Generation of entangled coherent states through cavity-assisted interaction*

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We propose schemes to generate an n-coherent-pulse GHZ state and a cluster state via the interaction between n coherent pulses and a two-sided cavity. In these schemes, a strong coupling condition is not needed, which makes the protocols possibly able to be implemented based on the current experiment technology.

Keywords: n-coherent-pulse GHZ state, n-coherent-pulse cluster state, two-sided cavities

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1. Introduction

Entanglement is considered to be one of the most profound features of quantum mechanics and is useful in quantum information processing, such as quantum cryptography,[1] quantum computing,[2] quantum teleportation,[3] and so on. Because of the importance of entanglement, entanglement state is well studied, including characterization,[4−6] classification,[5−7] and generation of entangled states.[8,9] However, most previous investigations have focused on the entanglement orthogonal state. Recent research shows that nonorthogonality plays an important role in quantum theory and quantum information such as quantum key distribution.[10] But entangled nonorthogonal states have not received sufficient attention.

The entangled coherent state is one of the most important nonorthogonal states. It can be used for quantum computation and quantum information processing.[11−15] Based on ion traps,[16,17] Kerr nonlinearities,[18] and Raman interaction,[19] some research has been starting to prepare entangled coherent states. In this paper, we propose schemes to generate entangled coherent states by simply transferring weak coherent optical pulses through an optical cavity trapping a three-level atom. Compared with the recent proposal of a photonic computation scheme based on single-sided cavities,[20−23] the schemes have the advantage of being directly built on state-of-the-art two-sided cavities.

2. Fundamental model

The fundamental model of the schemes is illustrated in Fig. 1. The coherent pulses are injected into a two-sided cavity which contains a single trapped atom with three levels. The states $|0\rangle$ and $|1\rangle$ are levels in the ground-state manifold with different hyperfine spins, while $|e\rangle$ is an excited state. The atomic

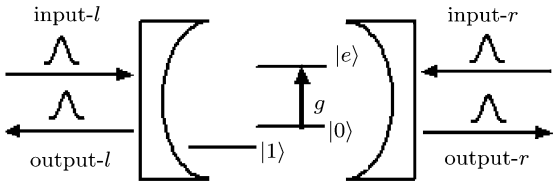


Fig. 1. Schematic setup of the fundamental model. Two pulses l and r are reflected or transmitted through the single-atom cavity.

transition $|0\rangle \rightarrow |e\rangle$ is resonantly coupled to the cavity mode a, which is resonantly driven by an optical pulse prepared in a weak coherent state $|\alpha\rangle$. The interaction between the atom and the cavity mode a is described by the Hamiltonian (setting $\hbar = 1$)

$$H = g(\sigma^{\dagger}a + \sigma a^{\dagger}), \tag{1}$$

where $\sigma = |0\rangle\langle e|$ is the atom lowering operator and g represents the atom–cavity coupling rate.

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The Heisenberg–Langevin equations for a and σ have the forms[24,25]

$$\dot{a} = -\mathrm{i}[a, H] - \frac{\kappa_l + \kappa_r}{2}a + \sqrt{\kappa_l}a_l^{\mathrm{in}}(t) + \sqrt{\kappa_r}a_r^{\mathrm{in}}(t), \quad (2)$$

$$\dot{\sigma} = -\mathrm{i}[\sigma, H] - \frac{\gamma}{2}\sigma + \sqrt{\gamma}\sigma_z\hat{N}, \quad (3)$$

where a_l^{in} and a_r^{in} denote one-dimensional quantum operators of the input pulses from left and right sides of the cavity, κ_l and κ_r are the corresponding cavity decay rates from the left side and right side (here we suppose $\kappa_l = \kappa_r = \kappa$ for convenience), γ is the spontaneous emission rate of the atom level $|e\rangle$, $\sigma_z = |e\rangle\langle e| - |0\rangle\langle 0|$, and $\hat{N}$ is the vacuum noise operator. Since the atom has a rare opportunity to stay in the state $|e\rangle$,[24] we can use the state projector $|0\rangle\langle 0|$ instead of $-\sigma_z$. The output fields a_j^{out} $(j = l, r)$, the input fields a_j^{in}, and the cavity mode a are connected through the following relation[25]

$$a_j^{\mathrm{out}} + a_j^{\mathrm{in}} = \sqrt{\kappa_j}a. \quad (4)$$

Equations (1)–(4) completely determine the evolution of the joint state of the atom, coherent pulse, and the cavity field.

