Adaptive local routing strategy on a scale-free network*

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Due to the heterogeneity of the structure on a scale-free network, making the betweennesses of all nodes become homogeneous by reassigning the weights of nodes or edges is very difficult. In order to take advantage of the important effect of high degree nodes on the shortest path communication and preferentially deliver packets by them to increase the probability to destination, an adaptive local routing strategy on a scale-free network is proposed, in which the node adjusts the forwarding probability with the dynamical traffic load (packet queue length) and the degree distribution of neighbouring nodes. The critical queue length of a node is set to be proportional to its degree, and the node with high degree has a larger critical queue length to store and forward more packets. When the queue length of a high degree node is shorter than its critical queue length, it has a higher probability to forward packets. After higher degree nodes are saturated (whose queue lengths are longer than their critical queue lengths), more packets will be delivered by the lower degree nodes around them. The adaptive local routing strategy increases the probability of a packet finding its destination quickly, and improves the transmission capacity on the scale-free network by reducing routing hops. The simulation results show that the transmission capacity of the adaptive local routing strategy is larger than that of three previous local routing strategies.

Keywords: local routing, scale-free networks, preferential probability, traffic load

PACC: 0590, 0540, 0210

1. Introduction

In the past few years, small-world[1] and scale-free properties were discovered in a variety of different social, natural and technological complex networks and systems, such as the Internet,[2] metabolic network, transportation network,[3] etc. This has stimulated a great deal of interest in studying the underlying organising principles, evolution mechanism and dynamics of various complex networks.

Recent works have concentrated on the interplay between traffic dynamics and structural properties of complex networks, in order to reduce traffic congestion of information flow in large communication networks such as the Internet and WWW. A lot of routing strategies are proposed to improve the efficiency of information transmission,[4−6] including global and local routing strategies. Most of the global routing algorithms aim at reducing link or node load by a judicious link weight assignment. The network transmission capacity is improved by them, which is defined as the rate of packet generation at which the network becomes jammed from free flow state. Network transmission capacity can be improved by a systematic adjustment of link weights to minimize the maximal betweenness of nodes on the network.

In the global routing strategies, it is necessary that each node knows the topology of the whole network.[7−9] But this may be practical for small or medium sized networks but not for very large networks in modern society such as the Internet or WWW. Moreover, the large amount of network structural information is a heavy communication cost to the network in real time. So a strategy with local information is favoured, especially for the search algorithms in a graph.

Compared with a lot of work on the global information transmission and routing strategy, studies on local strategy are rare, including the search and traffic transmission strategies on networks.

Generally, in a local routing strategy, the node does not know the shortest path to its destination, and only knows the degree distribution of its neighbouring nodes. Random walk is a primal local routing strategy,[10] and it is a probability-based routing

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strategy. But its transmission efficiency is very low. Wang *et al.*[11] proposed a local optimal routing strategy based on the local degree information of a scale-free network, and pointed out that maximizing network capacity can be obtained when the determinate optimal parameter $\alpha = -1$. But the network capacity of Wang's routing strategy is not large enough; it is essentially a static probability-based local routing strategy, in which the optimal parameter α is determinate and not tunable with the practical operation state and traffic load of the node.

However, for networks with power-law degree distribution, such as a scale-free network, due to the heterogeneity of the network structure, the betweennesses of the nodes with different degrees differ significantly, so it is very difficult to apply the usual methods of betweenness adjustment such as increasing edge weight. But the betweennesses of higher degree nodes are bigger, which implies that they are the core nodes in the backbone of shortest path communication. Therefore, one expects that higher degree nodes (so-called hubs) play important roles in the dynamics of the heterogeneous network. Hence one could use the heterogeneity to make efficient algorithms in a wide variety of problems, such as in a local search algorithm, which efficiently uses information of high degree nodes.[12–14]

In this paper, an adaptive local routing strategy is presented, in which a node automatically adjusts forwarding probability with the queue length and degree distribution of its neighbouring nodes. It significantly improves transport capacity by a systematic adjustment of local forwarding probability.

The paper is organised as follows. In Section 2, the basic network model is defined and the adaptive local routing strategy is presented. In Section 3, the performance analysis and selection of parameters are explained in detail, and the mechanism of improving network capacity is elaborated. In Section 4, simulation results and further explanations are given. We conclude the paper in Section 5.

2. Model and adaptive local routing strategy

Without loss of generality, the basic Barabasi–Albert (BA) scale-free network model[15] is adopted, in which the probability distribution function of nodes with degree k is inversely proportional to its rank, i.e. $p(k) \sim k^{-\gamma}$, where γ is a scaling parameter (always greater than 1, here we choose $\gamma = 3$).

