Stochastic resonance in a time-delayed asymmetric bistable system with mixed periodic signal*

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This paper studies the phenomenon of stochastic resonance in an asymmetric bistable system with time-delayed feedback and mixed periodic signal by using the theory of signal-to-noise ratio in the adiabatic limit. A general approximate Fokker–Planck equation and the expression of the signal-to-noise ratio are derived through the small time delay approximation at both fundamental harmonics and mixed harmonics. The effects of the additive noise intensity Q, multiplicative noise intensity D, static asymmetry r and delay time τ on the signal-to-noise ratio are discussed. It is found that the higher mixed harmonics and the static asymmetry r can restrain stochastic resonance, and the delay time τ can enhance stochastic resonance. Moreover, the longer the delay time τ is, the larger the additive noise intensity Q and the multiplicative noise intensity D are, when the stochastic resonance appears.

Keywords: stochastic resonance, time-delayed feedback, mixed periodic signal, signal-to-noise ratio
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1. Introduction

Since Benzi *et al.*[1] and C. Nicolis and G. Nicolis[2,3] first proposed the concept of stochastic resonance (SR) in 1981, SR has been widely investigated experimentally and theoretically. Fauve and Heslot[4] studied the experiment of an AC-driven Schmitt trigger, and they observed the SR through this experiment for the first time. Afterward, McNamara *et al.*[5,6] also observed the SR by a bidirectional ring laser and obtained the expression of signal-to-noise ratio (SNR) under the adiabatic limit. Dykman *et al.*[7] and Hu *et al.*[8,9] introduced the linear-response theory and perturbation theory. Zhou and Moss[10] employed the residence-time distribution to explain the SR phenomenon. Gammaitoni *et al.*[11] reported, interpreted and extended much of the understanding related to the theory and physics of SR, furthermore, they introduced the basic features of SR and its recent history to readers. By applying to these methods, the SR has been studied in a variety of linear and non-linear systems with different kinds of noises and the single periodic modulated signal, such as Refs. [12]–[18]. Later, the SR in a bistable system with mixed periodic modulated signal was investigated in Refs. [19]–[22], and the SR in neuronal systems and pulse-coupled electronic neurons systems driven by mixed frequency signals have been discussed in Refs. [23] and [24]. However, most studies of SR neglect the possible effects caused by time delays.

It is well known that time-delayed feedback plays a significant role in dynamical systems. The time delay usually arises due to the transport of information, matter, and energy with finite propagation time. Recently, there has been growing interest in the effects of noise on dynamical systems with delays.[25−32] For instance, a systematic statistical description of a certain class of stochastic delay-differential equations have been developed in the limit of small time delay in Refs. [28] and [29], and the SR in a bistable system with time delayed feedback and single periodic modulated signal has been studied in Refs. [30]–[32]. However, the phenomenon of SR in an asymmetric bistable systems with time-delayed feedback and mixed periodic signal has not been studied yet.

In this paper, the phenomenon of SR in a time-delayed asymmetric bistable system with mixed periodic signal is investigated. A general approximate Fokker–Planck equation and the expression of the SNR at both fundamental harmonics and mixed harmonics are derived through the small time delay approximation and two-state theory in Section 2. The effects of the additive noise intensity Q, multiplicative noise intensity D, static asymmetry r and delay time τ on the SNR are discussed in Section 3. Finally, some conclusions are drawn in Section 4.

