Geometric phases and quantum phase transitions in inhomogeneous XY spin-chains: Effect of the Dzyaloshinski–Moriya interaction*

Wang Lin-Cheng(王林成), Yan Jun-Yan(闫俊彦), and Yi Xue-Xi(衣学喜)

School of Physics and Optoelectronic Technology, Dalian University of Technology, Dalian 116024, *China*

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We study geometric phases of the ground states of inhomogeneous XY spin chains in transverse fields with Dzyaloshinski–Moriya (DM) interaction, and investigate the effect of the DM interaction on the quantum phase transition (QPT) of such spin chains. The results show that the DM interaction could influence the distribution of the regions of QPTs but could not produce new critical points for the spin-chain. This study extends the relation between geometric phases and QPTs.

Keywords: geometric phase, quantum phase transition, the Dzyaloshinski–Moriya interaction

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1. Introduction

Quantum phase transitions (QPTs), originating from quantum fluctuations at zero temperature in many-body systems, have been one of the most interesting topics of strongly correlated systems in recent years, and have drawn considerable interest in various fields of physics.[1] QPTs are characterized by dramatic changes in the ground-state properties of a many-body system. The traditional order parameter and symmetry breaking within the Landau–Ginzburg paradigm cannot describe completely the properties of the ground state of many-body system. Recently, the approaches to QPTs based on the concept of quantum information have been put forward, especially by the structure of the system's ground state.

The phase factor of a wave function is the source of all interference phenomena and one of the most fundamental concepts in quantum physics. Since Berry discovered that a system may gain a pure phase related to the geometric backgrounds of the parameters of the Hamiltonian,[2] geometric phases in quantum mechanics have been an important subject of theoretical and experimental investigations.[3−11] Recently, geometric phases have been used in quantum information theory, and have been suggested as an important tool to detect regions of criticality without having to undergo a QPT.[12−15] The fidelity, which describes a distance between two given quantum states with slightly different values of parameters, can also be used to characterize drastic changes in quantum states when systems undergo QPTs.[16,17] The fidelity of two states vanishes due to irrelevant local information in the same phase or relevant long distance information in different phases.[18] Actually, the above two approaches of fidelity and geometric phase have been pointed out to share the same origin and can be unified.[19] Progress based on the ground state fidelity has also been put forward, such as differential geometric theory[20] and fidelity susceptibility.[21] Other progress of QPTs has also been put forward consequently, such as the relation between QPTs and entanglement,[22−27] as well as the properties of auxiliary particles,[28−32] etc.

In these studies, the exactly solvable one-dimensional XY spin chains have attracted much attention; however, the one-dimensional spin chain with Dzyaloshinski–Moriya (DM) anisotropic antisymmetric interaction has rarely been considered.[33−35] The DM coupling describes an extended superexchange mechanism interaction by considering a term arising naturally in the perturbation theory due to the spin–orbit coupling,[36−39] which will arise from the weak intermolecular interactions. Nevertheless, DM interaction has been considered in studying the bipartite system and three-particle entanglements,[40−43] which shows that DM coupling may play an important role in the system's property. Since the DM anisotropic

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antisymmetric interaction may make the spin-chain's antisymmetric states and change their critical properties, it is useful to discuss this model. However, in the homogeneous Ising spin chain or the XY spin-chain, the DM anisotropic antisymmetric interaction can only change the energy level and could not affect the critical structure of the system. This result motivates us to consider the case of an inhomogeneous spin chain.[15] The inhomogeneous spin-chain has been studied in many ways, but the DM interaction has not been considered yet.

In this paper, we shall consider the inhomogeneous XY spin-chain in transverse fields with the Dzyaloshinski–Moriya (DM) anisotropic antisymmetric interaction, and establish the relation between the geometric phases of the ground state and the QPTs of this spin-chain. The effect of the DM interaction on the QPTs of the spin-chain will be obtained by this study.

2. Solution of the inhomogeneous XY spin chains with the Dzyaloshinski–Moriya interaction

We consider a one-dimensional inhomogeneous spin-1/2 XY chain with the DM interaction in an external magnetic transverse field, and this spin-chain consists of N cells with two sites in each cell (see Fig. 1). Such a spin chain can be described by

$$\begin{aligned} H &= H_{\rm XY} + H_{\rm D}, \\ H_{\rm XY} &= -\sum_{l=1}^{N}\{J_a(1+\gamma_a)\sigma_{l,a}^x\sigma_{l,b}^x + J_a(1-\gamma_a)\sigma_{l,a}^y\sigma_{l,b}^y \\ &\quad + J_b(1+\gamma_b)\sigma_{l,b}^x\sigma_{l+1,a}^x + J_b(1-\gamma_b)\sigma_{l,b}^y\sigma_{l+1,a}^y \\ &\quad + \lambda(\sigma_{l,a}^z + \sigma_{l,b}^z)\}, \\ H_{\rm D} &= -\sum_{l=1}^{N}[\boldsymbol{D}_a\cdot(\boldsymbol{\sigma}_{l,a}\times\boldsymbol{\sigma}_{l,b}) \\ &\quad + \boldsymbol{D}_b\cdot(\boldsymbol{\sigma}_{l,b}\times\boldsymbol{\sigma}_{l+1,a})], \end{aligned} \tag{1}$$

