The 2.7 K black-body radiation background reference frame*

López-Carrera Benjamín[a)], Rosales Marco Antonio[b)], and Ares de Parga Gonzalo[a)]

[a)] *Departamento de Física, Escuela Superior de Física y Matemáticas, Instituto Politécnico Nacional, U.P.A. López Mateos, Zacatenco* 07738, *México DF, México*

[b)] *Departamento de Física, Universidad de las Américas, Santa Catarina Mártir, C.P.* 72820, *Cholula, Puebla. México*

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This paper reports that the directional temperature is used to present a scheme for deducing the velocity of the reference frame where the black-body which produces the 2.7 K radiation background is at rest. The new renormalized relativistic thermodynamics lays the foundations of the method.

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1. Introduction

Since Planck[1] and Einstein[2] (PE) developed relativistic thermodynamics in 1907, a countless number of different proposals have appeared.[3] However, at the end of his life Einstein communicated to van Laue[4] his doubts about the validity of the laws of relativistic transformations of thermodynamics. Besides this, there were no important discrepancies in this respect and all the theory was resumed in the books of Tolman[5] and Pauli.[6] Nevertheless, in 1964, Ott[7] published his posthumous paper where he proposed a new set of relativistic transformation laws in thermodynamics and the subject newly called the attention of many researchers. More than a hundred articles appeared during the sixties and seventies of the last century, defending or contradicting the PE proposal. Many theories were presented incorporating new variants. For example, we can mention the works realized by Arzeliès,[8] Rohrlich[9] and Landsberg[10] among others. Landsberg[11] suggested that just the experiment could clarify the controversies. Nevertheless, in 1968, Balescu[3] presented a statistical theory of the PE proposal demonstrating that not only was it a unique model which preserves the invariant form of thermodynamics but that any other scheme could be obtained by using a gauge transformation. However, at the beginning of this century, most of the papers were close to the PE theory as is the case with Sieniutycz's article.[12] In the meantime, Landsberg and Matsas[13,14] published two works where they claimed to have demonstrated that due to the equivalence between the frequency number density for a moving black-body and the excitation rate of the Unruh–DeWitt detector,[15] a relativistic transformation law of the temperature cannot be defined. Their reasoning was based on the non-Planckian form of the frequency number density. Apparently, the Landsberg and Matsas papers explained the origin of all controversies. However, Ares de Parga *et al.*[16] (the AA proposal) integrated the frequency density number over all the particle frequencies and obtained the real number of particles. Then they calculated the internal energy multiplying the frequency number density by the frequency quantum of energy and integrating over all the frequencies, they got the PE transformation law of the energy. Using finite time thermodynamics, Ares de Parga *et al.*[16] found an exact differential which permitted them to define thermal quantities which preserve the invariant form of thermodynamics that was misconceived in the PE and Balescu proposals. The AA proposal was based on comparing the transformation law of the free Helmholtz energy[17] with the transformation law of the internal energy. The final conclusion consists in accepting that the renormalized relativistic thermodynamics permits not only a well definition of the temperature and its relativistic transformation but also an invariant form of the thermodynamics. In this order of ideas, we can assure the existence of a relativistic temperature, which transforms as: $T = \gamma^{-1} T_{\rm o}$, where T, $T_{\rm o}$ are the temperatures at the moving system and at the rest frame

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respectively and $\gamma = 1/\sqrt{1-u^2/c^2}$, where u represents the velocity between both systems. With this result, we are able to analyse the black-body distribution of 2.7 K radiation background. The idea consists in developing a method to deduce the direction and the magnitude of the velocity of the frame where the black-body which produces the 2.7 K radiation background (cosmic microwave background (CMB)) is at rest.

The article is organized as follows: Section 2 is advocated to connect the Henry expression of the particle number density and the excitation rate of the Unruh–DeWitt detector with the renormalized relativistic thermodynamics. A technique is developed in Section 3 in order to deduce the velocity of the reference frame where the 2.7 K black-body radiation background (or CMB) is at rest. In Section 4, some concluding remarks are exposed about the renormalized relativistic thermodynamics and the existence of more than one Big Bang.