In its most general form, the details of the dynamical information transmission process and the local routing strategy are described as follows:

(i) Generation of the data packets At each time step, there are R packets generated in the system with randomly chosen source and destination nodes, that is, the rate of packet generation or packet insertion is R. Each node has a large enough buffer to contain a data packet. The newly inserted data packet is added to the tail of the buffer queue.

(ii) Forwarding of a data packet All nodes choose C data packets from the front of their buffer queue and send them to the neighbouring nodes. If its queue length is less than C, then all packets in its buffer are transferred. Here C is called the forwarding capability of the node. If its destination is in the neighbouring nodes, the data packet is directly sent to its destination. Otherwise, choose neighbouring node i in accordance with the following unitary forwarding probability Π_i to transfer the data packet

$$\Pi_i = \frac{k_i^{\alpha_i}}{\sum_j k_j^{\alpha_j}}, \tag{1}$$

where k_i is the degree of node i, α_i is the optional index (also called preferential factor here) of neighbouring node i, $j = 1, 2, \ldots, n$, and n is the total number of neighbouring nodes. Π_i is the unitary forwarding probability of node i. At the beginning, the preferential factor α_i of each node is initialized to 0. In order to avoid searching destination nodes repeatedly in the same region, the rule of path iteration avoidance (PIA) was applied to the transmission process. The rule of PIA implies that a link between a pair of nodes cannot be continuously visited twice or more by the same packet. That is to say, traversals back and forth on the same link in successive time steps are forbidden.

(iii) Reception of the data packet For each new received data packet, if the current node is the destination, the packet is removed from the network, otherwise it is added to the tail of the buffer queue.

(iv) Updating the travel time and forwarding probability All nodes update in parallel the travel times of data packets in their buffer, that is, the travel time of each data packet increases a unit time. At the same time, node i adaptive adjusts and updates its preferential factor α_i as follows:

$$\alpha_i = \begin{cases} \alpha_i + \Delta\alpha, & \text{if} \quad q_i < q_{ic}, \\ \alpha_i - \Delta\alpha, & \text{else.} \end{cases} \tag{2}$$

Here $\triangle\alpha$ is the adjustment at each time step, q_i is the queue length of node i, q_{ic} is the critical queue

length of node i, which is a constant and proportional to its degree k.

These rules are applied to each node updating at each time step.

Here the above routing strategy is called an adaptive local routing (ALR) strategy on a scale-free network. In the adjustment process, it is required that $\alpha_{\min} \leq \alpha_i \leq \alpha_{\max}$. So if $\alpha_i + \Delta\alpha > \alpha_{\max}$ then $\alpha_i = \alpha_{\max}$; if $\alpha_i - \Delta\alpha < \alpha_{\min}$, then $\alpha_i = \alpha_{\min}$. Without loss of generality, it is assumed that $\alpha_{\max}= 1$ and $\alpha_{\min} = -1$. In general, the adjustment $\Delta\alpha$ is set to one tenth of the value range of α.

3. Performance analysis and selection of parameters

Network transmission capacity is one of the important performance aspects of network routing strategy, and is usually measured by the critical insertion rate $R_{\rm c}$ (also called critical rate) obtained by the maximal average inserted packets at a single time step in the free flow state.[16] In general, most optimal routing strategies for enhancing network capacity are to make the values of betweennesses (or effective betweennesses) of all nodes become close to each other, with little difference. Thus the loads of nodes are distributed in an almost equal way, with the queue lengths.[11]

However, for networks with power-law degree distribution, such as a scale-free network, due to the heterogeneity of the network structure, the betweennesses of nodes with different degrees differ significantly, so it is very difficult to apply the usual methods of betweenness adjustment such as increasing side weight. But one expects that high degree nodes play important roles in the traffic dynamics on the heterogeneous network, and could make the best of the heterogeneity to gain some efficient algorithms. When there is only local information available (just knowing the degrees of the neighbouring nodes), delivery preferring the high degree neighbouring node can shorten routing hops and improve transmission capacity. Adamic *et al.*[12] found that the most favorable searching was obtained by using the strategy of preferential utilizing of the higher connectivity nodes in a power-law network. In a power-law network, the betweennesses of high degree nodes are bigger. Then the backbone of shortest path communication is always constructed by high degree nodes; this may enhance the probability of a packet which can find its destination quickly and reduce routing hops by preferentially using them to deliver packets.

Therefore, in order to adapt to the structural heterogeneity of a scale-free network and make the best of the high degree nodes, it is necessary that the critical queue length of a node with high degree should be set larger than that of a node with low degree, so as to enhance the probability that the packet would find its destination quickly and to reduce average routing hops. Moreover, since the betweenness of a node with high degree is bigger, it is easier to become congested because of overload. At the same time, the preferential factor α_i and the receiving probability of neighbouring node i should be adjusted automatically with respect to its dynamical queue length, and the network flow should be adjusted dynamically according to the nodes' loads in real time.