By means of the Fourier transform method, the relation of input-output fields is given by the following form:

$$a_j^{\mathrm{out}}(\omega) = \frac{\mathrm{i}\,\omega + g^2|0\rangle\langle 0|/(\mathrm{i}\,\omega - \gamma/2)}{\kappa - \mathrm{i}\,\omega - g^2|0\rangle\langle 0|/(\mathrm{i}\,\omega - \gamma/2)}a_j^{\mathrm{in}}(\omega) + \frac{\kappa}{\kappa - \mathrm{i}\,\omega - g^2|0\rangle\langle 0|/(\mathrm{i}\,\omega - \gamma/2)}a_{\bar{j}}^{\mathrm{in}}(\omega) + \frac{\mathrm{i}\sqrt{\gamma\kappa}g|0\rangle\langle 0|/(\mathrm{i}\,\omega - \gamma/2)}{\kappa - \mathrm{i}\,\omega - g^2|0\rangle\langle 0|/(\mathrm{i}\,\omega - \gamma/2)}\hat{N}(\omega), \quad (5)$$

where $a_j^{\mathrm{in}}(\omega)$ and $a_j^{\mathrm{out}}(\omega)$ are the Fourier transforms of the input and the output field operators $a_j^{\mathrm{in}}(t)$ and $a_j^{\mathrm{out}}(t)$, ω depicts the frequency detuning between the cavity mode and the input pulse, $\{j,\bar{j}\} = \{l, r\}$ or $\{r, l\}$.

If the pulse bandwidth of $\delta\omega$ (the range of ω) is much smaller than the rates κ and g^2/κ, and $\gamma \ll g^2/\kappa$, we obtain $a_j^{\mathrm{out}}(\omega) = -a_j^{\mathrm{in}}(\omega)$ for the atom in the state $|0\rangle$, and $a_j^{\mathrm{out}}(\omega) = a_{\bar{j}}^{\mathrm{in}}(\omega)$ for the atom in the state $|1\rangle$.

When only a coherent light is injected from the left side of the cavity and interacts with the cavity trapping an atom with initial state $|\Phi_{\mathrm{ai}}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, the state of the coherent pulse and the atom evolves into

$$|\Psi\rangle = U|\alpha\rangle_l \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}(|-\alpha\rangle_l|0\rangle + |\alpha\rangle_r|1\rangle), \quad (6)$$

where U denotes the interaction between the cavity and the input pulse, the subscript "l" and "r" are the corresponding directions of the coherent pulse reflected or transmitted by the cavity. The whole effect is shown in Fig. 2.

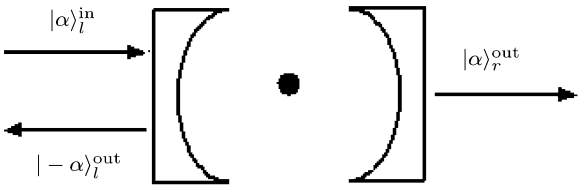


Fig. 2. Sketch of the output pulses after coherent light is injected from the left side of the cavity.