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2. The SNR of a time-delayed asymmetric bistable system

We consider a one-dimensional asymmetric bistable system with time-delayed feedback and mixed periodic modulated signal. The relevant Langevin equation is as follows:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = -\frac{\partial U(x_\tau)}{\partial x_\tau} + x(t)\xi(t) + \eta(t) + r + H(t),$$
$$U(x_\tau) = -\frac{a}{2}x_\tau^2 + \frac{b}{4}x_\tau^4,\ -\frac{\partial U(x_\tau)}{\partial x_\tau} = ax_\tau - bx_\tau^3, \quad (1)$$

where $x_\tau = x(t-\tau)$, τ is the delay time of the system (1), $\xi(t)$ and $\eta(t)$ are uncorrelated Gaussian white noise terms, constant r characterizes the bias of the bistable system, $H(t)$ denotes mixed periodic signal. The noise term is characterized by their mean and variance

$$\begin{aligned}
&\langle \xi(t)\rangle = \langle \eta(t)\rangle = 0,\\
&\langle \xi(t)\xi(t')\rangle = 2D\delta(t-t'),\\
&\langle \eta(t)\eta(t')\rangle = 2Q\delta(t-t'),\\
&\langle \xi(t)\eta(t')\rangle = \langle \eta(t)\xi(t')\rangle = 0,
\end{aligned} \quad (2)$$

where D and Q are the multiplicative and additive noise intensities, respectively. The mixed periodic signal $H(t)$ is given by

$$H(t) = f_1\cos(\omega_1 t) + f_2\cos(\omega_2 t). \quad (3)$$

Then equation (1) can be equivalently written as a stochastic equation

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = ax_\tau - bx_\tau^3 + B_0(x)\zeta(t) + r + H(t), \quad (4)$$

where $\zeta(t)$ is Gaussian white noise with

$$\begin{aligned}
&\langle \zeta(t)\rangle = 0,\\
&\langle \zeta(t)\zeta(t')\rangle = 2\delta(t-t'),
\end{aligned} \quad (5)$$

and

$$B_0(x) = \left(Dx^2 + Q\right)^{1/2}. \quad (6)$$

Using the small time delay approximation,[28,29] we can approximate Eq. (4) as

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = A(x) + B(x)\zeta(t), \quad (7)$$

where

$$\begin{aligned}
A(x) &= \left(ax - bx^3 + r + H(t)\right)\left(1 - a\tau + 3b\tau x^2\right),\\
B(x) &= B_0(x)\left(1 - a\tau + 3b\tau x^2\right).
\end{aligned} \quad (8)$$

Thus, the Fokker–Planck equation corresponding to Eq. (1) can be approximately written as

$$\begin{aligned}
&\frac{\partial P(x,t)}{\partial t}\\
&= -\frac{\partial}{\partial x}\left[F(x)P(x,t)\right] + \frac{\partial^2}{\partial x^2}\left[D(x)P(x,t)\right],
\end{aligned} \quad (9)$$

where

$$\begin{aligned}
F(x) &= A(x) + B(x)\frac{\mathrm{d}B(x)}{\mathrm{d}x},\\
D(x) &= B^2(x).
\end{aligned} \quad (10)$$

So the quasi-steady-state probability distribution of system (1) can be expressed as

$$\begin{aligned}
P_{\mathrm{st}}(x) &= \frac{N}{D(x)}\exp\left\{\int^x \frac{F(x')}{D(x')}\mathrm{d}x'\right\}\\
&= \frac{N}{B(x)}\exp\left\{\int^x \frac{A(x')}{B^2(x')}\mathrm{d}x'\right\}\\
&= \frac{N}{B(x)}\exp\left\{-\frac{\tilde{U}(x)}{D}\right\},
\end{aligned} \quad (11)$$

where N is the normalization constant, $\tilde{U}(x)$ is the rectified potential function. Letting $a = b = 1$, and $R = D/Q$, the form of $\tilde{U}(x)$ can be expressed as

$$\begin{aligned}
\tilde{U}(x) = &-\frac{(R+1)\ln\left(Rx^2+1\right)}{2(R - R\tau - 3\tau)} + \frac{R(1+2\tau)\ln\left(1-\tau+3\tau x^2\right)}{6\tau(R - R\tau - 3\tau)}\\
&- (r + H(t))\frac{R\sqrt{R(1-\tau)}\arctan\left(\sqrt{R}x\right) - R\sqrt{3\tau}\arctan\left(\sqrt{3\tau/(1-\tau)}x\right)}{(R - R\tau - 3\tau)\sqrt{1-\tau}}.
\end{aligned} \quad (12)$$