where $H_{\rm XY}$ is the Hamiltonian of the inhomogeneous XY spin-chain, σ_l^x, σ_l^y and σ_l^z are the Pauli operators act on the l-th site. J_a and J_b are coupling strengths of the spin coupling, while γ_a and γ_b are anisotropic parameters of the spin coupling in the cell and between the cells respectively. λ is the strength of the transverse magnetic field. The Hamiltonian $H_{\rm XY}$ encompasses several other Hamiltonians of spin-chain models, such as the Ising model in a transverse field in the case of $J_a = J_b$ and $\gamma_a = \gamma_b = 1$, the XY model in the case of $J_a = J_b$ and $0 \leq \gamma_a = \gamma_b \leq 1$, etc. $H_{\rm D}$ describes the DM interaction in the spin chain, and in the case of the vector $\boldsymbol{D}$ along the z axis, i.e., $\boldsymbol{D} = D\boldsymbol{z}$, the DM interaction term can be simplified as

$$\begin{aligned} H_{\rm D} = -\sum_{l=1}^{N}&[D_a(\sigma_{l,a}^x\sigma_{l,b}^y - \sigma_{l,a}^y\sigma_{l,b}^x) \\ &+ D_b(\sigma_{l,b}^x\sigma_{l+1,a}^y - \sigma_{l,b}^y\sigma_{l+1,a}^x)]. \end{aligned} \tag{2}$$

The periodic boundary condition of the spin chain is assumed in the following study, $\sigma_{N+1,m}^i = \sigma_{1,m}^i$ $(i = x, y, z; m = a, b)$, and the lattice parameter is $2a$.

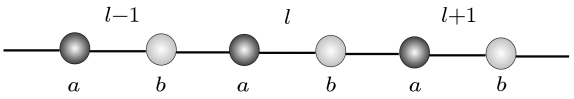


Fig. 1. The period-two inhomogeneous chain.

The Hamiltonian (1) of the system may be diagonalized by means of Jordan–Wigner transformation, i.e., $\sigma_{l,a}^z = 1 - 2c_{l,a}^\dagger c_{l,a}$,

$$\begin{aligned} \sigma_{l,a}^+ &= \Big[\prod_{l<l'}\prod_{m'=a,b}(1-2c_{l',m'}^\dagger c_{l',m'})\Big]c_{l,a}, \\ \sigma_{l,b}^+ &= \Big[\prod_{l<l'}\prod_{m'=a,b}(1-2c_{l',m'}^\dagger c_{l',m'})\Big](1-2c_{l,a}^\dagger c_{l,a})c_{l,b}, \end{aligned} \tag{3}$$

which maps spins to one-dimensional spinless fermions. In the above expression, $\sigma_j^\pm = (\sigma_j^x \pm {\rm i}\,\sigma_j^y)/2$. After Jordan–Wigner transformation, the Hamiltonian will be transformed to

$$\begin{aligned} H = -\sum_l&[-2(J_a + {\rm i}\,D_a)c_{l,a}c_{l,b}^\dagger + 2(J_a - {\rm i}\,D_a)c_{l,a}^\dagger c_{l,b} + 2J_a\gamma_a(c_{l,a}^\dagger c_{l,b}^\dagger - c_{l,a}c_{l,b}) + \lambda(1-2c_{l,a}^\dagger c_{l,a}) \\ &- 2(J_b + {\rm i}\,D_b)c_{l,b}c_{l+1,a}^\dagger + 2(J_b - {\rm i}\,D_b)c_{l,b}^\dagger c_{l+1,a} + 2J_b\gamma_b(c_{l,b}^\dagger c_{l+1,a}^\dagger - c_{l,b}c_{l+1,a}) + \lambda(1-2c_{l,b}^\dagger c_{l,b})]. \end{aligned} \tag{4}$$

Then, using the Fourier transformation,

$$c_{l,a}=\frac{1}{\sqrt{N}}\sum_{k=-M}^{M}\mathrm{e}^{\mathrm{i}\,kR_{la}}a_k,\quad c_{l,b}=\frac{1}{\sqrt{N}}\sum_{k=-M}^{M}\mathrm{e}^{\mathrm{i}\,k(R_{la}+a)}b_k \tag{5}$$