3. Generation of an n-coherent-pulse GHZ state

Now we present the generation of an n-coherent-pulse GHZ state in detail. The atom is initially prepared in the state $|\Phi_{\mathrm{ai}}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, and every coherent pulse is in the state $|\alpha\rangle$. The n coherent pulses are successively injected from the left side of the cavity. After n coherent pulses interacting with the cavity, the state of the system including the atom and n coherent pulses becomes

$$|\Psi\rangle_{1\mathrm{out}} = \prod_{i=1}^{n} U_i|\alpha\rangle_{il} \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\Big(\prod_{i=1}^{n}|-\alpha\rangle_{il}|0\rangle + \prod_{i=1}^{n}|\alpha\rangle_{ir}|1\rangle\Big). \quad (7)$$

Then the state of the atom is measured in the bases $\{|+\rangle, |-\rangle\}$, where $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. If the measurement outcome is $|+\rangle$, the n-coherent-pulse state is projected into

$$|\Psi\rangle_1 = \frac{1}{\sqrt{2}}(|-\alpha\rangle_{1l}|-\alpha\rangle_{2l}\ldots|-\alpha\rangle_{nl} + |\alpha\rangle_{1r}|\alpha\rangle_{2r}\ldots|\alpha\rangle_{nr}). \quad (8)$$

If the measurement outcome is $|-\rangle$, the n-coherent-pulse state is projected into

$$|\Psi\rangle_2 = \frac{1}{\sqrt{2}}(|-\alpha\rangle_{1l}|-\alpha\rangle_{2l}\ldots|-\alpha\rangle_{nl} - |\alpha\rangle_{1r}|\alpha\rangle_{2r}\ldots|\alpha\rangle_{nr}). \quad (9)$$

Apparently, both $|\Psi\rangle_1$ and $|\Psi\rangle_2$ are n-coherent-pulse GHZ states.

4. Preparation of an n-coherent-pulse cluster state

Next we describe how to create a two-coherent-pulse cluster state in detail. A single qubit rotation $R(a)$ is applied on the atom after each coherent pulse interacting with the cavity, where $R(a)|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $R(a)|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. After two $T_i = R(a)U_i$ operations are performed on the trapped atom and two input coherent pulses are injected from the left side of the cavity, the state of the system including two coherent pulses and the atom becomes

$$\begin{aligned}|\Psi\rangle_{2\text{out}} &= T_2 \otimes T_1 |\alpha\rangle_{2l}|\alpha\rangle_{1l} \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\\ &= \frac{1}{2}T_2|\alpha\rangle_{2l}[|0\rangle(|-\alpha\rangle_{1l} + |\alpha\rangle_{1r}) + |1\rangle(|-\alpha\rangle_{1l} - |\alpha\rangle_{1r})]\\ &= \frac{1}{2\sqrt{2}}[(|-\alpha\rangle_{1l}|-\alpha\rangle_{2l} + |\alpha\rangle_{1r}|-\alpha\rangle_{2l} + |-\alpha\rangle_{1l}|\alpha\rangle_{2r} - |\alpha\rangle_{1r}|\alpha\rangle_{2r})|0\rangle\\ &\quad + (|-\alpha\rangle_{1l}|-\alpha\rangle_{2l} + |\alpha\rangle_{1r}|-\alpha\rangle_{2l} - |-\alpha\rangle_{1l}|\alpha\rangle_{2r} + |\alpha\rangle_{1r}|\alpha\rangle_{2r})|1\rangle]. \end{aligned} \tag{10}$$

Then a measurement is executed on the atom in the bases $\{|0\rangle, |1\rangle\}$. If the measurement result is $|0\rangle$, the two-coherent-pulse state is projected into the cluster state

$$\begin{aligned}|\Psi\rangle_3 &= \frac{1}{2}(|-\alpha\rangle_{1l}|-\alpha\rangle_{2l} + |\alpha\rangle_{1r}|-\alpha\rangle_{2l} + |-\alpha\rangle_{1l}|\alpha\rangle_{2r} - |\alpha\rangle_{1r}|\alpha\rangle_{2r})\\ &= \frac{1}{2}(|-\alpha\rangle_{1l} + |\alpha\rangle_{1r}\sigma_z^2)(|-\alpha\rangle_{2l} + |\alpha\rangle_{2r}), \end{aligned} \tag{11}$$