3.1. Critical queue length

In order to take advantage of high degree nodes and adapt to the heterogenous degree distribution structure of the scale-free network, for simplicity it is assumed that the critical queue length of node i is proportional to its degree k_i,

$$q_{i\rm c} = \varepsilon \cdot k_i, \tag{3}$$

where ε is a proportional coefficient.

With the increase of the packet inserting rate R in the network, the data packets in the network will increase.

According to the idea that it is necessary to make the best of the high degree nodes in the adaptive routing strategy mentioned above and Eq. (3), it is found that the higher the degree of a node is, the sooner the queue length reaches a critical queue length $q_{\rm c}$. The lower the degree of a node is, the later the queue length reaches critical queue length. When the queue length is equal to or longer than the critical queue length $q_{\rm c}$, we call the node saturated. Therefore during the dynamic process when the rate of packet insertion R increases and reaches the critical rate $R_{\rm c}$, the queue lengths of high degree nodes increase to their critical queue lengths and they become saturated at first, then the low degree nodes begin to participate with the overfull packets in the high degree nodes till they become saturated too. The saturation process goes on in turn from high degree nodes to low degree nodes.

The critical queue length $q_{\rm c}$ reflects the maximal received packets of a node when the network is in a free flow state. So after the node having minimal degree

$k_{\min}$ becomes saturated, it has no more buffer to receive new inserted packets, and the surrounding high degree nodes have no buffers to receive more packets and participate in data packet traffic flow too. From the microcosmic perspective, for the node with minimal degree $k_{\min}$, if its critical queue length $q_{\rm c}$ is less than its forwarding capacity C, then all packets in its buffer will be forwarded and cleared once a time step, and its buffer cannot store too many packets or it becomes jammed, and thus the network cannot become congested. Therefore, the critical queue length of the node of minimal degree $k_{\min}$ is assumed to be the forwarding capability C in a conservative way as follows.

$$q_{k_{\min}{\rm c}} = \varepsilon \cdot k_{\min} = C. \tag{4}$$

Then we have the proportional coefficient

$$\varepsilon = \frac{C}{k_{\min}}. \tag{5}$$

Here the value of the parameter ε is determined by the condition that the node with minimum degree is not in congestion. So the critical queue length $q_{i{\rm c}}$ of node i is uniformly set as follows:

$$q_{i{\rm c}} = \frac{C}{k_{\min}} k_i, \tag{6}$$

where $k_{\min}$ is the minimal degree in the network.

3.2. Optimization parameter and adaptive mechanism

According to Eq. (1), it is obvious that when α_i is a positive constant ($i = 1, 2, \ldots, n$), the higher the degree of the neighbouring node is, the greater the probability of the packet to be transmitted is, and it is a degree-preferred local routing strategy. When all $\alpha_i = 0$, the neighbouring nodes have equal probability of the packet to be transmitted, and it is a degree-equal local routing strategy. When α_i is a negative constant, it is a degree-suppressed local routing strategy.

Wang *et al.*[11] pointed out that when the degree exponent α_i in Eq. (1) is fixed at constant α_f, the packet queue length of node i has the following free flow state,

$$\overline{q}(k_i) = \psi(R) \cdot k_i^{1+\alpha_{\rm f}}, \tag{7}$$

where $\psi(R)$ is a monotonically increasing function of the packet inserting rate R .

From Eq. (7), it is obtained that: 1) the degree-preferred strategy with all $\alpha_f > 0$ will cause a node with high degree to have a heavier load than a node with low degree. 2) Even if the uniformity degree-equal strategy with $\alpha_f = 0$ is adopted, because a node with high degree has more neighbouring nodes, it still could receive more packets and have a bigger queue length. 3) When a strong degree-suppressed strategy is adopted ($\alpha_f < -1$), the higher degree node has a lighter load.

The adaptive local routing strategy prefers to make the best of high degree nodes. Their critical queue lengths are set larger to store more packets so as to reduce routing hops and increase the arriving probability. When their queue lengths are not less than their critical queue lengths, more packets in the network will be delivered to middle degree or low degree nodes around them, avoiding the jammed state and not decreasing the network capacity.

When all $q_i \leq q_{i{\rm c}} = C \cdot k_i/k_{\min}$, all α_i are increasing and up to the upper limit $\alpha_{\max}$. When $0 < \alpha_i < \alpha_{\max}$, for each α_i the adaptive local routing strategy will adjust the forwarding probability according to the degree k_i and the queue length q_i in real time.

When all $\alpha_i = \alpha_{\max}$, the adaptive local routing strategy becomes the degree-preferred local strategy, and the nodes with higher degree are fully used to reduce routing hops and increase the arriving probability of packets.