with $M=(N-1)/2$, in momentum space, the Hamiltonian can be written as

$$\begin{aligned}H=-\sum_k\Big[&2(J_a+\mathrm{i}\,D_a)\mathrm{e}^{-\mathrm{i}\,ka}b_k^\dagger a_k+2(J_a-\mathrm{i}\,D_a)\mathrm{e}^{\mathrm{i}\,ka}a_k^\dagger b_k\\&-2J_a\gamma_a(\mathrm{e}^{\mathrm{i}\,ka}a_{-k}b_k+\mathrm{e}^{-\mathrm{i}\,ka}b_k^\dagger a_{-k}^\dagger)+\lambda(1-2a_k^\dagger a_k)\\&+2(J_b+\mathrm{i}\,D_b)\mathrm{e}^{-\mathrm{i}\,ka}a_k^\dagger b_k+2(J_b-\mathrm{i}\,D_b)\mathrm{e}^{\mathrm{i}\,ka}b_k^\dagger a_k\\&-2J_b\gamma_b(\mathrm{e}^{\mathrm{i}\,ka}b_{-k}a_k+\mathrm{e}^{-\mathrm{i}\,ka}a_k^\dagger b_{-k}^\dagger)+\lambda(1-2b_k^\dagger b_k)\Big].\end{aligned} \tag{6}$$

The above Hamiltonian has a quadratic form in fermion operators and can be exactly diagonalized by the following procedure. Obviously, the above Hamiltonian can be expressed as $H=\sum_k\Gamma_k^\dagger M_k\Gamma_k$, where $\Gamma_k^\dagger=(a_k^\dagger,a_{-k},b_k^\dagger,b_{-k})$ and M_k is a Hermitian matrix, i.e.,

$$M=\begin{pmatrix}\lambda&0&-A_k&\frac{1}{2}B_k\\0&-\lambda&\frac{1}{2}B_k&0\\-A_k^*&-\frac{1}{2}B_k^*&\lambda&0\\-\frac{1}{2}B_k^*&0&0&-\lambda\end{pmatrix}, \tag{7}$$

where A_k and B_k are defined as $A_k=2(J_a-\mathrm{i}\,D_a)\mathrm{e}^{\mathrm{i}\,ka}+2(J_b+\mathrm{i}\,D_b)2\mathrm{e}^{-\mathrm{i}\,ka}$, $B_k=-2J_a\gamma_a\mathrm{e}^{\mathrm{i}\,ka}+2J_b\gamma_b\mathrm{e}^{-\mathrm{i}\,ka}$. Then, we can use a unitary transformation U_k and insert $I=U_k^\dagger U_k$ in the Hamiltonian which may transform the matrix M_k into a diagonal one, $M_k'=U_kM_kU_k^\dagger$. Then the Hamiltonian can be expressed as

$$H=\sum_k\Gamma_k^\dagger U_k^\dagger U_kM_kU_k^\dagger U_k\Gamma_k=\sum_k\Gamma_k^{'\dagger}M_k'\Gamma_k', \tag{8}$$

and non-zero diagonal terms of M_k' are also the eigenvalues of the matrix M_k, which can be worked out as

$$\begin{aligned}\xi_{k,1}&=-\frac{1}{2}|A_k|-\frac{1}{2}\sqrt{(2\lambda-|A_k|)^2+|B_k|^2},\\\xi_{k,2}&=-\frac{1}{2}|A_k|+\frac{1}{2}\sqrt{(2\lambda-|A_k|)^2+|B_k|^2},\\\xi_{k,3}&=+\frac{1}{2}|A_k|-\frac{1}{2}\sqrt{(2\lambda+|A_k|)^2+|B_k|^2},\\\xi_{k,4}&=+\frac{1}{2}|A_k|+\frac{1}{2}\sqrt{(2\lambda+|A_k|)^2+|B_k|^2},\end{aligned} \tag{9}$$

where we have defined

$$\begin{aligned}|A_k|=2(&J_a^2+J_b^2+2J_aJ_b\cos 2ka\\&+D_a^2+D_b^2+2D_aD_b\cos 2ka\\&+2J_aD_b\sin 2ka-2D_aJ_b\sin 2ka)^{1/2},\\|B_k|=2\sqrt{&J_a^2\gamma_a^2+J_b^2\gamma_b^2-2J_aJ_b\gamma_a\gamma_b\cos 2ka}.\end{aligned} \tag{10}$$

The element of the new vector $\Gamma_k^{'\dagger}=(\gamma_{1,k}^\dagger,\gamma_{2,k}^\dagger,\gamma_{3,k}^\dagger,\gamma_{4,k}^\dagger)$ can be described by the transformation $\Gamma_k'=U_k\Gamma_k$. This transformation can be looked at as a generalization of Bogoliubov transformation,

