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Remote preparation of atomic and field cluster states from a pair of tri-partite GHZ states

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We propose two simple and resource-economical schemes for remote preparation of four-partite atomic as well as cavity field cluster states. In the case of atomic state generation, we utilize simultaneous resonant and dispersive interactions of the two two-level atoms at the preparation station. Atoms involved in these interactions are individually pair-wise entangled into two different tri-partite GHZ states. After interaction, the passage of the atoms through a Ramsey zone and their subsequent detection completes the protocol. However, for field state generation we first copy the quantum information in the cavities to the atoms by resonant interactions and then adapt the same method as in the case of atomic state generation. The method can be generalised to remotely generate any arbitrary graph states in a straightforward manner.

Keywords: quantum information, entanglement production, cavity quantum electrodynamics

PACC: 0367, 4250

1. Introduction

The concept of entanglement was framed to highlight the counterintuitive, nonlocal traits of the quantum theory.[1] However, the same phenomenon is now being utilized as the main resource for most quantum informatics tasks.[2] In this respect, recently discovered so-called cluster states,[3] a subgroup of more general graph states,[4,5] have provoked renewed interest in the quantum optics community. This keen interest is evidently linked with the proposal of a one-way computing model[6] that employs such states as the computational resource. The computation is carried out via local qubit measurement and has already been demonstrated experimentally through manipulation of a four-qubit polarisation entangled field cluster state.[7] Apart from one-way computation, the linear cluster states also exhibit some additional novel characteristics like rich nonlocality features[8] and resistance to decoherence as compared to their counterpart Greenberger–Horne–Zeilinger (GHZ) states.[9] Almost all major technologies involved in quantum state engineering and information processing have therefore been extensively explored for the generation of such states.[10−12]

We, in the present article, intend to show that a four-partite cluster state can be prepared by performing a controlled phase gate[13] over two qubits of the two independent tri-partite GHZ states. This task can be done remotely with the evident advantage of aiding the distributive computation/information processing in spatially separated quantum networks. The atomic and cavity field GHZ states being used as a resource here have been extensively investigated[14] and can be prepared relatively easily.[15] We assume that two parties, say Alice and Bob, are sharing the respective GHZ states such that Alice has access to only one qubit from each entangled GHZ state. Alice then performs controlled phase gate operation utilizing her parts of the entangled states. In the case of the atomic cluster state, the operation is achieved via simultaneous resonant and dispersive interactions of atoms that are available to Alice as a part of the shared GHZ states. These atoms, after culmination of simultaneous interactions, are then passed through a Ramsey zone. Remote state engineering is finally accomplished with the detection of these atoms through state-selective detectors.

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2. Scheme for atomic state generation

We suppose Alice and Bob share two pairs of tri-partite atomic GHZ states of the form

$$\left|\Psi_{AB}^{(j)}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|g_{A_j}, g_{B_j}^{(1)}, g_{B_j}^{(2)}\right\rangle + \left|e_{A_j}, e_{B_j}^{(1)}, e_{B_j}^{(2)}\right\rangle\right), \tag{1}$$

where $j = 1, 2$ corresponds to two sets of GHZ states. Here $|g\rangle$ ($|e\rangle$) designates the ground (excited) state of the two-level atoms and subscripts $A_j(B_j)$ denotes the qubits that are locally possessed by Alice (Bob). Now Alice, in order to remotely prepare the atomic cluster state at the Bob's end, sends her qubits, i.e., atom-1 and atom-2 belonging to two GHZ groups, simultaneously through an initially vacuum state cavity C. The initial product state of the system is given by

$$|\Psi_{AB}(0)\rangle = \frac{1}{2}\prod_{j=1}^{2}\left(\left|g_{A_j}, g_{B_j}^{(1)}, g_{B_j}^{(2)}\right\rangle + \left|e_{A_j}, e_{B_j}^{(1)}, e_{B_j}^{(2)}\right\rangle\right) \otimes |0_c\rangle\,. \tag{2}$$

We consider the case when the atoms are far apart and the dipole–dipole interaction is not present. Inside the cavity, atom-1 goes through resonant interaction whereas atom-2 at the same time performs dispersive interaction (Fig. 1). This can happen naturally if the two atoms are of different kinds. However, for identical atoms, we may employ a resonant cavity such that the part where the interactions of atom-2 happen is placed under a localised Stark field yielding sufficient detuning to transform the atom-field interactions from resonant to dispersive.[16] Alternatively, we may take a resonant L-shaped initially vacuum state cavity[17] such that atom-1 passes through one wing, say L_1, and atom-2 passes through the other wing L_2 where dispersive interactions can be induced using the Stark field.