When there are some saturated nodes in the network, the adaptive local routing strategy becomes the degree-suppressed one for the saturated nodes, but it is the degree-preferred local strategy for unsaturated nodes.

When $q_i \geq q_{i{\rm c}} = C \cdot k_i/k_{\min}$, the preferential factor α_i will decrease and the probability of receiving packets for node i is adjusted according to the local load and its previous value. For some high degree nodes, they will be in a dynamic load-balancing state and their queue lengths fluctuate around their critical queue lengths. The bigger the inserting rate R, the greater the amount of high degree nodes. This not only makes use of the delivering probability of nodes with high degree to shorten routing hops, but also controls queue length to avoid congestion.

Therefore, the adaptive local routing strategy takes advantage of degree-suppressed and degree-preferred local routing strategies. For the saturated node, the degree-suppressed routing strategy is adopted to prevent the packet queue length of its buffer from increasing constantly and causing congestion, while, for the unsaturated node, it adopts the degree-preferred strategy to make use of the high

degree nodes forwarding packets and shortening the routing hops.

4. Simulations and explanations

4.1. Network capacity

The network capacity is represented by the critical packet inserting rate R_c, which is the transition point from a free flow traffic state to a congested one. The phase transition is properly described by means of the following order parameter $\eta(R)$ introduced in Ref. [17]

$$\eta(R) = \lim_{t\to\infty} \frac{1}{R} \frac{\langle \Delta N_p \rangle}{\Delta t}. \tag{8}$$

In Eq. (8), $N(t)$ is defined as the number of total accumulated packets (traffic load) in the network at time t; $\Delta N_p = N(t + \Delta t) - N(t)$ and $\langle \cdot \rangle$ indicates the average over time windows of width Δt. This quantity represents the ratio between undelivered and generated packets over a long enough time period. When $R < R_c$, $\eta(R)$ tends to zero; when $R > R_c$, the network enters a congestion state and the load grows linearly in time, and $\eta(R)$ equals a finite value larger than 0; our simulation parameters are C=10, k_{min}=5, α_{max}=1 and $\alpha_{min} = -1$. Figure 1 plots the order parameter $\eta(R)$ as a function of inserting rate R for different routing strategies and network size. Because the R_c of the degree-preferred strategy cannot even reach 1 for network size $N = 5000$ and 10000, only curves for the other three strategies appear in Figs. 1(c) and 1(d).

From Fig. 1, we can find that: the ALR strategy has the biggest network capacity among the four local routing strategies; and the network capacity of the degree-suppressed strategy is inferior to ALR; the network capacities of the degree-equal strategy and the degree-preferred strategy are obviously lower than those of the other two. This is because in ALR, the preferential factor α can automatically adjust itself. Based on the current network communication traffic load, ALR can select and utilize different strategies. So actually, the other three strategies are adopted alternately according to the loads of nodes in ALR. If the load is light, α increases and the degree-preferred strategy for high degree nodes is applied to reduce routing hops. If the load becomes heavy, α decreases and the degree-suppressed strategy for high degree nodes is applied to disperse packets and avoid congestion. So ALR can effectively increase network capacity.

