A study of optical properties of a four-level atomic system via vacuum-induced coherence effects*

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This paper studies the effects of vacuum-induced coherence (VIC) in a four-level atomic system. The effects of VIC lead to the coherent hole burnings exhibited in the system at some certain points of the Rabi frequency. This is also the reason for the enhancement of the coherent population trapping. In addition, optical bistability occurs in the evolution curves of absorption versus the phase of Rabi frequencies.

Keywords: vacuum-induced coherence, quantum interference, coherent hole burning, optical bistability

PACC: 4250, 4265

1. Introduction

A multilevel atom having closely lying excited energy states can show strong quantum interference among the spontaneous emission channels from those upper levels coupled by the same continuum of the vacuum modes to a common ground level. The resulting coherence in the system is known as vacuum-induced coherence (VIC). It was first shown by Agarwal.[1] Occurrence of this coherence requires a stringent condition, i.e., the transition dipole matrix elements should be nonorthogonal.[2]

The effects of VIC in different types of atomic systems have been discussed extensively recently. Many interesting phenomena occur or are enhanced due to the effects of VIC, such as the narrowing of spontaneous emission,[3] quantum beat,[4] the cancellation of spontaneous emission,[5] the quenching of spontaneous emission,[6] the modification of absorption and dispersion properties of atomic systems,[7] photon-photon correlations,[8] velocity-selective coherent population trapping of moving atoms,[9] etc. Moreover, the VIC effects are observable in some realistic systems. Kiffner *et al.*[10] showed that an atomic system with degenerate transitions $j = 1/2 \leftrightarrow j = 1/2$ would be a suitable system where the effects of VIC are prominent. These systems consist of either a single $^{198}Hg^{+}$ ion in a trap[11] or a single $^{138}Ba^{+}$ ion in a trap.[12] In both cases the ground level is 6s $^2S_{1/2}$ and the excited level is 6p $^2P_{1/2}$.

Reference [13] successfully established that quantum interference affects the resonance fluorescence, the squeezing, and the population dynamics of a four-level atom and found that extremely narrow emission lines are obtained because of the decay-induced interferences. In the present paper, we consider the frequency separation and the phase of Rabi frequency with the same model. It is shown that coherent hole burning (CHB)[14] and coherent population trapping (CPT) are remarkably enhanced because of the effects of VIC. In addition, optical bistability occurs in the evolution curves of absorption versus the phase of Rabi frequencies.

2. Model and equations

The model of the four-level atomic system is shown in Fig. 1.[13] It has three closely lying excited

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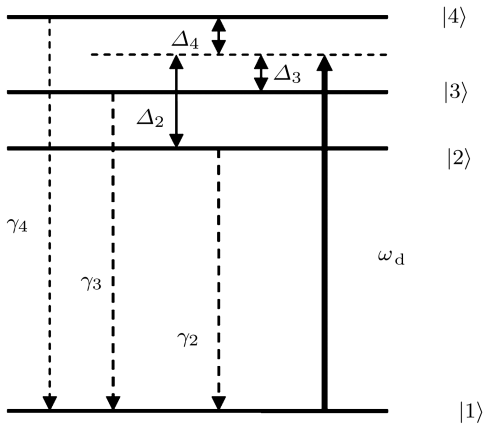


Fig. 1. The model of a four-level atomic system. The parameter $\omega_{\rm d}$ is the frequency of the driving field. Energy level $|1\rangle$ is the ground level. Energy levels $|2\rangle$, $|3\rangle$ and $|4\rangle$ are closely lying excited levels. The parameters γ_m $(m = 2, 3, 4)$ are the spontaneous emission coefficients of $|m\rangle \leftrightarrow |1\rangle$. The parameter Δ_m is the detuning of the driving field with the state $|m\rangle$.

levels $|2\rangle$, $|3\rangle$ and $|4\rangle$, which are coupled by a single-mode light field of frequency $\omega_{\rm d}$ to the ground level $|1\rangle$. γ_m $(m = 2, 3, 4)$ are the spontaneous emission coefficients from the excited levels $|m\rangle$ to the ground level $|1\rangle$. ω_{21}, ω_{31}, ω_{41} are the transition frequencies of $|m\rangle \leftrightarrow |1\rangle$.