Linearizing the unperturbed part of system (1) around $x_\pm = \pm 1$ and $x_0 = 0$ respectively and applying the stability analysis performed for the delayed linear equation presented in Refs. [28] and [29], we can know that when $\tau < \pi/4$ the fixed points $x_\pm = \pm 1$ are stable and the fixed point $x_0 = 0$ is unstable. Thus, in the following discussions, the time delay is assumed for $\tau < \pi/4$ to ensure that the system is still bistable.

Under the adiabatic limit condition, the transition rate out of $x_\pm$ can be obtained by

$$W_\pm = \frac{|U_0''(x_0)\,U_0''(x_\pm)|}{2\pi}\exp\left[\frac{\tilde{U}(x_\pm)-\tilde{U}(x_0)}{D}\right] = W_{\pm 0}\exp\left(\mp qH(t)\right), \tag{13}$$

where $U_0(x)$ denotes a symmetric bistable potential function of the unperturbed system (1),

$$W_{\pm 0} = \frac{1}{\sqrt{2}\pi}\exp\left[-\left(\frac{\Delta p}{2D}\pm rq\right)\right],\quad \Delta p = \left[\frac{(R+1)\ln(R+1)}{R-R\tau-3\tau} - \frac{R(1+2\tau)}{3\tau(R-R\tau-3\tau)}\ln\left(\frac{1+2\tau}{1-\tau}\right)\right],$$
$$q = \frac{R\sqrt{R(1-\tau)}\arctan\left(\sqrt{R}\right) - R\sqrt{3\tau}\arctan\left(\sqrt{3\tau/(1-\tau)}\right)}{D(R-R\tau-3\tau)\sqrt{1-\tau}}. \tag{14}$$

The bistable case can be regarded as a two-state system.[3,6] The master equation for occupation probabilities $n_\pm$ of the stable states $x_\pm$ is found to be

$$\frac{\mathrm{d}n_+}{\mathrm{d}t} = -\frac{\mathrm{d}n_-}{\mathrm{d}t} = W_-(t)\,n_- - W_+(t)\,n_+. \tag{15}$$

The distribution density $p(x,t)$ of the two-state system can be written as

$$p(x,t) = n_+(t)\,\delta(x-x_+) + n_-(t)\,\delta(x-x_-). \tag{16}$$

Based on the adiabatic limit condition, we can expand Eq. (13) in series with the small parameter $\varepsilon = qH(t)$, and then combining with Eq. (15), we can obtain the expressions of $n_\pm$. The averaged autocorrelation function is given by

$$\begin{aligned}&\langle\langle x(t)\,x(t+\tau)\rangle\rangle_t\\ &= \frac{\omega}{2\pi}\int_0^{2\pi/\omega}\lim_{t_0\to-\infty}\left[x_+^2 n_+(t+\tau|\,x_+,t)\,n_+(t|\,x_0,t_0)\right.\\ &\quad + x_+x_-n_+(t+\tau|\,x_-,t)\,n_-(t|\,x_0,t_0)\\ &\quad + x_+x_-n_-(t+\tau|\,x_+,t)\,n_+(t|\,x_0,t_0)\\ &\quad \left.+ x_-^2 n_-(t+\tau|\,x_-,t)\,n_-(t|\,x_0,t_0)\right].\end{aligned} \tag{17}$$

By performing the Fourier transform of the autocorrelation function, we can get the expressions of the power spectrum $S(\omega)$ for the fundamental harmonic in the case of $f_1 \neq 0$ and $f_2 = 0$, i.e.