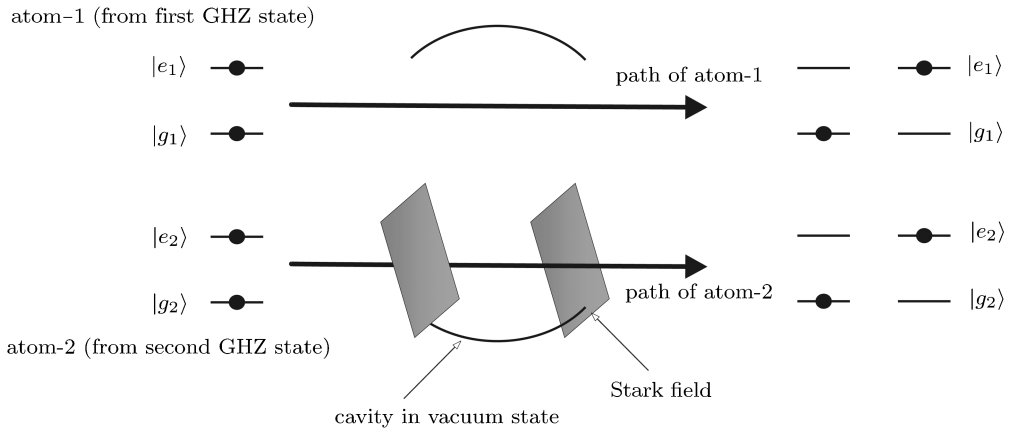


Fig. 1. Schematic representation of the atom–cavity system. The dispersive interactions in the lower region of the cavity are induced by subjecting it to a local Stark field.

We, however, consider the formal case here as it is simple and technically straightforward. The collective Hamiltonian H_c consists of two parts, i.e.,

$$H_c = \hbar\mu(a\,|e_{A_1}\rangle\langle g_{A_1}| + a^\dagger\,|g_{A_1}\rangle\langle e_{A_1}|) + \hbar\lambda(aa^\dagger\,|e_{A_2}\rangle\langle e_{A_2}| - a^\dagger a\,|g_{A_2}\rangle\langle g_{A_2}|), \tag{3}$$

one for the resonant interaction of atom-1 and the order for the off-resonant interaction of atom-2. The Hamiltonian is written under the dipole and rotating wave approximation as discussed in Ref. [18]. Here μ is the atom-field coupling constant in the resonant case and $\lambda = \mu^2/\delta$ is the effective coupling constant for the off-resonant case where δ is the detuning of atom-2 with the cavity mode. This part of the Hamiltonian is written under the large detuning limit, where $a^\dagger a$ is the photon number operator, i.e., $a^\dagger a\,|n\rangle = n\,|n\rangle$. The state vector after an interaction lasting for an arbitrary time t may be expressed as follows:

$$\begin{aligned}|\Psi_{AB}(t)\rangle = {} & C_{g_{A_1},g_{A_2}}^{B_{gg},0_c}(t)\,|0_c, g_{A_1}, g_{A_2}, B_{gg}\rangle + C_{g_{A_1},e_{A_2}}^{B_{ge},0_c}(t)\,|0_c, g_{A_1}, e_{A_2}, B_{ge}\rangle \\ & + C_{e_{A_1},g_{A_2}}^{B_{eg},0_c}(t)\,|0_c, e_{A_1}, g_{A_2}, B_{eg}\rangle + C_{e_{A_1},e_{A_2}}^{B_{ee},0_c}(t)\,|0_c, e_{A_1}, e_{A_2}, B_{ee}\rangle\end{aligned}$$

$$+ C^{B_{eg},1_c}_{g_{A_1},g_{A_2}}(t)\,|1_c, g_{A_1}, g_{A_2}, B_{eg}\rangle + C^{B_{ee},1_c}_{g_{A_1},e_{A_2}}(t)\,|1_c, g_{A_1}, e_{A_2}, B_{ee}\rangle\,, \tag{4}$$

where

$$|B_{gg}\rangle = \left|g^{(1)}_{B_1}, g^{(2)}_{B_1}, g^{(1)}_{B_2}, g^{(2)}_{B_2}\right\rangle,$$
$$|B_{ge}\rangle = \left|g^{(1)}_{B_1}, g^{(2)}_{B_1}, e^{(1)}_{B_2}, e^{(2)}_{B_2}\right\rangle,$$
$$|B_{eg}\rangle = \left|e^{(1)}_{B_1}, e^{(2)}_{B_1}, g^{(1)}_{B_2}, g^{(2)}_{B_2}\right\rangle,$$
$$|B_{ee}\rangle = \left|e^{(1)}_{B_1}, e^{(2)}_{B_1}, e^{(1)}_{B_2}, e^{(2)}_{B_2}\right\rangle.$$