The Hamiltonian of this system in the rotating wave approximation (RWA) is given by

$$\begin{aligned} H &= \hbar \sum_{m=1}^{4} \omega_m \sigma_{mm} + \hbar \sum_{k\lambda} \omega_{k\lambda} b_{k\lambda}^{\dagger} b_{k\lambda} \\ &\quad - \left[\hbar \sum_{m=2}^{4} \sum_{k\lambda} g_{mk} \sigma_{m1} b_{k\lambda} + \text{H.c.} \right] \\ &\quad - \left[\hbar \sum_{m=2}^{4} \Omega_m \, \mathrm{e}^{-\mathrm{i}\omega_{\rm d} t} \sigma_{m1} + \text{H.c.} \right], \end{aligned} \tag{1}$$

where $\sigma_{mn} = |m\rangle\langle n|$ are the usual Pauli matrices, $\hbar\omega_m$ are the energies of the atomic levels, and $b_{k\lambda}$ $(b_{k\lambda}^{\dagger})$ is the annihilation (creation) operator of the k-th mode of the vacuum field. The parameter g_{mk} is the coupling constant of the atomic transition $|m\rangle \to |1\rangle$ with the electromagnetic vacuum mode

$$g_{mk} = \sqrt{\frac{\omega_{k\lambda}}{2\hbar\varepsilon_0 V}} (\boldsymbol{\mu}_{1m} \cdot \boldsymbol{e}_{k\lambda}), \tag{2}$$

where $\boldsymbol{\mu}_{1m}$ is the dipolar moment of the transition and is defined by $\boldsymbol{\mu}_{1m} = \langle 1| - er|m\rangle$; $\Omega_m = \boldsymbol{\mu}_{1m} \cdot \boldsymbol{E}_0/(2\hbar)$ is the Rabi frequency of the transition $|m\rangle \to |1\rangle$. We take the complex Rabi frequency $\Omega_m = |\Omega_m| \exp(\mathrm{i}\phi_m)$, where ϕ_m is the phase of the Rabi frequency.

Considering the complex Rabi frequencies in the RWA, we have the following equations:

$$\begin{aligned}
\dot{\rho}_{22} &= -\gamma_2\rho_{22} - \eta_{42}(\rho_{42} + \rho_{24}) - \eta_{23}(\rho_{32} + \rho_{23}) - \mathrm{i}\Omega_2\rho_{12} + \mathrm{i}\Omega_2^*\rho_{21}, \\
\dot{\rho}_{33} &= -\gamma_3\rho_{33} - \eta_{32}(\rho_{32} + \rho_{23}) - \eta_{43}(\rho_{43} + \rho_{34}) - \mathrm{i}\Omega_3\rho_{13} + \mathrm{i}\Omega_3^*\rho_{31}, \\
\dot{\rho}_{44} &= -\gamma_4\rho_{44} - \eta_{43}(\rho_{34} + \rho_{43}) - \eta_{42}(\rho_{24} + \rho_{42}) - \mathrm{i}\Omega_4\rho_{14} + \mathrm{i}\Omega_4^*\rho_{41}, \\
\dot{\rho}_{12} &= -\left(\frac{\gamma_2}{2} - \mathrm{i}\Delta_2\right)\rho_{12} - p_{24}\gamma_{24}\rho_{14} - p_{23}\gamma_{23}\rho_{13} - \mathrm{i}\Omega_2^*(\rho_{22} - \rho_{11}) - \mathrm{i}\Omega_4^*\rho_{42} - \mathrm{i}\Omega_3^*\rho_{32}, \\
\dot{\rho}_{13} &= -\left(\frac{\gamma_3}{2} - \mathrm{i}\Delta_3\right)\rho_{13} - \eta_{23}\rho_{12} - \eta_{34}\rho_{14} - \mathrm{i}\Omega_3^*(\rho_{33} - \rho_{11}) - \mathrm{i}\Omega_2^*\rho_{23} - \mathrm{i}\Omega_4^*\rho_{43}, \\
\dot{\rho}_{14} &= -\left(\frac{\gamma_4}{2} - \mathrm{i}\Delta_4\right)\rho_{14} - \eta_{34}\rho_{13} - \eta_{24}\rho_{12} - \mathrm{i}\Omega_4^*(\rho_{44} - \rho_{11}) - \mathrm{i}\Omega_3^*\rho_{34} - \mathrm{i}\Omega_2^*\rho_{24}, \\
\dot{\rho}_{23} &= -\left(\frac{\gamma_2 + \gamma_3}{2} - \mathrm{i}\omega_{32}\right)\rho_{23} - \eta_{24}\rho_{43} - \eta_{34}\rho_{24} - \eta_{23}(\rho_{22} + \rho_{33}) - \mathrm{i}\Omega_2\rho_{13} + \mathrm{i}\Omega_3^*\rho_{21}, \\
\dot{\rho}_{24} &= -\left(\frac{\gamma_2 + \gamma_4}{2} - \mathrm{i}\omega_{42}\right)\rho_{24} - \eta_{23}\rho_{34} - \eta_{34}\rho_{23} - \eta_{24}(\rho_{22} + \rho_{44}) - \mathrm{i}\Omega_2\rho_{14} + \mathrm{i}\Omega_4^*\rho_{21}, \\
\dot{\rho}_{34} &= -\left(\frac{\gamma_3 + \gamma_4}{2} - \mathrm{i}\omega_{43}\right)\rho_{34} - \eta_{23}\rho_{24} - \eta_{24}\rho_{32} - \eta_{34}(\rho_{33} + \rho_{44}) - \mathrm{i}\Omega_3\rho_{14} + \mathrm{i}\Omega_4^*\rho_{31},
\end{aligned} \tag{3}$$

