Traffic flow of a roundabout crossing with an open boundary condition*

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This paper presents a cellular automaton traffic flow model with an open boundary condition to describe the traffic flow at a roundabout crossing with an inner roundabout lane and an outer roundabout lane. The simulation results show that the boundary condition, bottlenecks and the self-organization affect the traffic flow at the roundabout crossing. Because of the effect of bottlenecks, jams easily appear on the inner roundabout lane. To improve the capacity of the roundabout system, proper values of the enter probability α and the out probability β can be chosen.

Keywords: roundabout crossing, cellular automaton model, bottleneck, traffic flow
PACC: 0550, 0520

1. Introduction

Since the 1980s, cellular automaton (CA) has been applied to research into traffic flow. It can be modified to deal with the effects of kinds of realistic conditions easily, such as driver's behaviour, the style of the vehicles, and the road status, etc. Many CA traffic models have been presented, and a great deal of meaningful results have been given.[1−10] In the city traffic network, the traffic control of the road intersection is very important. There are two control modes: traffic light control and no signal control (e.g. the roundabout crossing). In this paper, we pay attention to the traffic flow of the roundabout crossing. In Ref. [11], a CA traffic model of the roundabout crossing is presented. It takes the roundabout crossing as a closed polygon and the traffic capacity of this model is investigated. The results are compared with the traffic light control intersections. In Refs. [12] and [13], the CA model is used with the offside-priority rule, and the traffic flows of a roundabout crossing with single-circle-lane and multi-circle-lane are discussed respectively. To the roundabouts with the same geometry structures, the different traffic rules will induce different capacities. Reference [14] adopts a determinate CA model and transforms the roundabout crossing into several bottlenecks. The traffic flow properties at the roundabout crossing are discussed. In the above references, the boundary condition is periodic. In this paper, we present a new CA traffic model of the roundabout crossing with an open boundary condition. The influence of the boundary condition is studied.

2. Model and evolution rules

The sketch map of the realistic roundabout crossing is shown in Fig. 1. The lane nearest the central island named the inner roundabout lane is for the vehicles moving round the island, and the outer lane named the outer roundabout lane is for the vehicles moving to the right. Based on the traffic rules and CA model rules, we present a CA traffic flow model of the roundabout crossing (see Fig. 2). This model consists of the inner roundabout lane, the outer roundabout lane, the driving-in lane and the driving-out lane. The inner roundabout lane is closed by eight segments; the outer roundabout lane consists of four separate segments; there are four driving-in lanes and four driving-out lanes of the roundabout crossing. The structure of the roundabout crossing is symmetric. According to the traffic rules of China, the driving direction is shown in Fig. 2. Therefore, there are 8 division points (hollow points) and 8 confluence points (solid points).

In this model, the evolution rules are as follows:

(i) The vehicles obey the updating rules of the NaSch model[15] at each time step $t \to t+1$. There are four substeps:

Step 1 Acceleration: $V_i \to \min(V_{\max}\,,\ V_i+1)$;

Step 2 Braking: $V_i \to \min(V_i\,,\ d_i)$;

Step 3 Randomization: $V_i \to \max(0\,,\ V_i-1)$ with probability P_d,

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Step 4 Driving: $x_i \to x_i + v_i$,

where x_i and V_i denote the position and velocity of the i-th vehicle respectively. Then, d_i is the number of empty cells in front of it. The maximum velocity and the brake parameter are denoted as $V_{\max}$ and P_d respectively. There is no lane changing or overtaking either on the inner lane or the outer lane.

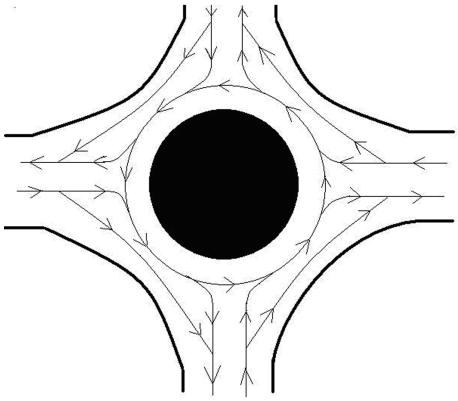


Fig. 1. Sketch map of a realistic roundabout crossing.

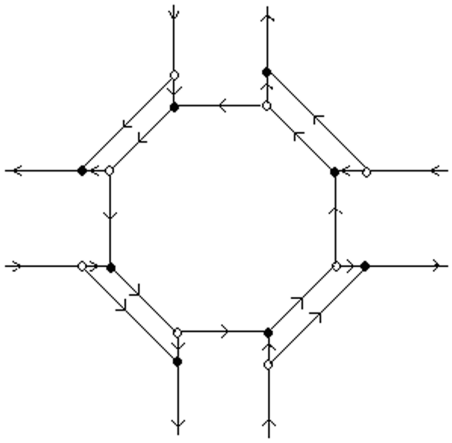


Fig. 2. CA traffic flow model of the roundabout crossing. The symbol ♦ denotes the confluence point; the symbol ◊ denotes the division point.

