1\rangle\,|g_2\rangle\,, \end{aligned} \tag{18}$$

where

$$\mathcal{N} = \sqrt{\frac{1}{2\mathrm{e}^{-(\kappa+\Gamma)t_1/2} \left[\cos(\alpha t_1) + \frac{\kappa-\Gamma}{4\alpha} \sin(\alpha t_1) \right]^2 + 2\mathrm{e}^{-\Gamma t_1}}}. \tag{19}$$

The corresponding concurrence and success probability are

$$\begin{aligned} C = {} & \frac{1}{\mathrm{e}^{-(\kappa+\Gamma)t_1/2} \left[\cos(\alpha t_1) + \frac{\kappa-\Gamma}{4\alpha} \sin(\alpha t_1) \right]^2 + \mathrm{e}^{-\Gamma t_1}} \\ & \times \left| \mathrm{e}^{-(\kappa+\Gamma)t_1/2} \left[\cos(\alpha t_1) + \frac{\kappa-\Gamma}{4\alpha} \sin(\alpha t_1) \right]^2 - \mathrm{e}^{-\Gamma t_1} \right|, \end{aligned} \tag{20}$$

and

$$P = \frac{1}{2}\eta \left\{ \mathrm{e}^{-(\kappa+\Gamma)t_1/2} \left[\cos(\alpha t_1) + \frac{\kappa-\Gamma}{4\alpha} \sin(\alpha t_1) \right]^2 + \mathrm{e}^{-\Gamma t_1} \right\}, \tag{21}$$

respectively.

We now give a brief discussion on the experimental issues. In recent cavity QED experiments,[12,13] Cs atoms were trapped in an optical cavity. The hyperfine levels $|F = 3, m = -2\rangle$ and $|F = 3, m = -1\rangle$ of $6S_{1/2}$ can act as the ground states $|g\rangle$ and $|f\rangle$, respectively, while the hyperfine level $|F' = 3, m' = -3\rangle$ of $6P_{1/2}$ can act as the state $|e\rangle$. The corresponding coupling strength is $g = 2\pi \times 34$ MHz. The decay rates for the atomic excited state and the cavity mode are $\Gamma = 2\pi \times 2.6$ MHz and $\kappa = 2\pi \times 4.1$ MHz, respectively. Suppose that the deviation from the desired interaction time during the first stage is $\delta t_1 = 0.02\pi/\alpha$. Then the concurrence is $C = 0.99$. In this case the two atoms are almost in a maximally entangled state. The success probability depends upon the detection efficiency: $P = 0.47\eta$. The transformation $|g\rangle \to |f\rangle$ can be achieved by using a pair of off-resonant classical fields with the same Rabi frequency Ω to drive the transitions $|g\rangle \to |h\rangle$ and $|f\rangle \to |h\rangle$, where $|h\rangle$ is an auxiliary excited state. The hyperfine level $|F' = 4, m' = -2\rangle$ of $6P_{1/2}$ can act as $|h\rangle$. The two classical fields are detuned from the respective transitions by the same amount Δ. Under the condition $\Delta \gg \Omega$ the upper level $|h\rangle$ can be adiabati-

cally eliminated and the two classical fields induce the Raman transition $|g\rangle \rightarrow |f\rangle$, with the effective coupling strength being Ω^2/Δ. If the classical fields are strong enough so that $\Omega^2/\Delta \gg g$, the atom–cavity interaction can be neglected during this transformation. The transformation $|g\rangle \rightarrow |e\rangle$ can be obtained by directly using a resonant strong classical field. It should be noted that the scheme is valid in the Lamb–Dicke regime. Recently, a single $^{40}Ca^{+}$ ion was localized at a fixed position in the cavity with high precision for a long time.[14] In a more recent cavity QED experiment,[15] the axial motion of a single Cs atom is localized to the Lamb–Dicke limit.

In conclusion, we have proposed a cavity QED based scheme for generating highly entangled states of two atoms with an auxiliary ground state not coupled to the cavity mode. Our scheme involves two stages. Conditional upon the detection of no photon, the two atoms are entangled via exchange of an excitation after the first stage. The state with the loss of the excitation is eliminated by the detection of a photon arising from the second interaction. Therefore, the entanglement has an immunity to the decoherence and imperfection of the photodetector. The required experimental techniques are within the scope of what can be obtained in the cavity QED experimental setup.

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