$$\begin{pmatrix}\gamma_{k,1}\\\gamma_{k,2}\\\gamma_{k,3}\\\gamma_{k,4}\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}\mathrm{e}^{\mathrm{i}\,\varphi_k^A}\sin\frac{\alpha_k}{2}&\mathrm{e}^{\mathrm{i}\,\varphi_k^B}\cos\frac{\alpha_k}{2}&\sin\frac{\alpha_k}{2}&-\mathrm{e}^{\mathrm{i}(\varphi_k^A-\varphi_k^B)}\cos\frac{\alpha_k}{2}\\\cos\frac{\alpha_k}{2}&-\mathrm{e}^{-\mathrm{i}(\varphi_k^A-\varphi_k^B)}\sin\frac{\alpha_k}{2}&\mathrm{e}^{-\mathrm{i}\,\varphi_k^A}\cos\frac{\alpha_k}{2}&\mathrm{e}^{-\mathrm{i}\,\varphi_k^B}\sin\frac{\alpha_k}{2}\\-\mathrm{e}^{\mathrm{i}\,\varphi_k^A}\sin\frac{\beta_k}{2}&\mathrm{e}^{\mathrm{i}\,\varphi_k^B}\cos\frac{\beta_k}{2}&\sin\frac{\beta_k}{2}&\mathrm{e}^{\mathrm{i}(\varphi_k^A-\varphi_k^B)}\cos\frac{\beta_k}{2}\\-\cos\frac{\beta_k}{2}&-\mathrm{e}^{-\mathrm{i}(\varphi_k^A-\varphi_k^B)}\sin\frac{\beta_k}{2}&\mathrm{e}^{-\mathrm{i}\,\varphi_k^A}\cos\frac{\beta_k}{2}&-\mathrm{e}^{-\mathrm{i}\,\varphi_k^B}\sin\frac{\beta_k}{2}\end{pmatrix}\begin{pmatrix}a_k\\a_{-k}^\dagger\\b_k\\b_{-k}^\dagger\end{pmatrix}. \tag{11}$$

In the above expressions, we have defined

$$\cos\alpha_k=\frac{2\lambda-|A_k|}{\sqrt{(2\lambda-|A_k|)^2+|B_k|^2}},\quad\cos\beta_k=\frac{2\lambda+|A_k|}{\sqrt{(2\lambda+|A_k|)^2+|B_k|^2}}, \tag{12}$$

and $\varphi_k^A=\arg A_k$, $\varphi_k^B=\arg B_k$.

To this end, we have obtained the diagonalization of the Hamiltonian as

$$H=\sum_{i=1,2,3,4}\sum_k\xi_{k,i}\Big(\gamma_{k,i}^\dagger\gamma_{k,i}-\frac{1}{2}\Big). \tag{13}$$

The energy spectrum $\xi_{k,2} \geq 0$, $\xi_{k,4} \geq 0$ and $\xi_{k,1} \leq 0$, $\xi_{k,3} \leq 0$, thus the state which satisfies $\gamma_{k,1}|G\rangle = 0$, $\gamma_{k,2}^{\dagger}|G\rangle = 0$, $\gamma_{k,3}|G\rangle = 0$, $\gamma_{k,4}^{\dagger}|G\rangle = 0$, will correspond to the lowest energy, i.e., the ground state $|G\rangle$, which consists of occupied fermions $\xi_{k,1}$ and $\xi_{k,3}$ but no $\xi_{k,2}$ and $\xi_{k,4}$ fermions. Thus, the ground state of the system may be expressed as

$$|G\rangle=\frac{1}{\sqrt{C}}\prod_{k>0}(\gamma_{-k,1}^{\dagger}\gamma_{k,1}^{\dagger}\gamma_{-k,2}\gamma_{k,2}\gamma_{-k,3}^{\dagger}\gamma_{k,3}^{\dagger}\gamma_{-k,4}\gamma_{k,4})|0\rangle_a|0\rangle_b, \tag{14}$$

where $|0\rangle_a$ and $|0\rangle_b$ are the vacuum states of the fermion operators a and b, and the corresponding ground state energy is given by

$$E_0 = \sum_k(\xi_{k,1} + \xi_{k,3}). \tag{15}$$

It is easy to verify that in the case of $J_a = J_b$, $\gamma_a = \gamma_b$ and $D_a = D_b = 0$, the above diagonalization will be reduced to the usual result of the homogeneous Ising spin-chain.[1] In the case of no DM interaction, the spin chain has been studied systematically.[44,45,15] In Ref. [15], the geometric phase has been used to discuss the property of QPTs of the inhomogeneous periodic XY spin chain. In the following study, we will also use the geometric phase[12,13] approach of QPTs to study the effect of the DM interaction in the system.