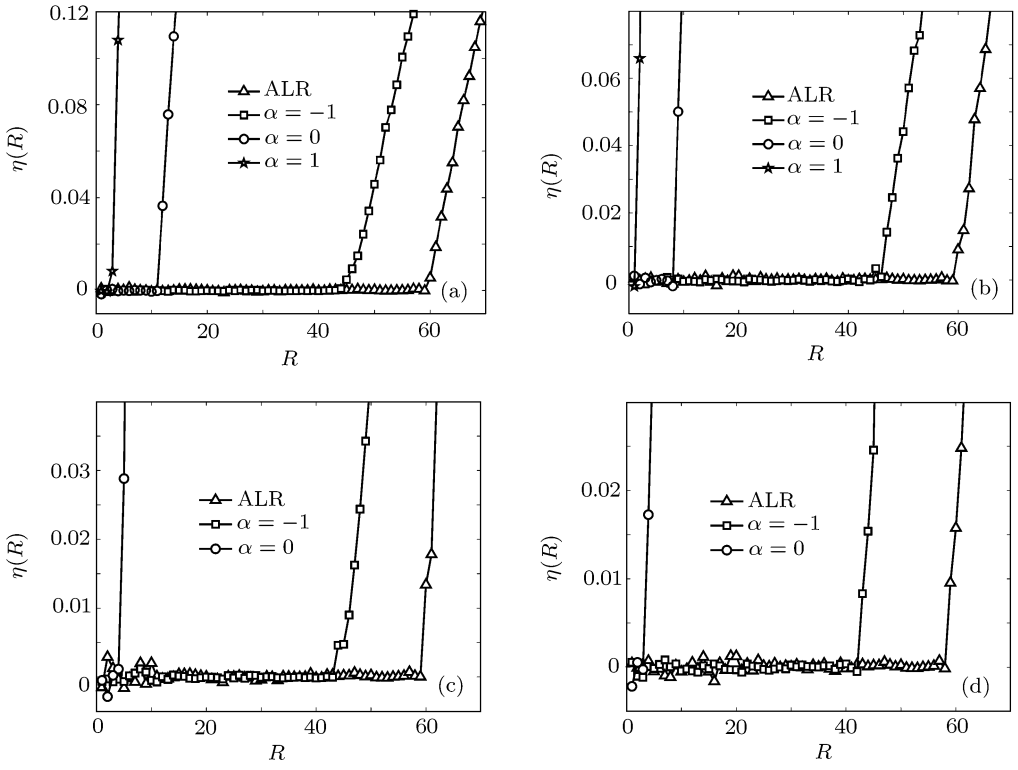


Fig. 1. Order parameter η versus R for different local routing strategies: ALR, degree-suppressed ($\alpha = -1$), degree-equal ($\alpha = 0$) and degree-preferred ($\alpha = 1$). Network sizes are (a) $N = 1000$, (b) $N = 2000$, (c) $N = 5000$, (d) $N = 10000$.

Figure 1 shows that with increasing network size, the network capacities of ALR and the degree-suppressed strategy with $\alpha = -1$ are stable, while the network capacity of the degree-preferred strategy and the degree-equal strategy gradually decreases. We will explain the facts in the next section.

4.2. Queue length

4.2.1. Queue length with static probability-based local routing strategy

With the static probability-based local routing strategy, the preferential factors α of all nodes are the same constant, and the forwarding probability Π_i of node i is a constant according to Eq. (1). When the network is in the free flow steady state, at every step average, the number of received data packets of each node is a constant, which is denoted as $\overline{p}_r$.

When the network is in the free flow steady state, that is $R < R_c$, we have $\overline{p}_r < C$ for all nodes. Otherwise, the node with $\overline{p}_r > C$ will become jammed, and congestion will spread to other nodes and then to the whole network. If there are some nodes with $\overline{p}_r = C$, with a tiny increase of R (R is still less than R_c), these nodes will receive more data packets than before, which will make their $\overline{p}_r$ bigger than C, i.e. $\overline{p}_r > C$. So while $R < R_c$, we have $\overline{p}_r < C$. Similarly, when $R = R_c$, we have $\overline{p}_r \leq C$.

When $\overline{p}_r < C$, the data packet queue of each node will be cleared after forwarding packets at each step, and it cannot accumulate more packets than C. So the average packet queue length is less than C, i.e. $\overline{q}(k) < C$. Similarly, when $R = R_c$, we have $\overline{q}(k) \leq C$. In the free flow state and critical state, for node i, we have

$$\overline{q}(k_i) \leq C. \tag{9}$$

When the preferential factor α is bigger than -1, i.e. $\alpha + 1 > 0$, according to Eq. (7), it is obvious that the node with high degree has a larger average queue length. The node with the highest degree $k_{\max}$ has the largest average queue length.

In the free flow state and critical state, we have

$$\overline{q}(k_{\max}) = \psi(R) \cdot k_{\max}^{1+\alpha} \leq C. \tag{10}$$

Here the equation holds in the critical state with the static probability-based local routing strategy, i.e.,

$$\overline{q}(k_{\max}) = C. \tag{11}$$

4.2.2. Queue length with ALR strategy

In the ALR strategy, the forwarding probabilities of nodes are dynamically changed with node load. If one node becomes saturated, its $\overline{p}_r$ will be restrained. As long as there are unsaturated nodes to hold more traffic packet flow from the saturated nodes, the network will not be jammed. The high degree nodes become saturated first of all, while the node with minimal degree $k_{\min}$ will become saturated last. If all nodes are saturated, according to Eq. (6), the node with the minimal degree $k_{\min}$ must have the smallest average queue length $\overline{q}_{k\min}$. Except for the nodes with $k_{\min}$, it is allowed that the critical queue lengths of other nodes exceed C. Consequently, if $\overline{q}_{k\min}$ exceeds the forwarding capability C, i.e. $\overline{q}_{k\min} > C$, no nodes will participate with overfull packets in high degree nodes and the network will be jammed.

So in the free flow steady state, $\overline{q}_{k\min}$ cannot exceed the small one between C and its critical queue length $q_c(k_{\min})$

$$\overline{q}_{k\min} \leq \min[q_c(k_{\min}), C] = \min(\varepsilon k_{\min}, C). \tag{12}$$

The equation holds in the critical state, i.e.,

$$\overline{q}_{k\min} = q_c(k_{\min}) = \varepsilon k_{\min} = C. \tag{13}$$

4.2.3. Comparison between queue lengths in the critical state

Equation (13) implies that the minimal critical queue length is equal to the forwarding capability C with the ALR strategy. But it implies that the maximal critical queue length is equal to the forwarding capability C with the static probability-based local routing strategy in Eq. (11).