The solution of the Schrödinger equation under initial conditions

$$C^{B_{gg},0_c}_{g_{A_1},g_{A_2}}(0) = C^{B_{ge},0_c}_{g_{A_1},e_{A_2}}(0) = C^{B_{eg},0_c}_{e_{A_1},g_{A_2}}(0) = C^{B_{ee},0_c}_{e_{A_1},e_{A_2}}(0) = \frac{1}{2}, \tag{5}$$

and

$$C^{B_{eg},1_c}_{g_{A_1},g_{A_2}}(0) = C^{B_{ee},1_c}_{g_{A_1},e_{A_2}}(0) = 0, \tag{6}$$

yields the following expressions for the probability amplitudes

$$C^{B_{gg},0_c}_{g_{A_1},g_{A_2}}(t) = \frac{1}{2}, \tag{7}$$

$$C^{B_{ge},0_c}_{g_{A_1},e_{A_2}}(t) = \frac{1}{2}\mathrm{e}^{-\mathrm{i}\,\lambda t}, \tag{8}$$

$$C^{B_{eg},0_c}_{e_{A_1},g_{A_2}}(t) = \frac{1}{2\beta}\mathrm{e}^{\mathrm{i}\,\lambda t/2}\{\beta\cos(\beta t/2) - \mathrm{i}\,\lambda\sin(\beta t/2)\}, \tag{9}$$

$$C^{B_{ee},0_c}_{e_{A_1},e_{A_2}}(t) = \frac{1}{2\beta}\mathrm{e}^{-3\,\mathrm{i}\,\lambda t/2}\{\beta\cos(\beta t/2) + \mathrm{i}\,\lambda\sin(\beta t/2)\}, \tag{10}$$

$$C^{B_{eg},1_c}_{g_{A_1},g_{A_2}}(t) = \frac{1}{2\beta}\mathrm{e}^{\mathrm{i}\,\lambda t/2}\left\{-\mathrm{i}\,2\mu\sin(\beta t/2)\right\}, \tag{11}$$

$$C^{B_{ee},1_c}_{g_{A_1},e_{A_2}}(t) = \frac{1}{2\beta}\mathrm{e}^{-\mathrm{i}\,3\lambda t/2}\{\mathrm{i}\,2\mu\sin(\beta t/2)\}, \tag{12}$$

where $\beta = \sqrt{4\mu^2 + \lambda^2}$.

It is clear from Eqs. (7) and (8) that the probabilities of these amplitudes are constant in time (i.e., as the $\left| C^{B_{gg},0_c}_{g_{A_1},g_{A_2}}(t) \right|^2$ and $\left| C^{B_{ge},0_c}_{g_{A_1},e_{A_2}}(t) \right|^2$ are statically equal to 1/4), whereas probabilities corresponding to the rest of the equations from Eq. (9) to Eq. (12) show oscillations in time. We plot all six probabilities in Fig. 2 showing the fact mentioned above. The probabilities $\left| C^{B_{eg},0_c}_{e_{A_1},g_{A_2}}(t) \right|^2$ and $\left| C^{B_{ee},0_c}_{e_{A_1},e_{A_2}}(t) \right|^2$ plotted in Figs. 2(c) and 2(d) bear the same trends. Similarly, probabilities $\left| C^{B_{eg},1_c}_{g_{A_1},g_{A_2}}(t) \right|^2$ and $\left| C^{B_{ee},1_c}_{g_{A_1},e_{A_2}}(t) \right|^2$ plotted in Figs. 2(e) and 2(f) exhibit identical patterns. In addition to this, the two sets of oscillatory probabilities are out of phase with each other but in both cases we have a maximum probability of 1/4 for different interaction times. The $\left| C^{B_{eg},1_c}_{g_{A_1},g_{A_2}}(t) \right|^2$ and $\left| C^{B_{ee},1_c}_{g_{A_1},e_{A_2}}(t) \right|^2$ have a minimum value of zero. The zero values of these probabilities occur at $t = n\pi/\beta$ with n being an even integer. This analysis leads to the conclusion that our requirement for the generation of an atomic cluster state is simply that the atoms, after interaction, should leave the cavity again into the vacuum state. This, in turn, implies that interaction time t should be selected such that the probabilities $\left| C^{B_{eg},1_c}_{g_{A_1},g_{A_2}}(t) \right|^2$ and $\left| C^{B_{ee},1_c}_{g_{A_1},e_{A_2}}(t) \right|^2$ both vanish. In this context, $n = 0$ corresponds to the case of the trivial solution where no entanglement is produced.