(ii) The vehicle which drives along the driving-in lane can turn right or left, or drive straight at the roundabout.

(iii) When the vehicles meet at the confluence point, the occupation rule is the same as that in Ref. [16]. That is, if two vehicles meet at the confluence point, the one arriving earlier has priority and will not be affected by the other vehicle. Then the later vehicle updates after the movement of the earlier vehicle in this time step. If two vehicles arrive at the confluence point simultaneously, the vehicle on the inner lane has priority.

(iv) To prevent the locked state, we also establish the following stipulations. If the vehicle at the division point has a velocity of 0, it will turn right with a certain probability P_c though it does not arrive at its intended outlet.

The boundary condition is defined as follows:[17,18] there will appear a vehicle with the enter probability α and velocity $V_i = V_{\max}$ at the beginning of the driving-in lane in each time step. If its velocity in the next time step is 0, it should be cancelled. The vehicle at the end of the driving-out lane simply moves out of the system with out probability β in each time step.

3. Numerical simulations and analysis

In our model, each lane of the roundabout crossing consists of L_i lattices (each lattice denotes 7.5 m). The distance of the driving-in lane and the driving-out lane is 7.5 m in the same direction. In the simulations, we let the length of each driving-in (denoted as $L_{\rm E}$) and driving-out (denoted as $L_{\rm G}$) lane be 200 lattices. The length of each segment of the outer roundabout lane is 5 lattices. So, there are 20 lattices for the whole outer roundabout lane. The length of the whole inner roundabout lane is 16 lattices. For a vehicle on a certain driving-in lane, we choose the other three driving-out lanes as its goal with the same probability. The maximal velocity $V_{\max} = 3$, the turning-probability $P_c = 0.25$, the brake probability $P_d = 0.20$.

In the roundabout system, there are 8 confluence points and 8 division points. They are similar to the on-ramp system and the off-ramp system respectively. According to the viewpoint of Ref. [19], the on-ramp and off-ramp are both bottlenecks. We can regard these 16 points as bottlenecks of the roundabout crossing system. Accounting for the structure of the inner roundabout lane, the distance between two neighbouring bottlenecks is so short that once a congested traffic flow is on the inner roundabout lane, an expanded congested pattern will occur (see Ref. [16]). The nonlinear interaction is stronger than that in other road structures. It is very important to expedite the traffic flow of the inner roundabout lane. On the other hand, the open boundary conditions will also strongly influence the whole system's traffic flow. So we will mainly discuss the traffic flow in the inner roundabout lane and driving-in and driving-out lanes under the open boundary conditions.

Before the analysis, we define the parameters as follows. On the i-th lane, the number of vehicles at time t is $N_i(t)$, the mean density

$$\rho_i(t) = N_i(t)/L_i, \tag{1}$$

the mean velocity

$$V_i(t) = \sum_{j=1}^{N_i(t)} V_i(j,t)/N_i(t), \tag{2}$$

and the mean flow

$$J_i(t) = \rho_i(t)V_i(t) = \sum_{j=1}^{N_i(t)} V_i(j,t)/L_i. \tag{3}$$

For the whole system, the mean density

$$\rho_0(t) = \sum_{i=1}^{16} N_i(i,t) \Big/ \sum_{i=1}^{16} L_i, \tag{4}$$

the mean velocity

$$V_0(t) = \sum_{i=1}^{16} \sum_{j=1}^{N_i(t)} V_i(j,t) \Big/ \sum_{i=1}^{16} N_i(t), \tag{5}$$

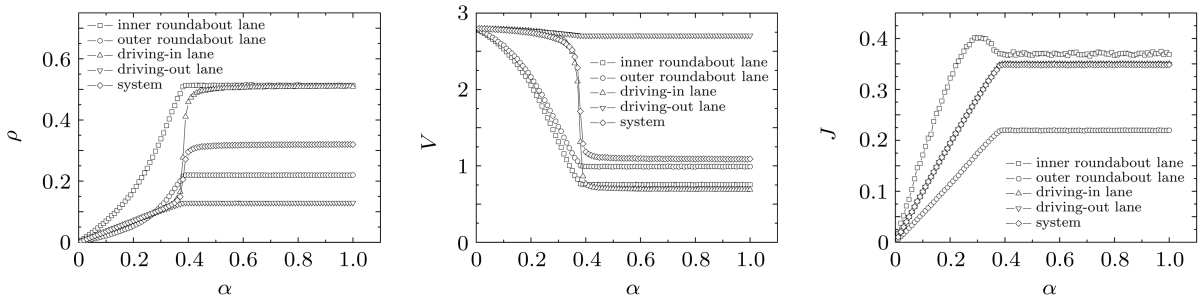
the mean flow

$$J_0(t) = \rho_0(t)\, V_0(t). \tag{6}$$

In the simulation, each sample runs 50000 time steps, the average of the numerical results versus time is taken from the last 10000 time steps, and 20 samples are taken for the assembly average to eliminate the influence of randomness.