$$S(\omega) = S_1(\omega) + S_2(\omega), \tag{18}$$

where

$$\begin{aligned}S_1(\omega) &= \left(\frac{1}{\cosh^2(rq)} - \frac{2\alpha^2}{\beta^2+\omega_1^2}\right)\cdot\frac{4\beta}{\beta^2+\omega^2},\\ S_2(\omega) &= \frac{4\pi\alpha^2}{\beta^2+\omega_1^2}\delta(\omega-\omega_1),\\ \alpha &= \frac{f_1 q}{\sqrt{2}\pi\cosh(rq)}\exp\left(-\frac{\Delta p}{2D}\right),\\ \beta &= \frac{\sqrt{2}}{\pi}\exp\left(-\frac{\Delta p}{2D}\right)\cosh(rq),\end{aligned} \tag{19}$$

where $S_1(\omega)$ is the power density associated with the noise background in the adiabatic limit case, $S_2(\omega)$ is the power density connected with the output signal.

Referring to the method of Refs. [15] and [21], we can get the expression of the SNR for the fundamental harmonic of the delayed asymmetric bistable system as

$$\mathrm{SNR} = \frac{\pi f_1^2 q^2}{2\sqrt{2}\cosh(rq)}\exp\left(-\frac{\Delta p}{2D}\right)\left[1 - \frac{2f_1^2q^2\exp(-\Delta p/D)}{\pi(\omega^2+2\exp(-\Delta p/D))\cosh^2(rq)/\pi^2}\right]^{-1}. \tag{20}$$

Similarly, the expression of SNR for higher harmonics in the case of weak modulation force $\omega_1, \omega_2 \ll W_{+0}+W_{-0}$ is obtained by

$$\begin{aligned}\mathrm{SNR}_1(\omega=\omega_1+\omega_2) &= \frac{\pi f_1^2 f_2^2 q^6 D^2}{\sqrt{2}\cosh(rq)}\exp\left(-\frac{\Delta p}{2D}\right)\left[1 - \frac{2f_1^2q^2\exp(-\Delta p/D)}{\pi\left(\omega^2+2\exp(-\Delta p/D)\cosh^2(rq)/\pi^2\right)}\right]^{-1}\\ &\quad\times\tanh^2(rq),\end{aligned} \tag{21}$$

$$\begin{aligned}\mathrm{SNR}_2(\omega=\omega_1+2\omega_2) &= \frac{\sqrt{2}\pi f_1^2 f_2^4 q^8 D^2}{\cosh(rq)}\exp\left(-\frac{\Delta p}{2D}\right)\left[1 - \frac{2f_1^2q^2\exp(-\Delta p/D)}{\pi\left(\omega^2+2\exp(-\Delta p/D)\cosh^2(rq)/\pi^2\right)}\right]^{-1}\\ &\quad\times\tanh^2\left[1+3\tanh^2(rq)\right],\end{aligned} \tag{22}$$

where Δp and q are given by Eq. (14). Then the SNR of system (1) can be calculated as a function of the multiplicative noise intensity D and the additive noise intensity Q.

3. The effects of the additive and multiplicative noise intensity, static asymmetry r and delay time τ on the SNR

The SNR for the fundamental harmonic has been discussed in Refs. [15] and [30]. Now we only discuss the effects of the additive noise intensity Q, multiplicative noise intensity D, static asymmetry r and delay time τ on the SNR for higher harmonics.

The effects of multiplicative noise intensity D on the SNR_1 and SNR_2 with different values of static asymmetry r and delay time τ are illustrated respectively in Figs. 1–4. From Figs. 1–4, it is seen that the curves of SNR_1 and SNR_2 all exhibit a single peak as D increases, this means that SR appears.