The state for $n = 2$ can be written as

$$\left|\Psi^{(\mathrm{cluster})}_{AB}\right\rangle = \frac{1}{2}[|g_{A_1}, g_{A_2}, B_{gg}\rangle + \mathrm{e}^{-2\,\mathrm{i}\,\pi\zeta}\,|g_{A_1}, e_{A_2}, B_{ge}\rangle - \mathrm{e}^{\mathrm{i}\,\pi\zeta}\,|e_{A_1}, g_{A_2}, B_{eg}\rangle - \mathrm{e}^{-3\,\mathrm{i}\,\pi\zeta}\,|e_{A_1}, e_{A_2}, B_{ee}\rangle], \tag{13}$$

where the interaction time is selected to be $t = 2\pi/\beta$. Also in Eq. (13) $\zeta = \lambda/\beta$ where we have traced out the cavity field, i.e., $|0_c\rangle$. Equation (13) gives a maximally entangled six-partite cluster state shared between Alice and Bob. Its entangled nature is evident from the fact that it does not satisfy factorization criteria, i.e.,

$$\left|C^{B_{gg},0_c}_{g_{A_1},g_{A_2}} * C^{B_{ee},0_c}_{e_{A_1},e_{A_2}}(t) - C^{B_{ge},0_c}_{g_{A_1},e_{A_2}}(t) * C^{B_{eg},0_c}_{e_{A_1},g_{A_2}}(t)\right| \neq 0, \tag{14}$$

for ζ ranging between 0 and 1 that may be found using experimentally acceptable interaction parameters μ and δ. In Fig. 2, $\zeta = \lambda/\beta = 0.0698$ satisfies the condition discussed above.

Having shown the maximally entangled nature of the engineered six-partite atomic cluster state, we now proceed to remotely prepare the four-partite two-dimensional (2D) atomic cluster state at Bob's end. A photonic state with similar tomography was employed for the demonstration of one-way computing.[7] Now, for remote preparation of such a state, Alice passes the atomic qubits in her possession through a Ramsey zone that in turn causes the following transformations

$$\left|g_{A_j}\right\rangle \rightarrow \frac{1}{\sqrt{2}}\left(\left|g_{A_j}\right\rangle + \left|e_{A_j}\right\rangle\right), \tag{15}$$

$$\left|e_{A_j}\right\rangle \rightarrow \frac{1}{\sqrt{2}}\left(\left|g_{A_j}\right\rangle - \left|e_{A_j}\right\rangle\right) \tag{16}$$

with $j = 1, 2$ for a time corresponding to half a Rabi cycle. Thus the state, after a bit of simplification, comes to be

$$\begin{aligned}\left|\Psi_B^{(\mathrm{cluster})}\right\rangle = \frac{1}{2}\Big[&\frac{1}{2}\{|B_{gg}\rangle + \mathrm{e}^{-2\,\mathrm{i}\,\pi\zeta}\,|B_{ge}\rangle - \mathrm{e}^{\mathrm{i}\,\pi\zeta}\,|B_{eg}\rangle - \mathrm{e}^{-3\,\mathrm{i}\,\pi\zeta}\,|B_{ee}\rangle\}\otimes|g_{A_1},g_{A_2}\rangle \\ &+\frac{1}{2}\{|B_{gg}\rangle - \mathrm{e}^{-2\,\mathrm{i}\,\pi\zeta}\,|B_{ge}\rangle - \mathrm{e}^{\mathrm{i}\,\pi\zeta}\,|B_{eg}\rangle + \mathrm{e}^{-3\,\mathrm{i}\,\pi\zeta}\,|B_{ee}\rangle\}\otimes|g_{A_1},e_{A_2}\rangle \\ &+\frac{1}{2}\{|B_{gg}\rangle + \mathrm{e}^{-2\,\mathrm{i}\,\pi\zeta}\,|B_{ge}\rangle + \mathrm{e}^{\mathrm{i}\,\pi\zeta}\,|B_{eg}\rangle + \mathrm{e}^{-3\,\mathrm{i}\,\pi\zeta}\,|B_{ee}\rangle\}\otimes|e_{A_1},g_{A_2}\rangle \\ &+\frac{1}{2}\{|B_{gg}\rangle - \mathrm{e}^{-2\,\mathrm{i}\,\pi\zeta}\,|B_{ge}\rangle + \mathrm{e}^{\mathrm{i}\,\pi\zeta}\,|B_{eg}\rangle - \mathrm{e}^{-3\,\mathrm{i}\,\pi\zeta}\,|B_{ee}\rangle\}\otimes|e_{A_1},e_{A_2}\rangle\Big]. \end{aligned} \tag{17}$$

Thus corresponding to any one of the four possibilities, i.e., $|g_{A_1},g_{A_2}\rangle$, $|g_{A_1},e_{A_2}\rangle$, $|e_{A_1},g_{A_2}\rangle$ and $|e_{A_1},e_{A_2}\rangle$, of the detection pattern of atoms through a state selective detector at Alice's end, we obtain an equally probable four-partite atomic cluster state prepared remotely at Bob's end. The scheme is inherently deterministic and an overall unit success probability is achievable in principle.