3. Geometric phase obtained by a cyclic rotation

In order to investigate the geometric phase in this system, we use a new family of Hamiltonians that can be described by applying a rotation of ϕ around the z direction to each spin as in the method in Refs. [12] and [13], i.e.,

$$H_\phi = g_\phi H g_\phi^{\dagger} \tag{16}$$

with $g_\phi = \prod_{j=-M}^{M}\exp(\mathrm{i}\,\phi\sigma_j^z/2)$. The corresponding ground state of the new Hamiltonian may be obtained as

$$|G_\phi\rangle = g_\phi|G\rangle. \tag{17}$$

The geometric phase of the ground state accumulated by adiabatically varying the angle ϕ from 0 to π,

$$\beta_g = -\frac{\mathrm{i}}{M}\int_0^{\pi}\langle G_\phi|\frac{\partial}{\partial\phi}|G_\phi\rangle \mathrm{d}\phi,$$

can be found to be

$$\begin{aligned}\beta_g &= \frac{\pi}{M}\langle G|\sum_l(\sigma_{l,a}^z + \sigma_{l,b}^z)|G\rangle \\ &= \frac{\pi}{M}\langle G|\sum_{k>0}(2 - a_k^{\dagger}a_k - a_{-k}^{\dagger}a_{-k} \\ &\quad - b_k^{\dagger}b_k - b_{-k}^{\dagger}b_{-k})|G\rangle.\end{aligned} \tag{18}$$

The above expression can be worked out with the help of the inverse matrix of U_k, $\Gamma_k = U^{-1}\Gamma_k'$, and U_k^{-1} can be expressed as

$$U_k^{-1} = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathrm{e}^{-\mathrm{i}\,\varphi_k^A}\sin\frac{\alpha_k}{2} & \cos\frac{\alpha_k}{2} & -\mathrm{e}^{-\mathrm{i}\,\varphi_k^A}\sin\frac{\beta_k}{2} & -\cos\frac{\beta_k}{2} \\ \mathrm{e}^{-\mathrm{i}\,\varphi_k^B}\cos\frac{\alpha_k}{2} & -\mathrm{e}^{\mathrm{i}(\varphi_k^A-\varphi_k^B)}\sin\frac{\alpha_k}{2} & \mathrm{e}^{-\mathrm{i}\,\varphi_k^B}\cos\frac{\beta_k}{2} & -\mathrm{e}^{\mathrm{i}(\varphi_k^A-\varphi_k^B)}\sin\frac{\beta_k}{2} \\ \sin\frac{\alpha_k}{2} & \mathrm{e}^{\mathrm{i}\,\varphi_k^A}\cos\frac{\alpha_k}{2} & \sin\frac{\beta_k}{2} & \mathrm{e}^{\mathrm{i}\,\varphi_k^A}\cos\frac{\beta_k}{2} \\ -\mathrm{e}^{-\mathrm{i}(\varphi_k^A-\varphi_k^B)}\cos\frac{\alpha_k}{2} & \mathrm{e}^{\mathrm{i}\,\varphi_k^B}\sin\frac{\alpha_k}{2} & \mathrm{e}^{-\mathrm{i}(\varphi_k^A-\varphi_k^B)}\cos\frac{\beta_k}{2} & -\mathrm{e}^{\mathrm{i}\,\varphi_k^B}\sin\frac{\beta_k}{2} \end{pmatrix}. \tag{19}$$

Substituting the above expression (19) into the geometric phase expression (18) and noting that $\gamma_{k,1}|G\rangle = 0, \gamma_{k,2}^{\dagger}|G\rangle = 0, \gamma_{k,3}|G\rangle = 0$, and $\gamma_{k,4}^{\dagger}|G\rangle = 0$, we finally obtained the geometric phase of the ground state,

$$\beta_g = \frac{\pi}{M}\sum_{k>0}(\cos\alpha_k + \cos\beta_k). \tag{20}$$

By this expression, we can numberically study the effect of the DM interaction on the geometric phase of the ground state. It has been pointed out in Ref. [13] that the derivative of the geometric phase with respect to the field strength could reflect the typical features of quantum criticality. In the following numerical studies, we will focus on the properties of the derivative of the geometric phase $\partial\beta_g/\partial\lambda$ as a function of different parameters in the Hamiltonian.

In Fig. 2, we plotted the derivative of the geometric phase as a function of λ and $\kappa_J = J_b/J_a = \kappa_{\rm D} = D_b/D_a$, where we have set $N = 501$, $\gamma_a = \gamma_b = 1$, i.e., the case of the Ising interaction. Comparing figures (a) and (b) in Fig. 2, we can find that the DM interaction could change the region of critical points, but it could not produce new critical points. This result is consistent with the case of a homogeneous chain, where the DM interaction can only change the energy level of the system but could not change the energy structure.[46,47] The DM interaction in the homogeneous chain will not change the Bogoliubov transformation parameter and could not affect the structure of QPTs,[47] while in the inhomogeneous case, the structure of QPTs has been changed. This effect can also be found in the case of fixed DM interaction $D_a = D_b = 1$, which has been shown in Fig. 3, and we can compare the case of existence of DM interaction with the case of absent DM interaction. The DM interaction could change the region of the critical point.