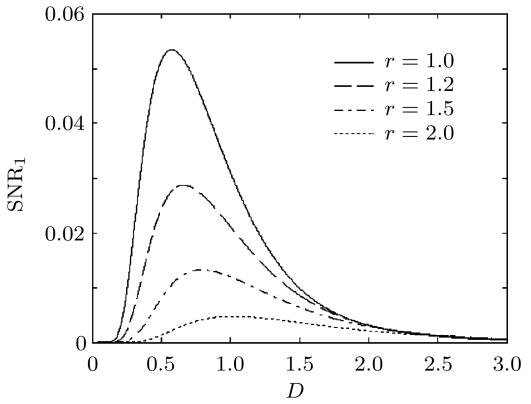


Fig. 1. The SNR_1 as a function of multiplicative noise intensity D for $f_1 = 0.4,\ f_2 = 0.1,\ \omega_1 = 0.3,\ \omega_2 = 0.1,\ R = 2.0,\ \tau = 0.5$ with different values of r.

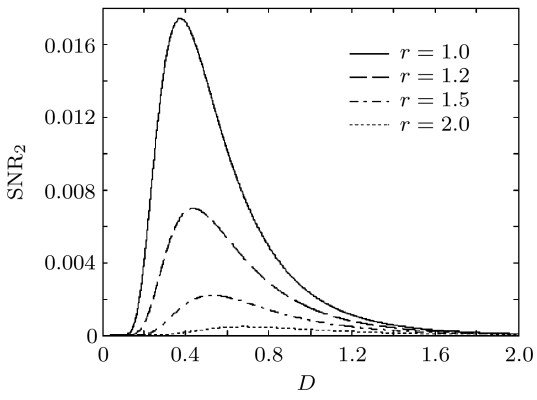


Fig. 2. The SNR_2 as a function of multiplicative noise intensity D for $f_1 = 0.4,\ f_2 = 0.1,\ \omega_1 = 0.3,\ \omega_2 = 0.1,\ R = 2.0,\ \tau = 0.5$ with different values of r.

Figure 1 is a plot of SNR_1 as a function of multiplicative noise intensity D for different values of r. As seen from Fig. 1, for small values of D, SNR_1 decreases with the increase of r; in contrast, SNR_1 increases with the increase of r for large values of D (not shown). Figure 2 is a plot of SNR_2 as a function of multiplicative noise intensity D for different values of r. As seen from Fig. 2, SNR_2 decreases monotonically as r increases. Moreover, the peaks of SNR_1 and SNR_2 become lower respectively and both move to the right with the increase of r.

Figure 3 is a plot of SNR_1 as a function of multiplicative noise intensity D for different values of τ. Figure 4 is a plot of SNR_2 as a function of multiplicative noise intensity D for different values of τ. From Figs. 3 and 4, it is clear that SNR_1 and SNR_2 decrease with the increase of τ only for very small values of D, and then SNR_1 and SNR_2 increase with the increase of τ. Meanwhile, the peaks of SNR_1 and SNR_2 become higher respectively and both move to the right.

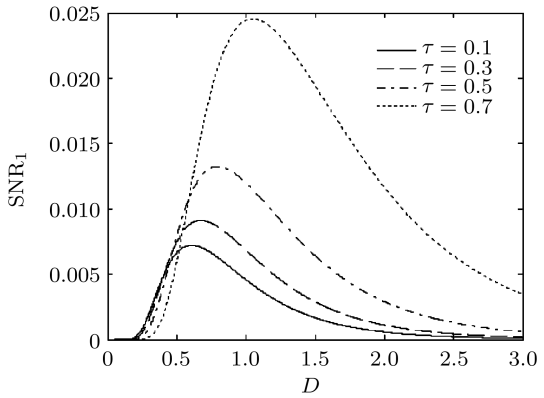


Fig. 3. The SNR_1 as a function of multiplicative noise intensity D for $f_1 = 0.4,\ f_2 = 0.1,\ \omega_1 = 0.3,\ \omega_2 = 0.1,\ R = 2.0,\ r = 1.5$ with different values of τ.