where $\sigma_z^i = |-\alpha\rangle_{il}\langle -\alpha| - |\alpha\rangle_{ir}\langle\alpha|$, whereas if the measurement result is $|1\rangle$, the two-coherent-pulse state is projected into the other cluster state

$$\begin{aligned}|\Psi\rangle_4 &= \frac{1}{2}(|-\alpha\rangle_{1l}|-\alpha\rangle_{2l} + |\alpha\rangle_{1r}|-\alpha\rangle_{2l} - |-\alpha\rangle_{1l}|\alpha\rangle_{2r} + |\alpha\rangle_{1r}|\alpha\rangle_{2r})\\ &= \frac{1}{2}(|-\alpha\rangle_{1l}\sigma_z^2 + |\alpha\rangle_{1r})(|-\alpha\rangle_{2l} + |\alpha\rangle_{2r}). \end{aligned} \tag{12}$$

Similarly, for preparing a three-coherent-pulse cluster state, we apply three T_i operations to the trapped atom and three input coherent pulses injected from the left side of the cavity. Thus, one has

$$\begin{aligned}|\Psi\rangle_{3\text{out}} &= T_3 \otimes T_2 \otimes T_1 \otimes |\alpha\rangle_{1l}|\alpha\rangle_{2l}|\alpha\rangle_{3l} \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\\ &= \frac{1}{4}[(|-\alpha\rangle_{1l}|-\alpha\rangle_{2l}|-\alpha\rangle_{3l} + |\alpha\rangle_{1r}|-\alpha\rangle_{2l}|-\alpha\rangle_{3l} + |-\alpha\rangle_{1l}|\alpha\rangle_{2r}|-\alpha\rangle_{3l}\\ &\quad - |\alpha\rangle_{1r}|\alpha\rangle_{2r}|-\alpha\rangle_{3l} + |-\alpha\rangle_{1l}|-\alpha\rangle_{2l}|\alpha\rangle_{3r} + |\alpha\rangle_{1r}|-\alpha\rangle_{2l}|\alpha\rangle_{3r}\\ &\quad - |-\alpha\rangle_{1l}|\alpha\rangle_{2r}|\alpha\rangle_{3r} + |\alpha\rangle_{1r}|\alpha\rangle_{2r}|\alpha\rangle_{3r})|0\rangle + (|-\alpha\rangle_{1l}|-\alpha\rangle_{2l}|-\alpha\rangle_{3l}\\ &\quad + |\alpha\rangle_{1r}|-\alpha\rangle_{2l}|-\alpha\rangle_{3l} + |-\alpha\rangle_{1l}|\alpha\rangle_{2r}|-\alpha\rangle_{3l} - |\alpha\rangle_{1r}|\alpha\rangle_{2r}|-\alpha\rangle_{3l}\\ &\quad - |-\alpha\rangle_{1l}|-\alpha\rangle_{2l}|\alpha\rangle_{3r} - |\alpha\rangle_{1r}|-\alpha\rangle_{2l}|\alpha\rangle_{3r} + |-\alpha\rangle_{1l}|\alpha\rangle_{2r}|\alpha\rangle_{3r}\\ &\quad - |\alpha\rangle_{1r}|\alpha\rangle_{2r}|\alpha\rangle_{3r})|1\rangle]. \end{aligned} \tag{13}$$