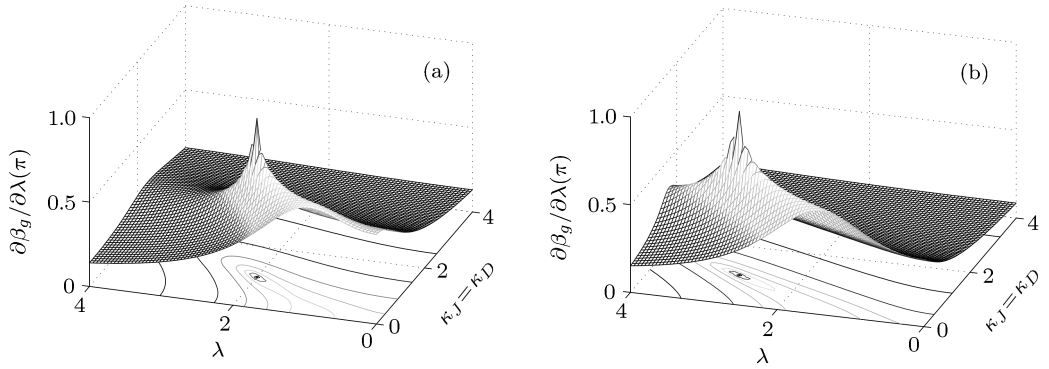


Fig. 2. The derivative of the geometric phase $\partial\beta_g/\partial\lambda$ versus the transverse strength λ and the inhomogeneous parameter κ_J of the chain. (a) $\kappa_J = J_a/J_b$, $D_a = D_b = 0$, $J_a = 1$; (b) $\kappa_J = J_b/J_a = \kappa_D = D_b/D_a$, $J_a = 1$, $D_a = 1$. The other parameters have been set as $\gamma_a = \gamma_b = 1$, and $N = 501$.

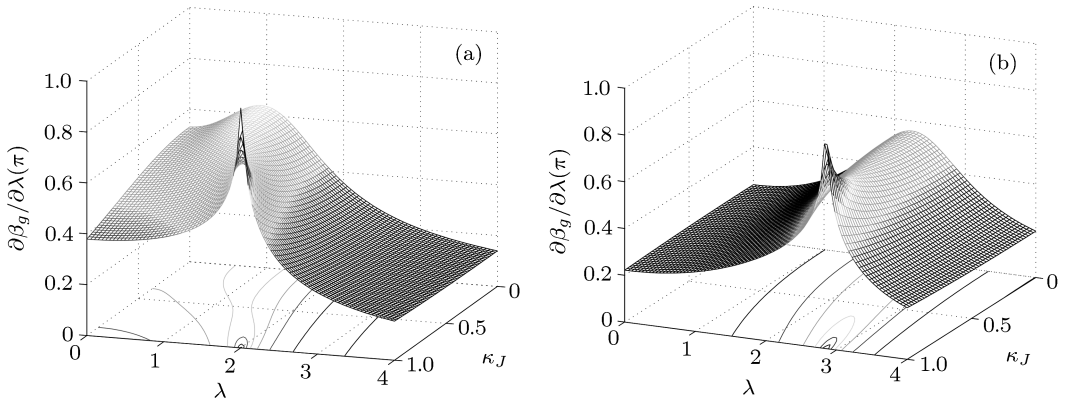


Fig. 3. The derivative of geometric phase versus the transverse strength λ and κ_J, where $N = 501$, $\kappa_J = J_b/J_a$, $\gamma_a = \gamma_b = 1$. (a) $D_a = D_b = 0$; (b) $D_a = D_b = 1$.

We can find the effect of the DM interaction more clearly from Fig. 4, where we have chosen the case of fixed J_a and J_b, and plot the geometric phase as a function of $\kappa_{\rm D} = D_a/D_b$ and the strength of the transverse field λ with fixed $\kappa_J = J_a/J_b = 1$ and $\gamma_a = \gamma_b = 1$. As the figures show, the peak of the curves of geometric phase will change with κ. But in all these cases, we can find that the DM interaction cannot introduce new critical points.

In the case of XY interaction, we can also study the geometric phase and the critical phenomenon with DM interaction, as was shown in Fig. 5, where we have plotted the case of different anisotropy parameters $\gamma_a(\gamma_b)$ and $D_a(D_b)$. It has been pointed out in Ref. [15] that the inhomogeneous XY spin chain may exist two quantum phase transition points in some parameter regions, which is quite different from those of the homogeneous Ising model or XY model. In Fig. 5, the effect of DM interaction on the two QPT points can be shown clearly in the derivative of the geometric phase. From Figs. 5(a)–5(d), we consider different kinds of DM interactions,

and it has been clearly shown that, when the DM interaction is added, the transverse field should be stronger and when the QPTs of the system happen, the distance between the two critical points will be shortened. This effect tells us that the DM interaction may influence the energy structure of the system. Unlike the case of the homogeneous spin chain where DM interaction only changes the energy level and does not change the coefficients of the Bogoliubov transformation,[47] both the energy level and the coefficient of Bogoliubov transformation of the inhomogeneous spin chain will be affected by DM interaction.