Figure 2 displays the dependence of queue lengths at critical insertion rate R_c on network size N. It is noticeable that in the same figure the critical insertion rates R_c of various strategies are different. Because the R_c of the degree-preferred strategy cannot even reach unity for network size $N = 5000$ and 10000, only curves for the other three strategies are given in Figs. 2(c) and 2(d).

With the static probability-based local routing strategy ($\alpha = -1, 0, 1$), the node with the highest degree $k_{\max}$ has the largest average queue length. Also, obviously, $\overline{q}(k_{\max})$ is equal to C at the critical insertion rate R_c. According to Eq. (7), we have $\log(\overline{q}(k_i)) \propto (1 + \alpha) \log(k_i)$. So in logarithmic coordinates, there are different slopes $(1 + \alpha)$ with different α in Fig. 2.

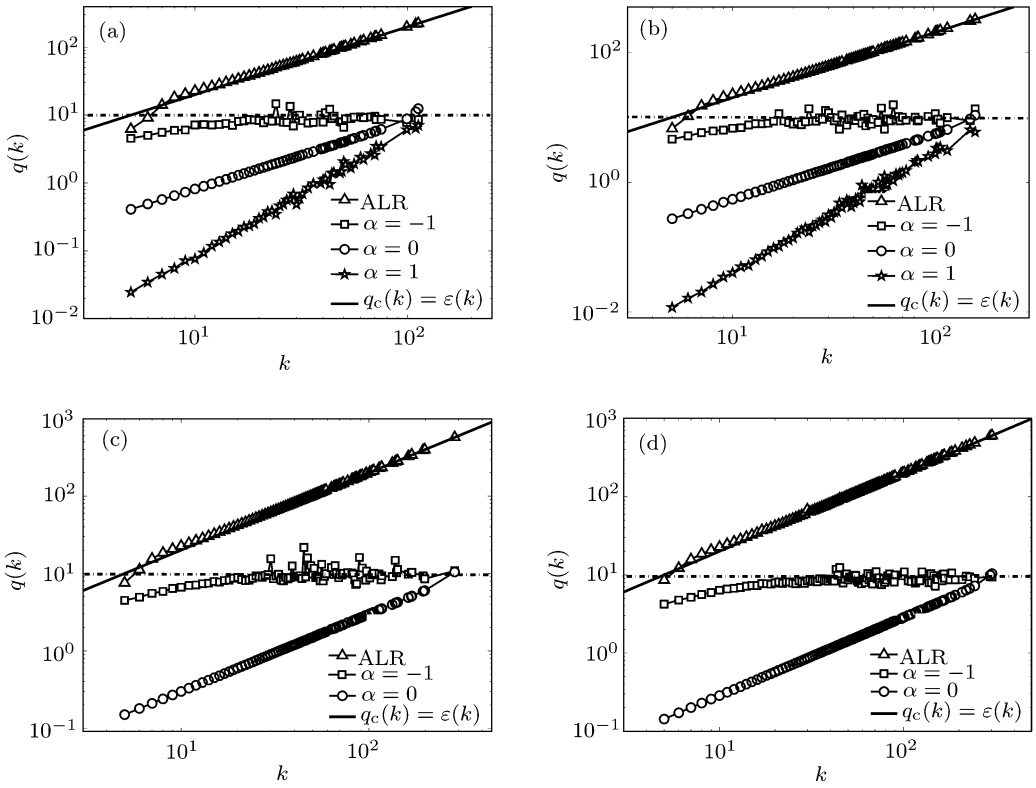


Fig. 2. Variation of average packet queue length $q(k)$ of nodes with degree k for four different routing strategies at each different critical inserting rate R_c: ALR, degree-suppressed ($\alpha = -1$), degree-equal ($\alpha = 0$) and degree-preferred ($\alpha = 1$). The horizontal dotted line represents node transmission ability C. The acclivitous solid line represents critical queue length $q_c(k)$. Network sizes are (a) $N = 1000$, (b) $N = 2000$, (c) $N = 5000$, (d) $N = 10000$.

In the ALR strategy, the nodes with the lowest degree $k_{\min}$ have the smallest average queue length, and $\overline{q}_{k\min}$ is equal to C at R_c. Because all nodes are saturated, queue lengths of all nodes reach their critical queue, i.e. $\log(\overline{q}(k_i)) = \log(q_c(k_i)) \propto \log(k_i)$. So in logarithmic coordinates, the slope with the ALR strategy is constant 1 in Fig. 2.