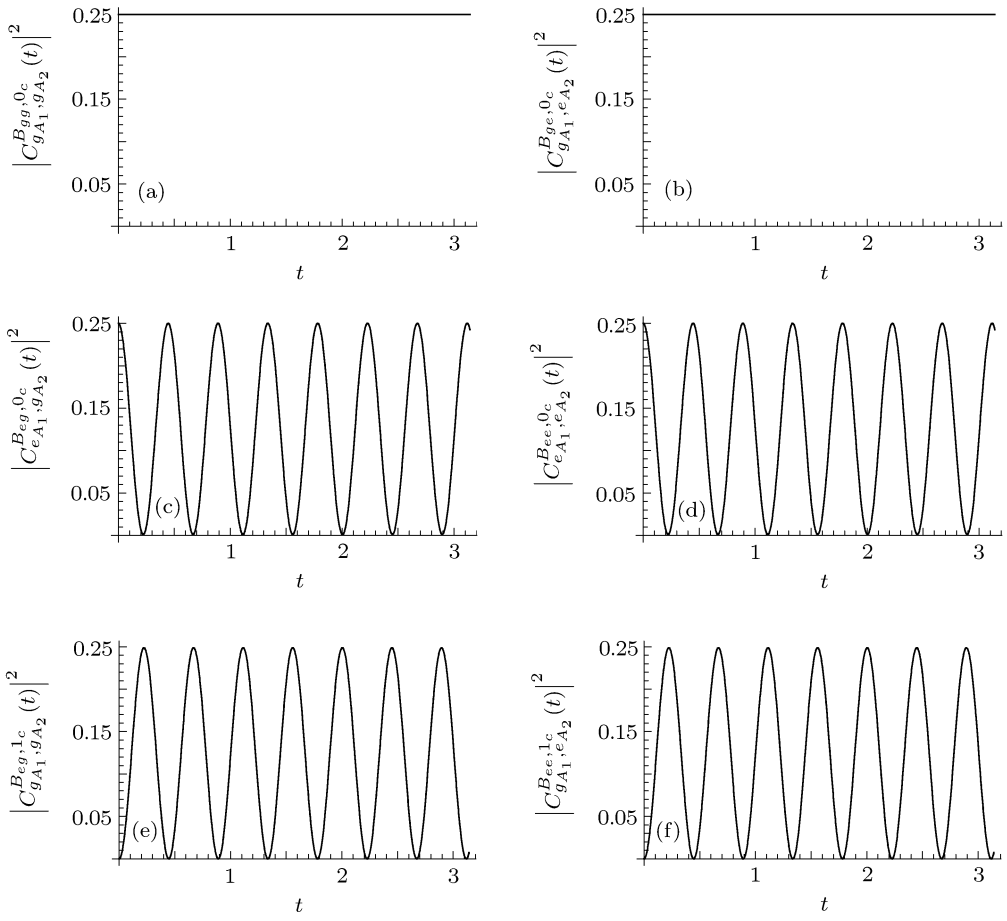


Fig. 1. The probabilities $\left|\, C^{B_{gg},0_c}_{g_{A_1},g_{A_2}}(t)\,\right|^2$, $\left|\, C^{B_{ge},0_c}_{g_{A_1},e_{A_2}}(t)\,\right|^2$, $\left|\, C^{B_{eg},0_c}_{e_{A_1},g_{A_2}}(t)\,\right|^2$, $\left|\, C^{B_{ee},0_c}_{e_{A_1},e_{A_2}}(t)\,\right|^2$, $\left|\, C^{B_{eg},1_c}_{g_{A_1},g_{A_2}}(t)\,\right|^2$ and $\left|\, C^{B_{ee},1_c}_{g_{A_1},e_{A_2}}(t)\,\right|^2$, plotted in (a), (b), (c), (d), (e) and (f) respectively, with respect to time. Here we have used $\mu = 2\pi\times 1.12\times 10^5$, $\Delta = 2\pi\times 8\times 10^5$ which gives $\lambda = \mu^2/\Delta = 9.85\times 10^3$.

