Generation of any superposition of coherent states along a straight line via resonant atom–cavity interaction*

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This paper proposes a scheme for generating arbitrary superpositions of several coherent states along a straight line for a cavity mode. In the scheme, several atoms are sent through a cavity initially in a strong coherent state. The superposition of several coherent states with desired coefficients may be generated if each atom is detected in the excited state after it exits the cavity. The scheme is based on resonant atom–cavity interaction and no classical field is required during and after the atom–cavity interaction. Thus, the scheme is very simple and the interaction time is very short, which is important in view of decoherence.

Keywords: superposition of coherent states, cavity mode, resonant interaction

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The generation of various quantum states occupies a central position in quantum optics and quantum information. The cavity quantum electrodynamics (QED) system is a qualified candidate for quantum state engineering. An atomic state is correlated with a field state via coherent energy exchange or conditional phase shift. Through suitable choice of the interaction time and manipulation of the atomic state one can force the field to evolve to the desired state. Vogel *et al.* proposed a scheme for generating any superposition of Fock states of a single-mode electromagnetic field by sending a sequence of two-level atoms through a high-Q superconducting cavity and then detecting their states.[1] Alternative schemes have also been presented by using a single atom.[2,3]

Superpositions of coherent states are another important class of quantum states. Due to the interference between the coherent components which are close to the classical ones, such superposition states may exhibit various nonclassical properties such as squeezing and sub-Poissonian statistics.[4,5] Furthermore, it has been shown that discrete coherent-state superpositions on a circle or along a straight line may approximate many quantum states,[6,7] which provides a new way for quantum state engineering. In the context of cavity QED various schemes have been proposed for the generation of superpositions of several coherent states on a circle or along a straight line.[8−12] For these schemes to be valid, the preparation time should be much shorter than the dissipation time. However, as these schemes are based on dispersive atom–cavity interaction to produce a rotation on the cavity field the atom–cavity coupling strength should be much smaller than the atom–cavity detuning and thus the required interaction time is relatively long.

In this paper, we propose an alternative scheme for generation of any superposition of coherent states along a straight line for a cavity mode. The distinct feature of the present scheme is that it is based on resonant atom–cavity interaction. Thus the required interaction time is greatly shortened, which is important in view of decoherence. Furthermore, unlike the previous schemes, no classical field is required during and after the atom–cavity interaction. The required experimental techniques are within the scope which can be obtained in the microwave cavity QED setup.

We consider that N two-level atoms are sequentially sent through a resonant single-mode cavity. In the rotating-wave approximation, the Hamiltonian describing the interaction between the j-th atom and the cavity mode is (assuming $\hbar = 1$)

$$H_j = g(a^\dagger S_j^- + a S_j^+), \tag{1}$$

where $S_j^+ = |e_j\rangle\langle g_j|$, $S_j^- = |g_j\rangle\langle e_j|$, with $|e_j\rangle$ and $|g_j\rangle$ being the excited and ground states of the j-th atom respectively, $a^\dagger$ and a are the creation and an-

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nihilation operators for the cavity mode, and g is the atom–cavity coupling strength. Assume that the cavity field is initially prepared in the coherent state $|\alpha\rangle$ by a microwave source. After the first atom exits the cavity the system evolves to the state

$$|\psi_1\rangle = U_1(t)D(\alpha)\,|\phi_1\rangle\,|0\rangle\,, \tag{2}$$

where $|\phi_1\rangle$ is the initial state of the first atom, $U(t)$ is the evolution operator

$$U_1(t) = \mathrm{e}^{-\,\mathrm{i}\,H_1 t}, \tag{3}$$

and $D(\alpha)$ is the displacement operator $D(\alpha) = \mathrm{e}^{\alpha a^{\dagger} - a^{*} a}$. We can rewrite Eq. (2) as

$$|\psi_1\rangle = D(\alpha)U_{1,d}(t)\,|\phi_1\rangle\,|0\rangle\,, \tag{4}$$

where

$$U_{1,d}(t) = D^{+}(\alpha)U_1(t)D(\alpha). \tag{5}$$

Substituting Eq. (3) into Eq. (5) we obtain

$$U_{1,d}(t) = \mathrm{e}^{-\,\mathrm{i}\,H_{1,d}t}, \tag{6}$$

where

$$H_{1,d} = \frac{1}{2}g[(a^{\dagger} + \alpha^{*})S_1^{-} + (a + \alpha)S_1^{+}]. \tag{7}$$