Then a measurement is performed on the atom in the bases $\{|0\rangle, |1\rangle\}$. If the measurement result is $|0\rangle$, the three-coherent-pulse state is projected into the cluster state

$$\begin{aligned}|\Psi\rangle_5 &= \frac{1}{2\sqrt{2}}(|-\alpha\rangle_{1l}|-\alpha\rangle_{2l}|-\alpha\rangle_{3l} + |\alpha\rangle_{1r}|-\alpha\rangle_{2l} \otimes |-\alpha\rangle_{3l} + |-\alpha\rangle_{1l}|\alpha\rangle_{2r}|-\alpha\rangle_{3l}\\ &\quad - |\alpha\rangle_{1r}|\alpha\rangle_{2r}|-\alpha\rangle_{3l} + |-\alpha\rangle_{1l}|-\alpha\rangle_{2l}|\alpha\rangle_{3r} + |\alpha\rangle_{1r}|-\alpha\rangle_{2l}|\alpha\rangle_{3r} - |-\alpha\rangle_{1l}|\alpha\rangle_{2r}|\alpha\rangle_{3r}\\ &\quad + |\alpha\rangle_{1r}|\alpha\rangle_{2r}|\alpha\rangle_{3r})\\ &= \frac{1}{2\sqrt{2}}(|-\alpha\rangle_{1l} + |\alpha\rangle_{1r}\sigma_z^2)(|-\alpha\rangle_{2l} + |\alpha\rangle_{2r}\sigma_z^3) \otimes (|-\alpha\rangle_{3l} + |\alpha\rangle_{3r}), \end{aligned} \tag{14}$$

whereas if the measurement result is $|1\rangle$, the three-coherent-pulse state is projected into the other cluster state

$$|\Psi\rangle_6 = \frac{1}{2\sqrt{2}}(|-\alpha\rangle_{1l} + |\alpha\rangle_{1r}\sigma_z^2)(|-\alpha\rangle_{2l}\sigma_z^3 + |\alpha\rangle_{2r}) \otimes (|-\alpha\rangle_{3l} + |\alpha\rangle_{3r}). \tag{15}$$

One can take a simple add-on strategy to generate an n-coherent-pulse cluster state, i.e., n $T_i = R(a)U_i$ operations are performed on the trapped atom and n input coherent pulses injected from the left side of the cavity. Then the atomic state is measured in the bases $\{|0\rangle, |1\rangle\}$. If the measurement outcome is $|0\rangle$, the n-coherent-pulse state is projected into the cluster state

$$\begin{aligned}|\Psi\rangle_{n1} \ = & \frac{1}{\sqrt{2}^n}(|-\alpha\rangle_{1l} + |\alpha\rangle_{1r}\sigma_z^2) \otimes (|-\alpha\rangle_{2l} + |\alpha\rangle_{2r}\sigma_z^3) \ldots \otimes (|-\alpha\rangle_{(n-1)l} + |\alpha\rangle_{(n-1)r}\sigma_z^n) \\ & \otimes (|-\alpha\rangle_{nl} + |\alpha\rangle_{nr}),\end{aligned} \tag{16}$$

whereas if the measurement result is $|1\rangle$, the n-coherent-pulse state is projected into the other cluster state

$$\begin{aligned}|\Psi\rangle_{n2} = & \frac{1}{\sqrt{2}^n}(|-\alpha\rangle_{1l} + |\alpha\rangle_{1r}\sigma_z^2) \otimes (|-\alpha\rangle_{2l} + |\alpha\rangle_{2r}\sigma_z^3) \ldots \otimes (|-\alpha\rangle_{(n-2)l} + |\alpha\rangle_{(n-2)r}\sigma_z^{n-1}) \\ & \otimes (|-\alpha\rangle_{(n-1)l}\sigma_z^n + |\alpha\rangle_{(n-1)r}) \otimes (|-\alpha\rangle_{nl} + |\alpha\rangle_{nr}).\end{aligned} \tag{17}$$

5. Conclusion

In summary, we propose schemes to achieve nonlinearity interaction among input coherent pulses via cavity QED. By measuring the atom state in the appropriate bases, the resultant motional state of n input coherent pulses is prepared in an n-coherent-pulse GHZ state or an n-coherent-pulse cluster state. The cavity involved in the proposals is a two-sided cavity, which is just the practical cavity in the current laboratory. A strong coupling condition is not required. All of these advantages make our protocols possibly able to be implemented experimentally in the near future.

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