3. Scheme for field state generation

Concerning the remote preparation of the cavity field cluster state at the receiving station (Bob's end), we assume that Alice and Bob are sharing two pairs of cavity field GHZ states of the type

$$\left|\Psi_{AB}^{(j)}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0_{A_j},0_{B_j}^{(1)},0_{B_j}^{(2)}\right\rangle + \left|1_{A_j},1_{B_j}^{(1)},1_{B_j}^{(2)}\right\rangle\right) \tag{18}$$

with $j = 1, 2$. For the generation of such states there are a number of proposals[15] given by many authors.

Proceeding towards the field cluster state generation, Alice sends two initially ground state two-level atoms through the cavities in her possession. These atoms perform resonant interactions for a time corresponding to half a Rabi cycle with their respective cavities. This effectively sweeps the quantum information from the cavities to the atoms while leaving cavities in the vacuum state. The cavities therefore can be subsequently traced out of the system. Thus, after interaction, each GHZ state transforms to

$$\left|\Psi_{AB}^{(j)}\right\rangle = \frac{1}{\sqrt{2}}\Big(\left|g_{A_j}, 0_{B_j}^{(1)}, 0_{B_j}^{(2)}\right\rangle - \mathrm{i}\left|e_{A_j}, 1_{B_j}^{(1)}, 1_{B_j}^{(2)}\right\rangle\Big), \tag{19}$$

where $\left|g_{A_j}\right\rangle$ ($\left|e_{A_j}\right\rangle$) stands for the ground (excited) state of atom-1 or atom-2. The initial product state of the two GHZ states therefore becomes

$$|\Psi_{AB}\rangle = \frac{1}{2}\prod_{j=1}^{2}\left(\left|g_{A_j}, 0_{B_j}^{(1)}, 0_{B_j}^{(2)}\right\rangle - \mathrm{i}\left|e_{A_j}, 1_{B_j}^{(1)}, 1_{B_j}^{(2)}\right\rangle\right) \otimes |0_c\rangle\,. \tag{20}$$

The remaining state engineering procedure is similar to the one described earlier for atomic cluster state generation which is being abbreviated here. Atom-1 and atom-2 interact simultaneously with an initially vacuum state cavity C. Atom-1 performs resonant interaction whereas at the same instant atom-2 goes through dispersive interactions caused by an applied Stark field. Finally the atoms are passed through a Ramsey zone prior to detection. The four-partite cavity field cluster state prepared remotely at Bob's end will be

$$\begin{aligned}\left|\Psi_B^{(\mathrm{cluster})}\right\rangle = \frac{1}{2}\Big[&\frac{1}{2}\{|B_{00}\rangle - \mathrm{i}\,\mathrm{e}^{-2\,\mathrm{i}\,\pi\zeta}\,|B_{01}\rangle + \mathrm{i}\,\mathrm{e}^{\mathrm{i}\,\pi\zeta}\,|B_{10}\rangle + \mathrm{e}^{-3\,\mathrm{i}\,\pi\zeta}\,|B_{11}\rangle\} \otimes |g_{A_1}, g_{A_2}\rangle \\ &+ \frac{1}{2}\{|B_{00}\rangle + \mathrm{i}\,\mathrm{e}^{-2\,\mathrm{i}\,\pi\zeta}\,|B_{01}\rangle + \mathrm{i}\,\mathrm{e}^{\mathrm{i}\,\pi\zeta}\,|B_{10}\rangle - \mathrm{e}^{-3\,\mathrm{i}\,\pi\zeta}\,|B_{11}\rangle\} \otimes |g_{A_1}, e_{A_2}\rangle \\ &+ \frac{1}{2}\{|B_{00}\rangle - \mathrm{i}\,\mathrm{e}^{-2\,\mathrm{i}\,\pi\zeta}\,|B_{01}\rangle - \mathrm{i}\,\mathrm{e}^{\mathrm{i}\,\pi\zeta}\,|B_{10}\rangle - \mathrm{e}^{-3\,\mathrm{i}\,\pi\zeta}\,|B_{11}\rangle\} \otimes |e_{A_1}, g_{A_2}\rangle \\ &+ \frac{1}{2}\{|B_{00}\rangle + \mathrm{i}\,\mathrm{e}^{-2\,\mathrm{i}\,\pi\zeta}\,|B_{01}\rangle - \mathrm{i}\,\mathrm{e}^{\mathrm{i}\,\pi\zeta}\,|B_{10}\rangle + \mathrm{e}^{-3\,\mathrm{i}\,\pi\zeta}\,|B_{11}\rangle\} \otimes |e_{A_1}, e_{A_2}\rangle\Big].\end{aligned} \tag{21}$$