2. The renormalized relativistic thermodynamics

As we have noticed, Landsberg and Matsas[13,14] claimed the non-existence of a relativistic transformation of the temperature. Let us summarize how they arrived at this conclusion. If we consider a black-body at rest in a frame $K_{\rm o}$ with temperature $T_{\rm o}$, the particle number density is a Planckian function. On the other hand, when the black-body is observed from another reference frame K at a constant motion with respect to $K_{\rm o}$, the particle number density between frequencies ω and $\omega + {\rm d}\omega$ for a solid angle (${\rm d}\Omega$) is expressed by[18−20]

$$n(\omega, T_\theta){\rm d}\omega{\rm d}\Omega = \frac{\omega^2/c^3}{2\pi^2\left(\left(\exp\dfrac{\hbar\omega}{kT_\theta}\right)-1\right)}{\rm d}\omega{\rm d}\Omega$$

with

$$T_\theta = \frac{T_{\rm o}\sqrt{1-\dfrac{u^2}{c^2}}}{1-\dfrac{u}{c}\cos\theta}, \tag{1}$$

where T_θ represents the directional temperature, θ the angle between the axis of the motion and the direction of observation in the frame K and u the speed between both systems. After integrating over the solid angle, we arrive at

$$n(\omega, T_{\rm o}, u){\rm d}\omega = \frac{\omega k T_{\rm o}\sqrt{1-\dfrac{u^2}{c^2}}}{2\pi^2c^2u\hbar}\ln\frac{1-\exp-\left(\dfrac{\hbar\omega\sqrt{1+\dfrac{u}{c}}}{kT_{\rm o}\sqrt{1-\dfrac{u}{c}}}\right)}{1-\exp-\left(\dfrac{\hbar\omega\sqrt{1-\dfrac{u}{c}}}{kT_{\rm o}\sqrt{1+\dfrac{u}{c}}}\right)}{\rm d}\omega, \tag{2}$$

which is proportional to the excitation rate of the Unruh–DeWitt detector.[15] Landsberg and Matsas[13,14] asserted that since the frequency particle number density does not present a Planckian form it is impossible to define a universal relativistic transformation of the temperature even if averages of the directional temperature are tried. Nevertheless, by integrating over all the frequencies, Ares de Parga *et al.*[16] found that the number of particles is conserved, that is:

$$N = V\int_0^\infty n(\omega, T_{\rm o}, u){\rm d}\omega = \frac{2\zeta(3)}{\pi^2}\left(\frac{T_{\rm o}}{\hbar c}\right)^3, \tag{3}$$

as was expected.[16] Due to this last result, they calculated the internal energy and obtained a surprising result:

$$E = \frac{4}{c}V\sigma T_{\rm o}^4\gamma^2\left(1+\frac{u^2}{3c^2}\right), \tag{4}$$

where σ is the Stefan–Boltzmann constant. Equation (4) coincides with the PE relativistic transformation of the energy[1,2] for the black-body case. This result demonstrated not only the possible existence of a relativistic transformation law for the energy but also for the temperature. On the other hand, by using finite time thermodynamics, Ares de Parga *et al.*[16] showed that the efficiency of the thermal cycle was a relativis-

tic invariant under any kind of relativistic transformation law for the different proposal summarized in Balescu's paper.[3] Moreover, analysing the adiabatic free expansion of an ideal gas from a moving frame, it is a straightforward result to demonstrate that the PE relativistic transformation law of the temperature is the correct one. Nevertheless, although Balescu[3] claimed the invariant form of thermodynamics under the PE theory, the relation between the Helmholtz free energy and the internal energy fails to be conserved. For this reason we have to introduce what we have called the process of renormalization in relativistic thermodynamics.[16] The more important results are the following ones: first of all we define a renormalized energy or thermal energy as[16]

$$\xi = E - W_{o\to E_o}$$

where

$$W_{o\to E_o} = \gamma\frac{u^2}{c^2}(E_o + PV_o). \tag{5}$$

The thermal energy density is defined by $\varsigma = \xi/V$. A set of simple transformations is obtained leading to an invariant form of thermodynamics. Once we have a consistent relativistic thermodynamics, we can analyse the black-body case. The Stefan–Boltzmann law is now expressed in an interesting form:

$$\varsigma = \varsigma_o = \varepsilon_o = \frac{4}{c}\sigma T_o^4 = \frac{4}{c}\sigma\gamma^4 T^4, \tag{6}$$

where ς_o and ε_o denote the thermal and internal energy densities in the rest frame, respectively. Equations (6) and (1) and the certainty of having a consistent relativistic thermodynamics theory will permit us to analyse the 2.7 K black-body radiation background.