Define the new atomic basis[13,14]

$$|+_1\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle + |g_1\rangle), \quad |-_1\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle - |g_1\rangle). \tag{8}$$

Then we can rewrite $H_{1,d}$ as

$$\begin{aligned} H_{1,d} = {} & \frac{1}{2}g[a^{\dagger}(2\sigma_{1,z} + \sigma_1^{+} - \sigma_1^{-}) \\ & + a(2\sigma_{1,z} + \sigma_1^{-} - \sigma_1^{+})] + 2\Omega\sigma_{1,z}, \end{aligned} \tag{9}$$

where $\sigma_{1,z} = \frac{1}{2}(|+_1\rangle\langle+_1| - |-_1\rangle\langle-_1|)$, $\sigma_1^{+} = |+_1\rangle\langle-_1|$, $\sigma_1^{-} = |-_1\rangle\langle+_1|$, and $\Omega = \alpha g$. We can rewrite the displaced evolution operator $U_{1,d}(t)$ as

$$U_{1,d}(t) = \mathrm{e}^{-\,\mathrm{i}\,2\Omega\sigma_{1,z}t}\mathcal{T}\mathrm{e}^{-\,\mathrm{i}\,H_{1,i}t}, \tag{10}$$

where

$$\begin{aligned} H_{1,i} = {} & \frac{1}{2}g[a^{\dagger}(2\sigma_{1,z} + \mathrm{e}^{\mathrm{i}\,2\Omega t}\sigma_1^{+} - \mathrm{e}^{-\,\mathrm{i}\,2\Omega t}\sigma_1^{-}) \\ & + a(2\sigma_{1,z} + \mathrm{e}^{-\,\mathrm{i}\,2\Omega t}\sigma_1^{-} - \mathrm{e}^{\mathrm{i}\,2\Omega t}\sigma_1^{+})], \end{aligned} \tag{11}$$

and $\mathcal{T}$ denotes the time-ordering operator. Assuming that $\Omega \gg g$, i.e., $\alpha \gg 1$, we can neglect the terms oscillating fast. Then $H_{1,i}$ reduces to

$$\begin{aligned} H_{1,i} &= g(a^{\dagger} + a)\sigma_{1,z} \\ &= \frac{g}{2}(a^{\dagger} + a)(S_1^{-} + S_1^{+}). \end{aligned} \tag{12}$$

Assume that the atom is initially in the superposition state:

$$|\phi_1\rangle = \frac{1}{\sqrt{1 + |\varepsilon_1|^2}}(|-_1\rangle + \varepsilon_1\,|+_1\rangle),$$

where ε_1 is a complex number. Using Eqs. (4), (10) and (12), we obtain evolution of the system after an interaction time τ

$$\begin{aligned} |\psi_1\rangle = {} & \frac{1}{\sqrt{1 + |\varepsilon_1|^2}}D(\alpha)(\mathrm{e}^{i\theta}\,|\beta\rangle\,|-_1\rangle \\ & + \varepsilon_1\mathrm{e}^{-\,\mathrm{i}\,\theta}\,|-\beta\rangle\,|+_1\rangle), \end{aligned} \tag{13}$$

where $|\beta\rangle$ is the coherent state $|\beta\rangle = D(\beta)\,|0\rangle$, and the parameters θ and β are given by $\theta = \Omega\tau$ and $\beta = \mathrm{i}\,g\tau/2$. The state evolution is valid under the condition $|\beta| \ll |\alpha|$. If we now find the atom in the state $|e_1\rangle$ the field will collapse into

$$|\varphi_1\rangle = \mathcal{N}_1 D(\alpha)(\mathrm{e}^{\mathrm{i}\,\theta}\,|\beta\rangle + \varepsilon_1\mathrm{e}^{-\,\mathrm{i}\,\theta}\,|-\beta\rangle), \tag{14}$$

where

$$\mathcal{N}_1 = [1 + |\varepsilon_1|^2 + 2\mathrm{Re}(\varepsilon_1)\mathrm{e}^{-2|\beta|^2}]^{-1/2}. \tag{15}$$