3.1. Traffic flow for $\beta = 1$ and α changing

When $\beta = 1$, the vehicle can drive out of the roundabout system freely. Then, we analyse the influence of the enter probability α on the traffic flow. Figure 3 shows the mean density, velocity and flux of the system and each lane changes with the increase of α.



(a) density (b) velocity (c) flux

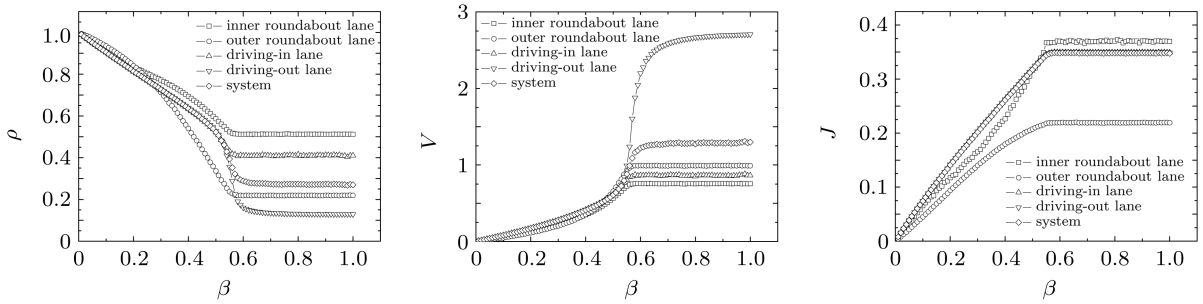
Fig. 3. Mean density, velocity, flux of system and each lane versus α while $\beta = 1$, $P_d = 0.20$.

When α is less than 0.39, the traffic flows of the system and each lane change with different α. When α increases, the mean densities and fluxes increase, and the mean velocities decrease. When α is larger than 0.39, the traffic parameters (ρ, V, J) do not change with different α. Their values have a platform for α from 0.39 to 1.0.

Looking at Fig. 3 in detail, we can see that the density of the driving-out lane is always lower, the vehicles can move freely with high velocity (about 2.8). This is because of the boundary condition $\beta = 1$. The density of the driving-in lane is also lower when $\alpha < 0.39$, but it changes to a high value when $\alpha > 0.39$. This means that the lane is congested. The changing trends of traffic flow on the inner and outer roundabout lanes are the same with increasing α. But the density and flux of the inner roundabout lane is always higher than that of the outer roundabout lane. This is opposite to their velocities. From Fig. 3, we can see that the inner roundabout lane and driving-in lane are more easily congested than the outer roundabout and driving-out lanes. The enter probability α strongly influences the inner lane and driving-in lane.

3.2. Traffic flow for $\alpha = 0.39$ and change of β

According to the above results, we let $\alpha = 0.39$ and change β to investigate the influence of the out-probability of exit boundary conditions. Figure 4 shows the mean density, velocity and flux of the system and each lane changes with increasing β. We can see that with the increase of β, the mean densities decrease for $\beta < 0.56$ and then become constants for $\beta > 0.56$, but the mean velocities and fluxes increase and then become constants. These changes relate to β. When β is small, the vehicles cannot move out of the system freely. They will assemble at the exits. So jams appear; the mean velocities are very low. When β increases, the driving-out lane will be expedited. So its density decreases rapidly and velocity increases a lot. Then its upstream traffic (inner lane, outer lane and driving-in lane) will be expedited too. For $\beta > 0.56$, the flux of the inner lane is the highest, the outer lane is the lowest.



(a) density (b) velocity (c) flux

Fig. 4. Mean densities, velocities, fluxes of the system and each lane versus β when $\alpha = 0.39$, $P_d = 0.20$.

4. Conclusions

A CA traffic flow model with an open boundary condition for a roundabout system is presented. The characters of the traffic flow are investigated. The simulation results show that the boundary conditions and road bottlenecks are important influencing factors on the traffic flow of the roundabout crossing. To improve the capacity of the roundabout system, we can choose proper values of α and β, such as those above: $\beta = 1$, $\alpha > 0.39$ or $\alpha = 0.39$, $\beta > 0.56$.

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