3. Calculation of the velocity of the 2.7 K black-body radiation background reference frame

It is necessary to comment on two important aspects. The first one consists in analysing Dirac's considerations about the existence of a frame where the black-body which produces the 2.7 K radiation background is at rest. Indeed, in 1979, during an Einstein's centennial commemorative congress at the University of Illinois at Carbondale, Dirac[21] claimed that due to the dipole anisotropy of the radiation background, it will be possible to find the velocity of such a frame. By extrapolation, we will be able to determine the reference frame where the Big Bang occurred and it will represent a privileged frame contradicting the general theory of relativity. In the same congress, differing from Dirac's argument, Wigner[21] commented that general relativity just assures the equivalence of the laws of the nature in each reference frame and it indicates nothing about the initial conditions and so the discovery of a reference frame at rest with the Big Bang will not contradict the general theory of relativity. The second comment corresponds to the note that Henry *et al.*,[19] by the time when the 2.7 K radiation background was discovered, assured that the sole effect of a uniform motion through black-body cavity is to introduce an effective temperature which replaces the rest frame cavity temperature for each angle. Both comments and the uncertainty about the correct relativistic transformation law of the temperature, left to the side the problem of searching for the mentioned frame. Considering the black-body radiation background frame as being at rest in the middle of the Milky Way, Henry explains Partridge and Wilkinson's anisotropy[22] of the 2.7 K radiation background by a $\cos 2\theta$ effect. The technique was based on considering that the relative motion between the solar system and the 2.7 K radiation background was small in order to consider $u/c \ll 1$ which simplifies the calculation for obtaining the angular effects. Nevertheless, the assumption that the 2.7 K radiation background reference frame coincides with the motion of the middle of the Milky Way is very strong. Indeed, it implies that the Big Bang frame was the same as that of the Milky Way. But following Wigner's ideas and accepting that the renormalized relativistic thermodynamics represents the way of describing the relativistic transformation laws of thermodynamics, we are in the position of dealing with the search of the mentioned reference frame without assuming or making any approximation.

We will base our reasoning in that the high skill of the experimentalist could design a detector for radiation coming from a chosen direction and to measure the particle frequency number density between frequencies ω and $\omega + \mathrm{d}\omega$ over the solid angle $\mathrm{d}\Omega$, $n(\omega, T_\theta)$, represented by Eq. (1). The anisotropy of the space, the galactic background, sun's radiation, the red shifts, and, in the case of earth's detector, atmosphere effects among others, must be considered and subtracted from the result of the detector in order to obtain $n(\omega, T_\theta)$.[23–25]

Multiplying Eq. (1) by $\hbar\omega$, we obtain a density E which is not exactly the energy density since for

having it, it would be necessary to integrate over the solid angle and the result would be proportional to the excitation rate of the Unruh–DeWitt detector. Nevertheless, the obtained density E will be exact as the energy density of a black-body, i.e. the corresponding energy density of a Planckian number particle density with temperature T_θ. Therefore, it is straightforward to notice that when the angle coincides with the velocity of the black-body, the density is a maximum with respect to the frequency. We call this angle the principal angle and we denote it as $\theta_{\rm p} = 0$. This is because the corresponding directional temperature is the biggest among all the other directional temperatures and consequently $E(\omega, T_{\theta=0})$ is the maximum particle frequency number density over all the angles θ. Thus the direction of the searched reference frame may be found just by choosing the direction for which the density $E(\omega, T_\theta)$ is a maximum and defining this angle as $\theta = 0$. Once we get what we will call the principal angle or direction, we can know the principal directional temperature $T_{\theta=0}$ by simply measuring the frequency $\omega_{m,\theta=0}$ which maximizes the density E.[16] That is

$$\omega_{m,\theta=0} \quad \text{is such that} \quad \frac{{\rm d}E_{\omega,\theta=0}}{{\rm d}\omega} = 0 \quad \text{being}$$

$$E_{\omega,\theta=0} = \hbar\omega \left[n(\omega, T_{\theta=0})\right]. \tag{7}$$

Since $E_{\omega,\theta=0}$ has a Planckian form, the maximum frequency will satisfy

$$\frac{\hbar\omega_{m,\,\theta=0}}{T_{\theta=0}} = 2.822. \tag{8}$$

We relate the velocity u with the directional temperature $T_{\theta=0}$, by using Eq. (1), the result is

$$T_{\theta=0} = T_{\rm o}\frac{\sqrt{1-u^2/c^2}}{1-u/c} = T_{\rm o}\sqrt{\frac{1+u/c}{1-u/c}}. \tag{9}$$