For a given interaction time τ we could appropriately choose the amplitude α of the initial coherent state so that $\theta = 2n\pi$, with n being an integer. In this case we obtain

$$|\varphi_1\rangle = \mathcal{N}_1 D(\alpha)(|\beta\rangle + \varepsilon_1\,|-\beta\rangle). \tag{16}$$

We then send a second atom through the cavity. Assume that the second atom is initially prepared in the superposition state:

$$\frac{1}{\sqrt{1 + |\varepsilon_2|^2}}(|-_2\rangle + \varepsilon_2\,|+_2\rangle).$$

The interaction time of the second atom with the cavity field is equal to that of the first atom with the cavity field. When this atom exits the cavity, the system is in the entangled state

$$\begin{aligned} |\psi_2\rangle = {} & \frac{\mathcal{N}_1}{\sqrt{(1 + |\varepsilon_2|^2)}}D(\alpha)[|-_2\rangle\,(|2\beta\rangle + \varepsilon_1\,|0\rangle) \\ & + \varepsilon_2\,|+_2\rangle\,(|0\rangle + \varepsilon_1\,|-2\beta\rangle)]. \end{aligned} \tag{17}$$

If this atom is also detected in the state $|e_2\rangle$ we obtain

$$|\varphi_2\rangle = \mathcal{N}_2 D(\alpha)(|2\beta\rangle + (\varepsilon_1 + \varepsilon_2)\,|0\rangle + \varepsilon_1\varepsilon_2\,|-2\beta\rangle), \tag{18}$$

where

$$\begin{aligned} \mathcal{N}_2 = {} & [1 + |\varepsilon_1 + \varepsilon_2|^2 + |\varepsilon_1\varepsilon_2|^2 \\ & + 2\,\mathrm{Re}[(\varepsilon_1 + \varepsilon_2)(1 + \varepsilon_1\varepsilon_2)]\mathrm{e}^{-2|\alpha|^2} \\ & + 2\,\mathrm{Re}(\varepsilon_1\varepsilon_2)\mathrm{e}^{-8|\alpha|^2}]^{-1/2}. \end{aligned} \tag{19}$$

Now the superposition state involves three components along a straight line with an equal distance of 2β. The weighting factors are controllable by the parameters ε_1 and ε_2.

We repeat the procedure N times and assume that we detect each atom in the same state $|e\rangle$. By this way we can convert an initial coherent state into a superposition of $N+1$ coherent states along a straight line with the equal distance of 2β. Before the N-th atom is injected into the cavity, the field is assumed to be in the following state

$$|\varphi_{N-1}\rangle = \mathcal{N}_{N-1} D(\alpha) \sum_{k=0}^{N-1} C_k^{N-1} |(N-1-2k)\beta\rangle . \quad (20)$$

After the N-th atom initially prepared in the state: $\frac{1}{\sqrt{1+|\varepsilon_N|^2}}(|g_N\rangle + \varepsilon_N |e_N\rangle)$ has interacted with the cavity field and then is detected in its state $|e_N\rangle$, the pure field state reads as

$$|\varphi_N\rangle = \mathcal{N}_N \sum_{k=0}^{N} C_k^N |(N-2k)\alpha\rangle , \quad (21)$$

where

$$\begin{aligned} &C_0^N = C_0^{N-1}, \\ &\cdots \\ &C_k^N = C_k^{N-1} + \varepsilon_N C_{k-1}^{N-1}, \quad k \neq 0, N, \\ &\cdots \\ &C_N^N = \varepsilon_N C_{N-1}^{N-1}, \end{aligned} \quad (22)$$

$$\mathcal{N}_N = \sum_{j=0}^{N} \sum_{k=0}^{N} C_j^N (C_k^N)^* \mathrm{e}^{-2(j-k)^2|\beta|^2}. \quad (23)$$