where $\Delta_m = \omega_{m1} - \omega_{\rm d}$ $(m = 2, 3, 4)$ are the detunings of the field, $\omega_{mn} = \omega_{m1} - \omega_{n1}$ is the frequency separation between the $|m\rangle$ and $|n\rangle$ $(m \neq n \neq 1)$. The equation satisfies the normalization condition $\rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1$. The parameter $\eta_{mn} = p_{mn}\sqrt{\gamma_m\gamma_n}/2$ represents the VIC effects, where p is defined as $p_{mn} = p_{nm} = \boldsymbol{\mu}_{1m} \cdot \boldsymbol{\mu}_{1n}/(|\boldsymbol{\mu}_{1m}||\boldsymbol{\mu}_{1n}|) = \cos\theta_{mn}$. The intensity of quantum interference of the spontaneous emission channels is decided by θ_{mn} which is the angle between dipole matrix elements

$\boldsymbol{\mu}_{1m}$ and $\boldsymbol{\mu}_{1n}$. If $p_{mn} = 0$ ($\theta_{mn} = \pi/2$), the dipole matrix elements $\boldsymbol{\mu}_{1m}$ and $\boldsymbol{\mu}_{1n}$ are orthogonal to each other, the interference phenomenon does not exist, so the VIC effects vanish. If $p_{mn} = 1$ ($\theta_{mn} = 0$), the dipole matrix elements $\boldsymbol{\mu}_{1m}$ and $\boldsymbol{\mu}_{1n}$ are parallel to each other, and the VIC effects are at maximum.

3. Results and analysis

We solve the density matrix equation (3) for steady-state values to study the characteristics of the light field in the system. Here the excited levels are degenerated and coupled via a single driving field to a common lower state. It is safe to assume $\gamma_2 = \gamma_3 = \gamma_4$ and $\Omega_2 = \Omega_3 = \Omega_4$.[15] In this system, the transition of $|4\rangle \leftrightarrow |1\rangle$ affects the main characteristics of the system, the imaginary parts of ρ_{14} can stand for the absorption (gain) of the system. In Fig. 2, we choose $p_{23} = p_{24} = p_{34} = p_{mn}$ and take the real-valued Rabi frequency Ω as a variable parameter. Figure 2(a) shows that an absorption peak occurs when there are no VIC effects ($p_{mn} = 0$). Correspondingly, figures 2(b), 2(c), 2(d) show that when $p_{mn} \neq 0$, peculiar CHB[14] will be exhibited in the system. The CHBs even transform to a gain peak from an original absorption peak at certain points of Ω, which equals 0.168, 0.431 and 0.590 respectively. In this case the curve exhibits a peak with an extremely steep slope which looks like a needlepoint. It is also shown that the negative depth of the peak increases almost 14 times (from -0.481 to -6.79) as p_{mn} increases from 0.1 to 0.9. This means that the gain of the field has a great increase when the VIC effects become more and more obvious. In addition, the peak of the gain is rightward when p_{mn} is increasing. The CHBs result from the combination of strong saturating excitation and dynamically induced quantum coherence;[16] thus our results show that in this system CHBs occur, the slopes of which become extremely steep due to the effects of VIC.