Here, again for the sake of mathematical brevity we have taken

$$|B_{00}\rangle = \left|0_{B_1}^{(1)}, 0_{B_1}^{(2)}, 0_{B_2}^{(1)}, 0_{B_2}^{(2)}\right\rangle, \quad |B_{01}\rangle = \left|0_{B_1}^{(1)}, 0_{B_1}^{(2)}, 1_{B_2}^{(1)}, 1_{B_2}^{(2)}\right\rangle, \quad |B_{10}\rangle = \left|1_{B_1}^{(1)}, 1_{B_1}^{(2)}, 0_{B_2}^{(1)}, 0_{B_2}^{(2)}\right\rangle$$

and $|B_{11}\rangle = \left|1_{B_1}^{(1)}, 1_{B_1}^{(2)}, 1_{B_2}^{(1)}, 1_{B_2}^{(2)}\right\rangle$. It is interesting to note that the procedure described above can be equally utilized to locally engineer a four-partite atomic/cavity field cluster state using two pairs of Bell states, i.e., $\left|\phi_{j,k}^{(+)}\right\rangle = (|g_j, g_k\rangle + |e_j, e_k\rangle)/\sqrt{2}$ with $j, k = 1, 2$ $(3, 4)$. When Ising interaction based on the above described controlled phase operation is performed on qubit-2 and qubit-3, the engineered four-partite atomic cluster state will be of the type

$$|\Psi\rangle = \frac{1}{2}\{|g_1, g_2, g_3, g_4\rangle + |g_1, g_2, e_3, e_4\rangle + |e_1, e_2, g_3, g_4\rangle - |e_1, e_2, e_3, e_4\rangle\}, \tag{22}$$

which is a standard cluster state.

4. Success probability and fidelity

The success probability and fidelity of the schemes depend on many factors including delays in simultaneous injection of atoms into the cavity, velocity spread around the mean velocity of the atoms, operational imprecision in the interaction times of the atoms with the quantised as well as classical fields and coupling dispersion inside the cavity. Another somewhat relevant parameter that might reduce success probability is spontaneous emission. This is, however, being ignored here, keeping in view the recent technological developments of high-Q cavities[19], including those of microcavities[20] having very high mode densities. This effectively suppresses the spontaneous emission probability to a level where it can be safely ignored in the mathematical treatment of the problem. Furthermore, in order to avoid free space atomic evolution with a slightly higher spontaneous emission probability, we can handle the same task using a single bimodal cavity.

Anyhow, the major factors that may appreciably decrease the success probability of the proposal are delay time $\Delta t_{\rm d}$ in simultaneous injection and uncontrollable imprecision in the atom-field interaction time $\Delta t_{\rm int}$ inside the cavity. This is because most of the parameters like deviation from the monochromaticity of atomic velocities and coupling dispersion can be operationally incorporated into $\Delta t_{\rm int}$. The mathematical expression for success probability $P_{\rm success}$ versus $\Delta t_{\rm d}$ and $\Delta t_{\rm int}$ is given by

$$P_{\rm success} = \frac{3(4\delta^2+\mu^2)+\cos(2\mu\Delta t_{\rm d})(\mu^2+4\delta^2\cos\left(\sqrt{4\mu^2+\frac{\mu^4}{\delta^2}}\Delta t_{\rm int}\right)}{4(4\delta^2+\mu^2)}. \tag{23}$$

In Fig. 3, we show the success probability for the remote generation of a four-partite cluster state versus delay $\Delta t_{\rm d}$ in simultaneous injection of atoms and uncontrollable imprecision in quantised atom-field interaction time $\Delta t_{\rm int}$. Similarly, fidelity snapshot sketches versus variations in $\Delta t_{\rm d}$ for various fixed $\Delta t_{\rm int}$ are shown in Fig. 4.

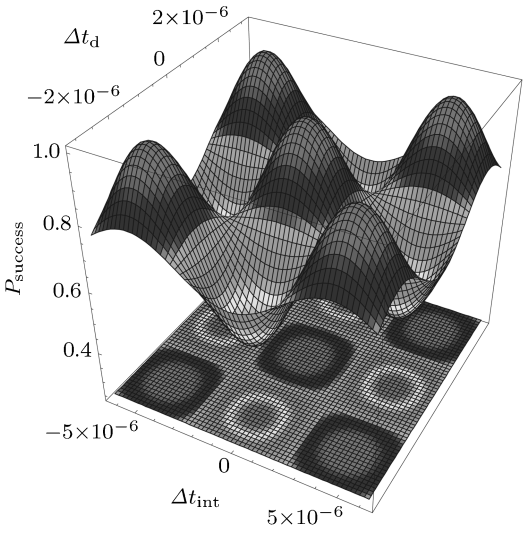


Fig. 3. Success probability when the atoms are injected simultaneously. The figure shows explicitly the maximality of the entangled state (13), here we plot for Δ and μ as stated earlier.