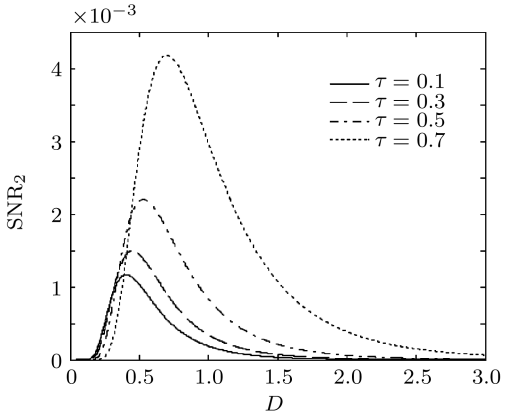


Fig. 4. The SNR_2 as a function of multiplicative noise intensity D for $f_1 = 0.4,\ f_2 = 0.1,\ \omega_1 = 0.3,\ \omega_2 = 0.1,\ R = 2.0,\ r = 1.5$ with different values of τ.

The effects of additive noise intensity Q on the SNR_1 and SNR_2 with different values of static asymmetry r and delay time τ are illustrated respectively in Figs. 5–8. From Figs. 5–8, it is seen that the curves of SNR_1 and SNR_2 all exhibit a single peak as Q in-

creases, which is the typical characteristic of the SR phenomenon.

Figure 5 is a plot of SNR_1 as a function of additive noise intensity Q for different values of r. As seen from Fig. 5, for small values of Q, SNR_1 decreases with the increase of r; in contrast, SNR_1 increases with the increase of r for large values of Q (not shown). Figure 6 is a plot of SNR_2 as a function of additive noise intensity Q for different values of r. As seen from Fig. 6, SNR_2 decreases monotonically as r increases. From Figs. 5 and 6, we can also see that the peaks of SNR_1 and SNR_2 become lower respectively and both move to the right with the increase of r, which is consistent with the phenomenon presented in Figs. 1 and 2. This shows that the static asymmetry r can restrain SR.

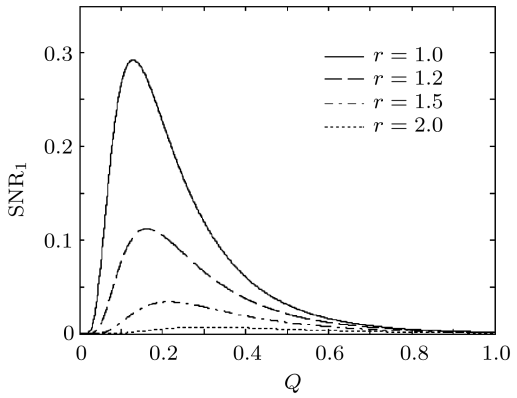


Fig. 5. The SNR_1 as a function of additive noise intensity Q for $f_1 = 0.4$, $f_2 = 0.1$, $\omega_1 = 0.3$, $\omega_2 = 0.1$, $D = 1.0$, $\tau = 0.5$ with different values of r.

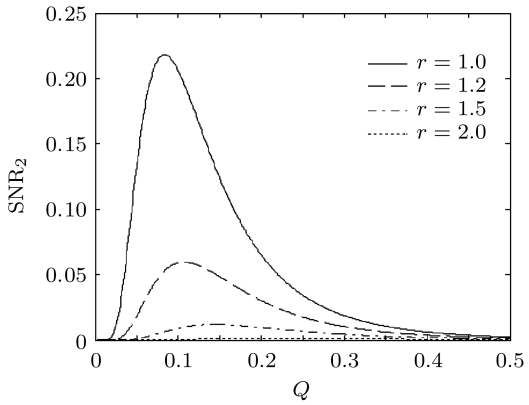


Fig. 6. The SNR_2 as a function of additive noise intensity Q for $f_1 = 0.4$, $f_2 = 0.1$, $\omega_1 = 0.3$, $\omega_2 = 0.1$, $D = 1.0$, $\tau = 0.5$ with different values of r.