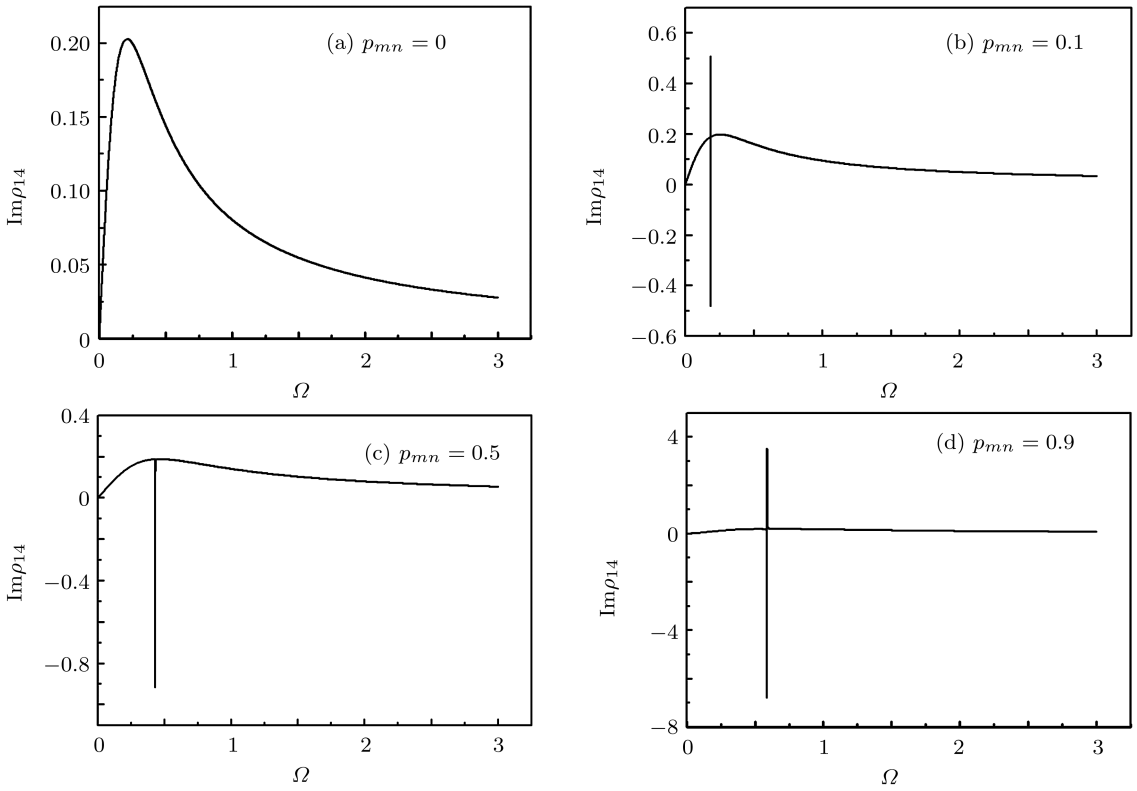


Fig. 2. The imaginary parts of ρ_{14} versus the Rabi frequency ($\Omega_2 = \Omega_3 = \Omega_4 = \Omega$, $\phi_m = 0$) with different p_{mn}. The following quantities take the same unit as the spontaneous emission γ_m, and their values are: $\gamma_2 = \gamma_3 = \gamma_4 = 1$, $\Delta_2 = -0.1$, $\Delta_4 = 0.1$, $\Delta_3 = 0$.

Because of the strong VIC effects, the curves of the $\sum_{i=2}^{4} \rho_{ii}$ can have a special shape along with the change of frequency separation as shown in Fig. 3. Here we take the phase of Rabi frequencies $\phi_m = 0$ also, and the

values of θ_{mn} ($\theta_{23} = \theta_{24} = \theta_{34}$) are 0 and $\pi/2$, respectively. The maximum VIC effect appears at $\theta_{mn} = 0$ ($p_{mn} = 1$). At this point, it can be seen that as $\omega_{43} \to 0$, almost all the population is captured in the upper levels, namely $\sum_{i=2}^{4} \rho_{ii} \approx 1$, so the population of the ground energy level is almost zero. It is shown that the CPT occurs intensively at this point. Moreover, the $\sum_{i=2}^{4} \rho_{ii}$ always decreases with the increase of ω_{43} and finally drops quickly to the value 0.45. Comparatively, when $\theta_{mn} = \pi/2$ ($p_{mn} = 0$), the effects of VIC disappear, and the population $\sum_{i=2}^{4} \rho_{ii}$ drops down obviously. The peak of the curve measures the value to be just 0.64 at $\omega_{43} = 1.81$. So, clearly the CPT is remarkably enhanced due to the effects of VIC in the system.