Now the dominant contribution in the deterioration of success probability and fidelity through both the injection time delay as well as interaction time indeterminacy comes from the initial velocity spread of the atoms employed in the engineering procedure. Oven-based atomic beam sources have been demonstrated to yield a velocity spread of only ±2 m/s when the mean velocity is taken to be about 503 m/s.[16] However, much better control over atomic motion with production of almost monochromatic atomic beams with minimum velocity spread can now be achieved using pre-cooled atoms out of a magneto-optical trap.[21]

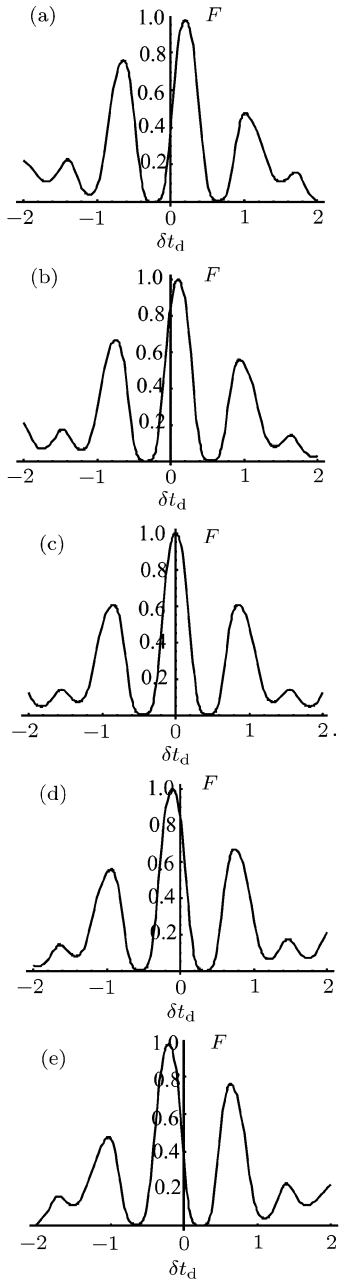


Fig. 4. Fidelity plot showing that it reaches the ideal value of 1 when the atoms have zero injection time difference as well as zero interaction time difference. In Figs. 4(a), 4(b), 4(c), 4(d), and 4(e) $\delta t_{\rm d}$ has a multiplicative factor of 10^{-5}, and $\delta t_{\rm int}$ takes values of -2×10^{-5}, -1×10^{-5}, 0, 1×10^{-5} and 2×10^{-5}, respectively. Here again the values of Δ and μ are taken as stated earlier.

5. Results and discussions

Our proposed schemes for the generation of atomic as well as field cluster states bears good features for experimental execution. They do not require any excessive atomic or cavity resources and are based on standard cavity QED techniques. Experimental cavity QED has acquired the status of a mature technology[22] and by now cavity quality factors in the range of 10^{12} have been reported leading to life-times up to seconds that are many orders of magnitude higher than the usual atom-field interaction times.[23] Implementation of Ramsey zones and induced dispersive interactions through an applied Stark field have already been mastered and used in many experiments[17,24] including successful demonstration of a quantum phase gate.[25] Moreover, cavity QED performance have already been demonstrated at the lowest quantum level.[26] Keeping all these favourable parameters in view, we may safely conclude that the proposed schemes are experimentally feasible and may yield results with overall good success probabilities and fidelities when assessed in the background of the currently available technology.

It is interesting to note that the schemes presented above can be generalised to the remote preparation of any arbitrary graph state either in atomic or field variables. This is owing to the novelty of the phase gate operation being utilized here. The basic building blocks for engineering of any arbitrary graph morphology are taken to be a Bell state $|\Psi\rangle = (|\alpha_A, \alpha_B\rangle + |\beta_A, \beta_B\rangle)/\sqrt{2}$, shared between Alice and Bob. Here α, β can be taken as either atomic internal state or cavity field variables.

Now if Alice and Bob are assumed to be in possession of n such states, i.e., $|\Psi\rangle = \left(\left|\alpha_A^{(j)}, \alpha_B^{(j)}\right\rangle + \left|\beta_A^{(j)}, \beta_B^{(j)}\right\rangle\right)/\sqrt{2}$ with $j = 1, 2, \ldots, n$, then Alice can prepare any desired graph state at Bob's end through LOCC. These local operations are the utilization of consecutive phase gates described earlier in a sequence deemed fit for the remote preparation of the desired graph state.

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