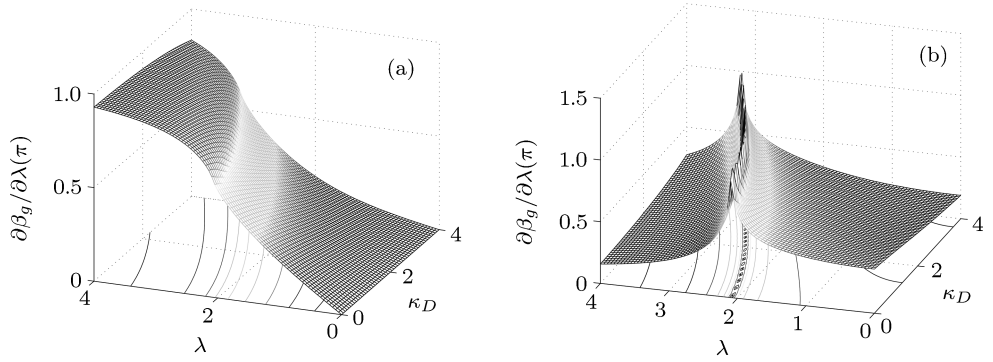


Fig. 4. Geometric phase (a) and its derivative (b) versus the transverse strength λ and κ_D. The other parameters have been set as $N = 501$, $J_a = J_b = 1$, $D_a = 0.5$, $\kappa_D = D_b/D_a$, $\gamma_a = \gamma_b = 1$.

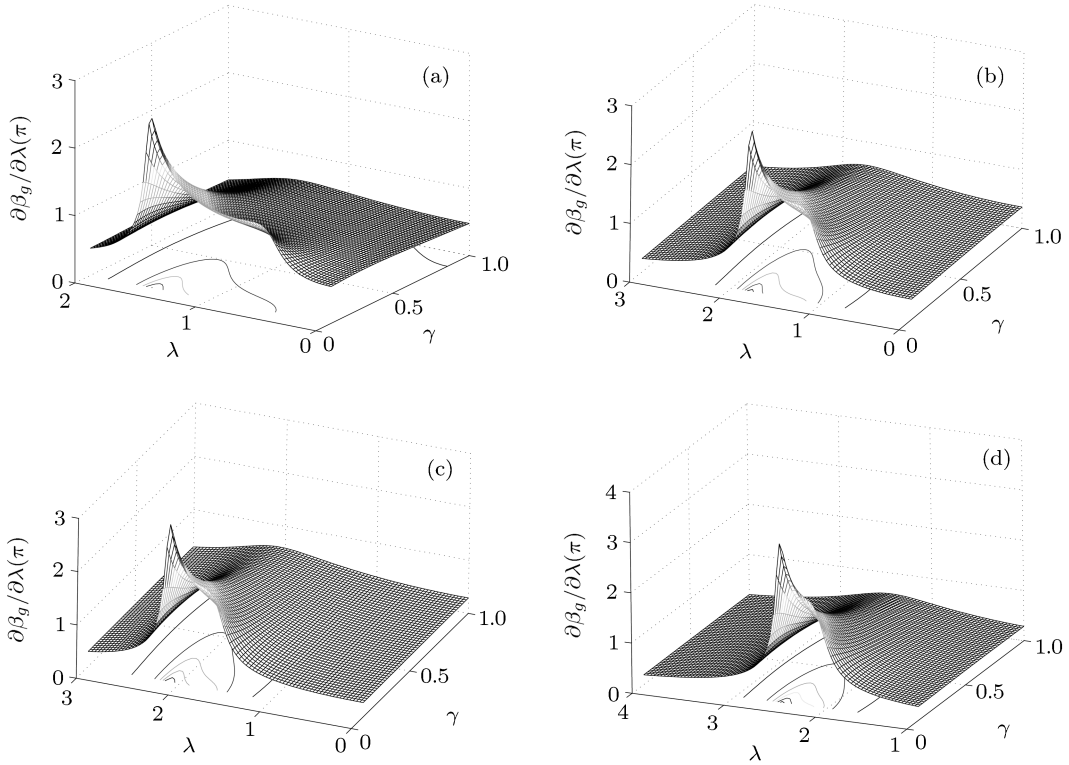


Fig. 5. The derivative of geometric phase versus λ and γ. (a) $D_a = D_b = 0$; (b) $D_a = 0.5, D_b = 0.5$; (c) $D_a = 0.5, D_b = 1.0$; (d) $D_a = D_b = 1$. The other parameters have been set as $N = 501$, $J_a = 1, J_b = 0.5$, $\gamma = \gamma_a = \gamma_b$.

4. Conclusion

In summary, we have studied the geometric phases of the ground states of inhomogeneous XY spin chains in transverse fields with the DM interaction in this paper. By this approach, we have obtained general properties of QPTs for the model, and have shown that the DM interaction may change the location of the critical region while it could not introduce any new critical points of the spin chain. This result is also an extension of the study on the relation between geometric phase and QPTs in one-dimensional spin chains.