Figure 2 shows the relation between network size and capacity of different local routing strategies. At the critical state $R = R_c$, according to Eq. (7), with the degree-suppressed strategy ($\alpha = -1$), all nodes have the same packet queue length, the packet queue expression is $q(k) = \psi(R_c) = C$. With the degree-equal strategy ($\alpha = 0$), it is $q(k) = \psi(R_c) \cdot k_{\max} = C$. With the degree-preferred strategy ($\alpha = 1$), it is $q(k) = \psi(R_c) \cdot k_{\max}^2 = C$.

From Eq. (7), it is noticed that $\psi(R)$ is a monotonically increasing function of R, and the maximal degree $k_{\max}$ increases with increasing network size N; but the forwarding capacity C is a constant. So for the degree-equal and degree-preferred strategies, their critical insertion rate R_c decreases with increasing network size N. But for the degree-suppressed strategy, the packet queue length expression does not include $k_{\max}$, so its R_c is independent of the network size N.

As for the ALR strategy, because the increase of network size N does not change the minimal degree $k_{\min}$ and affect the critical packet queue length expression (13), the critical insertion rate R_c with the ALR strategy is independent of network size N.

4.3. Critical inserting rate R_c

With the ALR strategy, the relation between critical inserting rate R_c and the proportional coefficient ε is displayed in Fig. 3 for different forwarding capacities, minimal degrees and network sizes.

From Fig. 3, it is found that at the beginning the critical inserting R_c (network capacity) in the ALR strategy linearly increases with the increase of proportional coefficient ε. There is a critical transition point, above which the critical insertion rate R_c is a constant, and the critical transition value of ε is $C/k_{\min}$, as

described in Eq. (11).

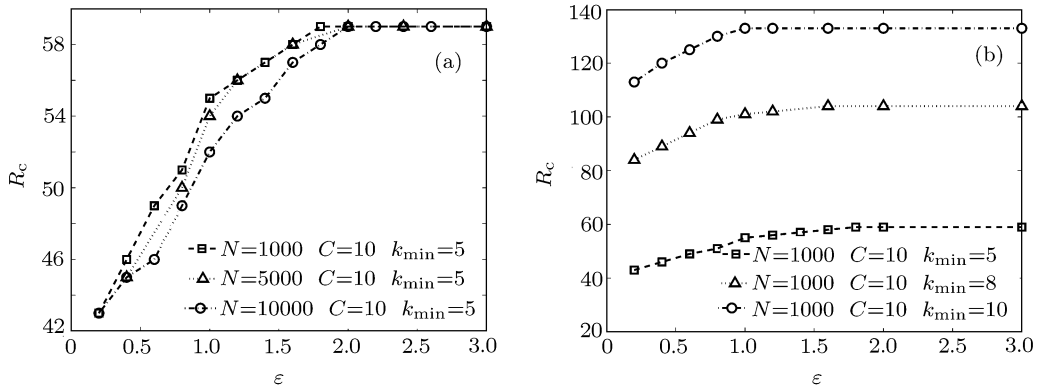


Fig. 3. The variation of critical inserting rate R_c of the ALR strategy with the proportional coefficient ε, forwarding capacity and the minimal degree k_{min} for different network sizes.

Figure 3(a) shows that the critical packet insertion rate R_c varies with the different minimal degree k_{min} at the same forwarding capacity C, and the critical insertion rate R_c is independent of network size N and only dependent on the proportional coefficient ε.

Figure 3(b) shows the relation between the critical insertion rate R_c and the minimal degree at the same network size N and forwarding capacity C, and it shows that the critical insertion rate R_c is proportional to the minimal degree. So we can extend network capacity by increasing the minimal degree k_{min} at the same network size N. The larger the minimal degree k_{min}, the greater the number of links between nodes. So more packets can arrive at their destinations more easily, and the network capacity increases.

4.4. Travel time and routing hops

The average travel time and routing hops of packets versus insertion rate are shown in Figs. 4 and 5 respectively.