In order to generate the state $|\varphi_N\rangle$ with the desired coefficients C_k^N we express the unknown coefficients C_k^{N-1} in terms of C_k^N by

$$C_k^{N-1} = \sum_{j=0}^{k} C_j^N (-\varepsilon_N)^{k-j}. \quad (24)$$

Substituting the obtained coefficient C_{N-1}^{N-1} into the last equation of Eq. (22) we obtain the characteristic equation for ε_N

$$C_N^N + \sum_{j=0}^{N-1} C_j^N (-\varepsilon_N)^{N-j} = 0.$$

Solve this equation and choose one value ε_N from the N roots. Substituting the obtained ε_N into Eq. (24) we obtain the coefficients C_k^{N-1} of the state $|\varphi_{N-1}\rangle$. For the state $|\psi_{N-1}\rangle$ we perform the same calculations and obtain the parameter ε_{N-1} and coefficients C_k^{N-2} for the state $|\varphi_{N-2}\rangle$. Repeating the procedure until we reach the coherent state $|\alpha\rangle$, we obtain a series of complex parameters $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N$ which determine the initial states of N atoms injecting into the cavity in order to generate the desired state $|\psi_N\rangle$. The probability of success is

$$P = \frac{1}{2(N_1)^2(1+|\varepsilon_1|^2)} \prod_{k=2}^{N} \frac{(N_{k-1})^2}{2(N_k)^2(1+|\varepsilon_k|^2)}. \quad (25)$$

The success probability P decreases as N increases. Therefore, the method is restricted to the small value of N. However, it has been shown that many quantum states can be well approximated by superpositions of even a small number of coherent states.[5] As an example, we consider the following superposition state

$$|\varphi\rangle = \mathcal{N} D(\alpha)(|2\beta\rangle + |-2\beta\rangle + p\,|0\rangle), \quad (26)$$

where $\mathcal{N} = [2 + 2\exp(-8\,|\beta|^2) + 4p\exp(-2\,|\beta|^2) + p^2]^{-1/2}$. For such a state the maximum degree of squeezing is about 74% at $|\beta| = 0.785$ and $p = 1.35$.[15] In order to produce this target state we use two atoms with the parameters $\varepsilon_1 = \mathrm{e}^{\mathrm{i}\,\phi}$ and $\varepsilon_2 = \mathrm{e}^{-\mathrm{i}\,\phi}$ with $\phi = \arccos(p/2)$. The success probablity is about 0.34.

We now give a brief discussion on the experimental feasiblity of the proposed scheme. For the Rydberg atoms with principal quantum numbers 50 and 51, the radiative time is $T_r = 3\times10^{-2}$ s, and the coupling constant is $g = 2\pi \times 25$ kHz.[16] For the generation of the state of Eq. (26) the interaction time of each atom with the cavity field is $t = 1.57/g = 1 \times 10^{-5}$ s. The total time to complete the procedure is on the order of $T = 10^{-4}$ s. The decoherence time of the superposition of different coherent components is $T_c' = 2T_c/\bar{d}^{\,2}$, where $\bar{d}$ is the average distance between the components in phase space. Very recently, an ultrahigh fineness Fabry–Perot resonator with a damping time $T_c = 0.13$ s has been built.[17] Then, the decoherence time for the cavity field is about $T_c' \sim 4.84 \times 10^{-2}$ s. The infidelity induced by the decoherence is on the order of $T/T_r + T/T_c' \simeq 1.27 \times 10^{-3}$.

In summary, we have proposed a scheme for generating arbitrary superpositions of several coherent states along a straight line for a cavity field. In the scheme a sequence of suitably prepared atoms is sent through the cavity. The detection of all the atoms in the excited state collapses the field to the desired state. The scheme uses resonant atom–cavity interaction and thus the required interaction time is short, which is important in view of decoherence. Futhermore, no classical field is required during and after the atom–cavity interaction.

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