Thus by experimentally knowing the frequency $\omega_{m,\theta=0}$, we deduce the directional temperature $T_{\theta=0}$ and then we obtain the magnitude of the velocity u as a function of $T_{\rm o}$ and $T_{\theta=0}$. But we do not know the value of $T_{\rm o}$. We need another relation. This is easy to find since for $\theta = \pi$ the directional temperature is the lowest among all the angles. So the density $E(\omega, T_{\theta=\pi})$ is the lower one. We can now repeat the same process for $\theta = \pi$ and by knowing experimentally the maximum frequency $\omega_{m,\theta=\pi}$ for this angle, we deduce $T_{\theta=\pi}$ and then we have

$$T_{\theta=\pi} = T_{\rm o}\frac{\sqrt{1-u^2/c^2}}{1+u/c} = T_{\rm o}\sqrt{\frac{1-u/c}{1+u/c}}. \tag{10}$$

Using Eqs. (9) and (10), we obtain

$$\frac{\omega_{\theta=\pi}}{\omega_{m,\theta=0}} = \frac{T_{\theta=\pi}}{T_{\theta=0}} = K = \frac{1-u/c}{1+u/c} \Rightarrow u = c\frac{1-K}{1+K}, \tag{11}$$

where K can be called the ratio between the lower directional temperature and the bigger directional temperature and so if we experimentally know the frequencies $\omega_{\theta=0}$ and $\omega_{\theta=\pi}$ and the principal direction, we will be able to deduce the velocity of the black-body which produces the 2.7 K radiation background. We have to notice that this method permits the searched velocity to be obtained without making any kind of approximation. That is, the 2.7 K reference frame does not have to be at a slow speed with respect to the detector.

4. Concluding remarks

Starting by considering that an experimental data bank of the frequency particle number density over each differential solid angle ($n(\omega, T_\theta)$) is available, we could obtain the direction and the velocity between the detector and the frame where the black-body which produces the 2.7 K radiation background is at rest. Of course, as we mentioned in Section 3, the high skill of the experimentalist is required since problems such as the anisotropy of the space, the galactic background and red shift effects among others[23–25] must be taken into account in the detector's measurements.

Henry *et al.*[18] believed that due to the symmetry of the Planckian form of $n(\omega, T_\theta)$, the directional temperature was just a parameter whose sole effect was replacing the rest temperature of the black-body. That is, for each angle $n(\omega, T_\theta)$ will be a Planckian function and T_θ will play the role of an effective temperature. Despite Henry's argument, due to renormalized thermodynamics, the transformed temperature represents a meaningful physical concept and not just a convention. Thus considering the temperature on a moving system, it is possible to find the direction and the speed of the frame where the black-body which produces the 2.7 K radiation background is at rest without assuming its slow motion with respect to the detector.

It is important to notice an interesting result, which is straightforwardly obtained. Indeed, the appropriate measurement of the radiation background temperature would have to take into account Eq. (6)

and consequently as renormalized thermodynamics predicts

$$T_{\rm o} = \gamma T. \tag{12}$$

If we review Henry's paper,[18] then we notice that the only effect of the moving system was considered in the anisotropy of the radiation background. Consequently, by using the approximation of $u/c \ll 1$, it implies that $T = T_{\rm o}$ and both systems were at temperature 2.7 K. Without this consideration, we cannot assure that the radiation background was at temperature $T_{\rm o} = 2.7$ K. In order to obtain the temperature of the radiation background we need to use the methodology of Section 3. What we know is that T is close to 2.7 K and since $\gamma > 1$, the radiation background temperature $T_{\rm o}$ must be bigger than 2.7 K and consequently the universe could be younger.

Nevertheless, by using a different method, Lineweaver[21] has shown that the direction of the reference frame, such that the black-body which carries on the 2.7 K radiation background is at rest, corresponds to the Hydra–Centaur direction with a speed of 627 km·s^{-1}. Therefore, since the speed of the reference frame is 627 km·s^{-1}, which is very small in comparison with the speed of light, we can be assured that the temperature 2.7 K is correct; that is: it represents the temperature of the CMB at rest since $\gamma \approx 1$. If the detector is put in the Hydra–Centaur direction with a speed of 627 km·s^{-1}, the particle number density will have to be independent of the angle, that is, the dipolar effect should disappear. If not, Wigner will be wrong and Dirac will possess the truth.[21]

On the other hand, it has to be noticed that the renormalized relativistic thermodynamics represents a consistent theory which is supported by a statistical theory[27] and a covariant theory.[28] The Henry expression and the excitation rate of the Unruh–DeWitt detector can be derived by using renormalized relativistic statistical mechanics.[29] Other theories proposed by van Kampen and Israel[30] or recently by Nakamura,[31] even if they are very close to the renormalized relativistic thermodynamics, do not leave invariant under relativistic transformations; the thermodynamics consequently presents some inconsistencies.

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