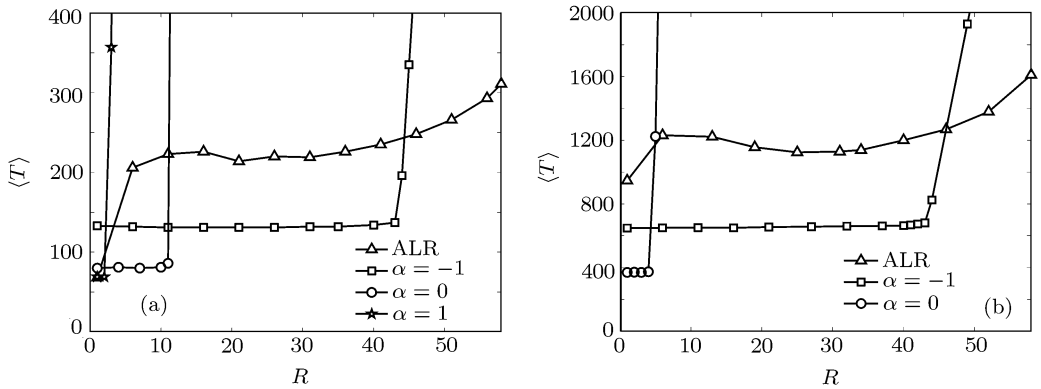


Fig. 4. Average packet travel time $\langle T \rangle$ versus insertion rate R for different strategies. Network sizes are (a) $N = 1000$, (b) $N = 5000$.

In Fig. 4, it is shown that in the free-flow state, the ALR strategy has the largest average packet travel time $\langle T \rangle$ among the four routing strategies. Actually, it is a local routing strategy that improves network capacity by increasing average travel time $\langle T \rangle$. The travel time is composed of routing hops and waiting time in the buffer. Intuitively, in the ALR strategy the high degree nodes hold more packets in their buffers, and these packets have to wait for more time in the buffer than in the static probability-based local routing strategies.

In Fig. 5, it is shown that in the free-flow state, the ALR strategy has the minimal average packet routing hops $\langle \text{hops} \rangle$ among the four local routing strategies. The network capacity is improved by preferentially utilizing the higher degree nodes with the ALR strategy. The nodes with higher degree have important effects on the shortest path communication, and reduce unnecessary transmission. The average routing hops $\langle \text{hops} \rangle$ are reduced too.

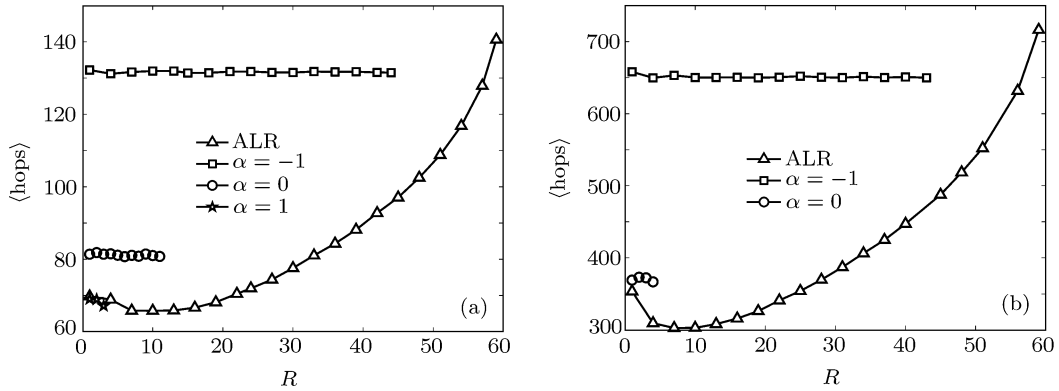


Fig. 5. Average packet routing hops ⟨hops⟩ versus insertion rate R for different strategies. Network sizes are (a) $N = 1000$, (b) $N = 5000$.

In the ALR strategy, the higher degree nodes could contain more packets in their buffers due to their larger critical queue lengths, and these packets could effectively utilize short hop routing. Then their arriving probabilities increase and thus reduce unnecessary iteration transmission. As a result, the critical insertion rate R_c and network capacity are improved.

5. Conclusion and discussion

For the heterogeneous scale-free network, the adjustment to homogenize nodes betweennesses is not effective, even though all nodes have the same forwarding capacity. It is obvious that the higher degree nodes play important roles in traffic dynamics on the heterogeneous network, and hence this may be available to make the best of the heterogeneity to attain some efficient local routing algorithms.

An adaptive local routing strategy is presented in this paper, in which the node automatically adjusts the forwarding probability with the dynamical traffic loads and degree distribution of neighbouring nodes. At first, it increases the probability of a packet for finding its destination and reduces routing hops by preferentially using the higher degree nodes, which have larger critical queue lengths to store and forward more packets. After the higher degree nodes are saturated, more packets will be delivered by the lower degree nodes around them. At the same time, it realizes dynamic control of the traffic load on the network by adjusting the preferential factor in time.

The adaptive local routing strategy increases the critical queue length of the high degree node, and improves the transmission capacity by reducing routing hops and taking advantage of its important effect on the shortest path communication. It increases the travel time to some extent, because the average waiting time in the buffer increases.

The adaptive local routing strategy is simulated with different network sizes (from $N = 1000$ to $N = 10000$), different forwarding capacities and different minimal degrees. It is shown that the adaptive local routing strategy is more efficient than the previous local routing strategies.

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