Figure 7 is a plot of SNR_1 as a function of additive noise intensity Q for different values of τ. Figure 8 is a plot of SNR_2 as a function of additive noise intensity Q for different values of τ. From Figs. 7 and 8, it is clear that SNR_1 and SNR_2 decrease with the increase of τ only for very small values of Q, and then SNR_1 and SNR_2 increase with the increase of τ. Meanwhile, we can also see that the peaks of SNR_1 and SNR_2 become higher respectively and both move to the right with the increase of τ, which is consistent with the phenomenon presented in Figs. 3 and 4. This shows that the delay time τ can enhance SR.

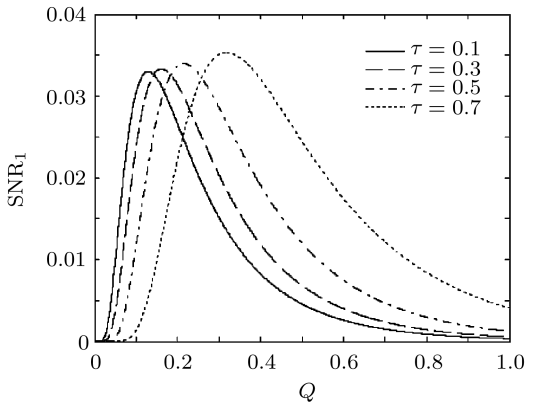


Fig. 7. The SNR_1 as a function of additive noise intensity Q for $f_1 = 0.4$, $f_2 = 0.1$, $\omega_1 = 0.3$, $\omega_2 = 0.1$, $D = 1.0$, $r = 1.5$ with different values of τ.

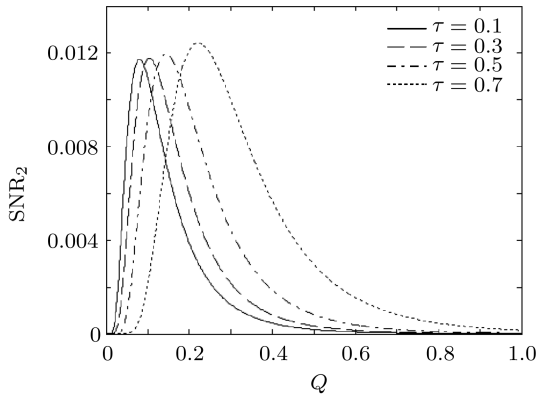


Fig. 8. The SNR_2 as a function of additive noise intensity Q for $f_1 = 0.4$, $f_2 = 0.1$, $\omega_1 = 0.3$, $\omega_2 = 0.1$, $D = 1.0$, $r = 1.5$ with different values of τ.

Comparing Figs. 1–4 with Figs. 5–8 respectively, we can see that the effects of the multiplicative noise intensity D and additive noise intensity Q are different on SR; adjusting the additive noise intensity Q is more sensitive than that of the multiplicative noise intensity D in controlling SR. Comparing Figs. 1, 3, 5 and 7 with Figs. 2, 4, 6 and 8 respectively, we can see that the values of SNR_2 are smaller than those of SNR_1 when the other parameters choose the same values, which shows that the higher mixed harmonics can restrain SR. Moreover, from Figs. 1–8, we can see that the longer the delay time τ is, the larger the additive noise intensity Q and the multiplicative noise intensity D are, when the SR appears.

4. Conclusions

In this paper, we have studied the SR phenomenon in an asymmetric bistable system with time-delayed feedback and mixed periodic signal. A general approximate Fokker–Planck equation is obtained by using the small time delay approximation proposed by Refs. [28] and [29]. The expressions of the SNR at both fundamental harmonics and mixed harmonics are derived in the adiabatic limit. The effects of the additive noise intensity Q, multiplicative noise intensity D, static asymmetry r and delay time τ on SNR are discussed. It is found that the higher mixed harmonics and the static asymmetry can restrain SR, which is consistent with the result of Refs. [18], [21] and [22], and the delay time τ can enhance SR. That is, the static asymmetry r and delay time τ play opposite roles in SR. In addition, the longer the delay time τ is, the larger the additive noise intensity Q and the multiplicative noise intensity D are, when the SR appears.

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