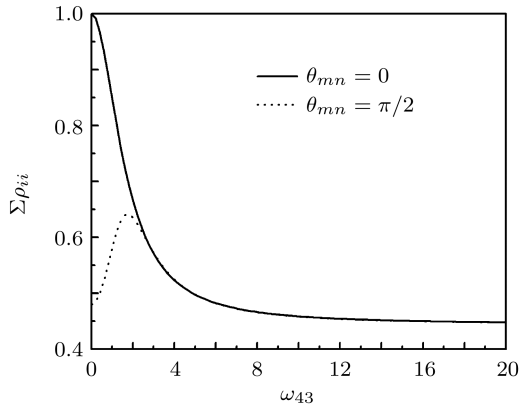


Fig. 3. The evolution of population of excited energy levels $\sum_{i=2}^{4} \rho_{ii}$ versus the frequency separation ω_{43}. The θ_{mn} is chosen as 0, $\pi/2$. Other parameters are chosen as $\gamma_2 = \gamma_3 = \gamma_4 = 1$, $\Omega_m = |\Omega_m| \exp\ (\mathrm{i}\phi_m)$ $(m = 2, 3, 4)$, $\phi_m = 0$, $|\Omega_2| = |\Omega_3| = |\Omega_4| = 1$, $\Delta_2 = -\Delta_4$, $\Delta_3 = 0$, and $\omega_{43} = \omega_{32}$.

The dynamical behaviours of the atom and the quality of the steady-state are related to the phase of the transition in a closed system.[17] The phases of Rabi frequency always play an important role in the phenomena of quantum interference.[18–24] For convenience, the Rabi frequencies are all chosen to be real-valued in Ref. [13]. In the present work, we consider the phase of Rabi frequency and take the complex value with $\Omega_m = |\Omega_m| \exp(\mathrm{i}\phi_m)$ $(m = 2, 3, 4)$. Figure 4 shows that the imaginary part of ρ_{14} versus the phase of Rabi frequency ϕ_m $(m = 2, 3, 4)$ with different ω_{43}. It shows that the absorption curves exhibit sine (cosine) type and the phase of the curves lags with the increase of ω_{43}. If ϕ_m changes an angle of π, the curve overturns, which indicates that the characteristics of the system change with each π change of ϕ_m. For instance, when $\omega_{43} = 0.1$, the curve exhibits a gain peak in the range of $\phi_m = 1.52$–4.75, but it turns to an absorption peak in the range of $\phi_m = 4.75$–7.88. In this case, it shows the property of the optical bistability.

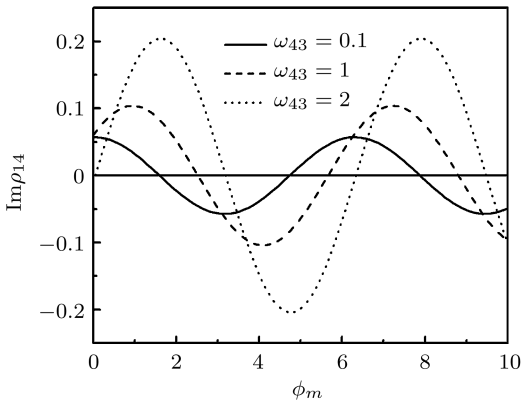


Fig. 4. The imaginary parts of ρ_{14} versus the phase of Rabi frequency ϕ_m $(m = 2, 3, 4)$. Other parameters are $\gamma_2 = \gamma_3 = \gamma_4 = 1$, $\Delta_2 = -\Delta_4$, $\Delta_3 = 0$, $\omega_{43} = \omega_{32}$, $\Omega_m = |\Omega_m| \exp(\mathrm{i}\phi_m)$ $(m = 2, 3, 4)$, $|\Omega_m| = 1$, and $\theta_{34} = 0$, $\theta_{24} = \theta_{23} = \pi$.

4. Conclusions

With the effects of VIC in a four-level atomic system, many interesting phenomena can be observed and enhanced. The CHB shows when the effects of VIC occur ($p_{mn} \neq 0$) in the system at some points of Ω ($\Omega = 0.168$, 0.431 and 0.590). The slope of the peak of the CHB is extremely steep. When the maximum VIC effects appear ($p_{mn} = 1$), almost all the populations are captured in the upper excited levels i.e., $\sum_{i=2}^{4} \rho_{ii} \approx 1$ as $\omega_{43} \to 0$, the CPT occurs intensively in the system. In addition, the optical bistability occurs when the phase of Rabi frequencies is changed continuously.

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