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References

[1] Sachdev S 1999 *Quantum Phase Transition* (Cambridge: Cambridge University Press)
[2] Berry M V 1984 *Proc. R. Soc. London* A **392** 45
[3] Aharonov Y and Anandan J 1987 *Phys. Rev. Lett.* **58** 1593
[4] Samuel J and Bhandari R 1988 *Phys. Rev. Lett.* **60** 2339
[5] Mukunda N and Simon R 1993 *Ann. Phys.* (N.Y.) **228** 205
[6] Mukunda N and Simon R 1993 *Ann. Phys.* (N.Y.) **228** 269
[7] Pati A K 1995 *Phys. Rev.* A **52** 2576
[8] Sjöqvist E, Pati A K, Ekert A, Anandan J S, Erisson M, Oi D K L and Vedral V 2000 *Phys. Rev. Lett.* **85** 2845
[9] Bhandari R 2002 *Phys. Rev. Lett.* **89** 268901
[10] Anandan J 2002 *Phys. Rev. Lett.* **89** 268902
[11] Tong D M, Sjöqvist E, Kwek L C and Oh C H 2004 *Phys. Rev. Lett.* **93** 080405
[12] Carollo A C M and Pachos J K 2005 *Phys. Rev. Lett.* **95** 157203
[13] Zhu S L 2006 *Phys. Rev. Lett.* **96** 077206
[14] Chen G, Li J Q and Liang J Q 2006 *Phys. Rev.* A **74** 054101
[15] Ma Y Q and Chen S 2009 *Phys. Rev.* A **79** 022116
[16] Zanardi P and Paunković N 2006 *Phys. Rev.* E **74** 031123
[17] Zanardi P, Quan H T, Wang X and Sun C P 2007 *Phys. Rev.* A **75** 032109
[18] Zhou H Q and Barjaktarevič J P 2007 arXiv: 0701608 [cond-mat]
[19] Campos Venuti L and Zanardi P 2007 *Phys. Rev. Lett.* **99** 095701
[20] Zanardi P, Giorda P and Cozzini M 2007 *Phys. Rev. Lett.* **99** 100603
[21] You W L, Li Y W and Gu S J 2007 *Phys. Rev.* E **76** 022101
[22] Osborne T J and Nielsen M A 2002 *Phys. Rev.* A **66** 032110
[23] Wu L A, Sarandy M S and Lidar D A 2004 *Phys. Rev. Lett.* **93** 250404
[24] Gu S J, Deng S S, Li Y Q and Lin H Q 2004 *Phys. Rev. Lett.* **93** 086402
[25] Cui H T, Song H S and Yu C S 2008 *Chin. Phys.* B **17** 2795 2799
[26] Cheng W W, Shan C J, Huang Y X, Li H and Liu T K 2008 *Chin. Phys.* B **17** 4002 4008
[27] Tong P Q and Zhu X 2008 *Chin. Phys.* B **17** 1623
[28] Quan H T, Song Z, Liu X F, Zanardi P and Sun C P 2006 *Phys. Rev. Lett.* **96** 140604
[29] Yi X X, Cui H T and Wang L C 2006 *Phys. Rev.* A **74** 054102
[30] Yuan Z G, Zhang P and Li S S 2007 *Phys. Rev.* A **75** 012102
[31] Yi X X and Wang W 2007 *Phys. Rev.* A **75** 032103
[32] Wang L C, Huang X L and Yi X X 2008 *Euro. Phys. J.* D **46** 345 349
[33] Li Y C and Li S S 2009 *Phys. Rev.* A **79** 032338
[34] Jafari R, Kargarian M, Langari A and Siahatgar M 2008 arXiv: 0804.4579 [cond mat]
[35] Kargarian M, Jafari R and Langari A 2009 arXiv: 0903.0630 [quant ph]
[36] Dzyaloshinskii I 1958 *J. Phys. Chem. Solids* **4** 241
[37] Moriya T 1960 *Phys. Rev. Lett.* **4** 228
[38] Moriya T 1960 *Phys. Rev.* **120** 91
[39] Anderson P W 1959 *Phys. Rev.* **115** 2
[40] Zhang G F 2007 *Phys. Rev.* A **75** 034304
[41] Gurkan Z N and Pashev O K 2008 arXiv: 0804.0710 [quant ph]
[42] Kwan M K, Gurkan Z N and Kwek L C 2008 *Phys. Rev.* A **77** 062311
[43] Gao F, Tang S Q, Xie L J, Zhan X G and Zhang D Y 2009 *Chin. Phys.* B **18** 3203
[44] Lima J P, Goncalves L L and Alves T F A 2007 *Phys. Rev.* B **75** 214406
[45] Lima J P and Goncalves L L 2008 *Phys. Rev.* B **77** 214424
[46] Derzhko O, Verkholyak T, Krokhmalskii T and Büttner H 2006 *Physica* B **378** 443
[47] Cheng W W and Liu J M 2009 *Phys. Rev.* A **79** 052320