Opinion evolution based on cellular automata rules in small world networks

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In this paper, we apply cellular automata rules, which can be given by a truth table, to human memory. We design each memory as a tracking survey mode that keeps the most recent three opinions. Each cellular automata rule, as a personal mechanism, gives the final ruling in one time period based on the data stored in one's memory. The key focus of the paper is to research the evolution of people's attitudes to the same question. Based on a great deal of empirical observations from computer simulations, all the rules can be classified into 20 groups. We highlight the fact that the phenomenon shown by some rules belonging to the same group will be altered within several steps by other rules in different groups. It is truly amazing that, compared with the last hundreds of presidential voting in America, the eras of important events in America's history coincide with the simulation results obtained by our model.

Keywords: cellular automata rules, truth table, memory, simulation

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1. Introduction

Cellular automata (CA), previously introduced by Von Neumann in the early 1950s, are a class of spatially and temporally discrete mathematical systems characterised by local interactions and an inherently parallel form of evolution. Due to the simple mathematical constructions and distinguishing features, CA have been used as abstract models for systems in physics and other areas of science, to explain lots of complex systems, including reaction-diffusion processes,[1,2] growth phenomena,[3] memory training,[4,5] and some others.[6−9] In 1969, Hedlund, who viewed one-dimensional cellular automata (1D CA) in the context of symbolic dynamics as endomorphisms of the shift dynamical system, set out to study the topological dynamics of CA.[10] In 1983, Wolfram worked on the analysis of dynamical systems and studied details for CA. He described the process of evolution by the concept of entropy and divided the $2^{2^3} = 256$ elementary 1D CA rules into four classes in his book, 'A New Kind of Science'.[11] Based on this work, Chua *et al.* provided a rigorous nonlinear dynamical approach to Wolfram's empirical observations from the viewpoint of mathematical analysis.[12−17]

As a method for collecting information concerning people's choices for the same question or the results of a special event with a running time, tracking surveys have been widely used in many areas, such as the American election. Time must be greatly valued during the tracking survey. Since people's attitudes toward the same thing do change as time goes by, spending some periods of time for the survey can help us to understand the cause of change better. Every person would keep an array of feedback within a particular time. With these data, we can analyse the developments of people's opinions in a social investigation, or medical workers would confirm whether a drug has a chronic side-effect.

In this work, a tracking survey is combined with CA rules to evaluate the evolution of people's attitudes to the same question. The paper is outlined as follows: Section 2 presents the basic concepts of 1D CA, and our model could be raised to be able to classify 256 CA rules into 20 sorts by the varieties in evolution. Section 3 shows that the sequence of American presidents has a periodic change, which is identical with the simulation results of our model. Finally, Section 4 gives the conclusion and prospects for future studies.

2. The model

A cellular automaton is specified by a d-dimensional regular discrete lattice L with boundary conditions and S a finite set of states; x_i assigned to

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each node or cell C_i of the lattice. A local rule f acting on the cells in the range r of the neighbourhood N_i^r of each cell C_i determines the dynamics of the cellular automaton in discrete time steps starting from an initial condition.

We consider one-dimensional elementary cellular automata (1D ECA) in a finite case, that is, a CA with n a finite number of cells, $S = \{0, 1\}$, and nearest neighbourhood $r = 3$. Thus, each cell C_i can be assumed to be a dynamical system, with an intrinsic state x_i, an output y_i, and three inputs u_{i-1}, u_i, and u_{i+1}, where u_{i-1} denotes the input from the left neighbouring cell C_{i-1}, u_i denotes the 'self' input and u_{i+1} denotes the input from the right neighbouring cell C_{i+1}. In this case, there are 256 different possible local rules $y_i = f(u_{i-1}, u_i, u_{i+1})$. The n cells are subject to periodic boundary conditions and their states x_i are updated synchronously by the local rule.

Local rules are given by a truth table, and the rule number can be identified. For a cell C_i, there are 2^3 possible combinations of input values and indicating, for each combination, the output $y_i = (\gamma_7, \gamma_6, \gamma_5, \gamma_4, \gamma_3, \gamma_2, \gamma_1, \gamma_0)$. The corresponding decimal number is simply the integer $N = \gamma_7 \cdot 2^7 + \gamma_6 \cdot 2^6 + \gamma_5 \cdot 2^5 + \gamma_4 \cdot 2^4 + \gamma_3 \cdot 2^3 + \gamma_2 \cdot 2^2 + \gamma_1 \cdot 2^1 + \gamma_0 \cdot 2^0$, called its rule number. For example, if $y_i = (\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7) = (0, 1, 0, 1, 1, 0, 1, 0)$, then the rule number is $N = 0 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 90$.

We assume that each input is a constant integer $u_i \in \{-1, 1\}$, and the output $y_i(t)$ converges to a constant $y_i \in \{-1, 1\}$ from a zero initial condition $x_i(0) = 0$. For simplicity we use –1 instead of 0 as the value of variables. Thus, we can utilise a 'Boolean cube' with grey–black vertices to represent the truth table (see Table 1) as Chua did in Refs. [12] and [13].

Table 1. Truth table for a local rule.

n	u_{i-1}	u_i	u_{i+1}	y_i
7	1	1	1	γ_7
6	1	1	0	γ_6
5	1	0	1	γ_5
4	1	0	0	γ_4
3	0	1	1	γ_3
2	0	1	0	γ_2
1	0	0	1	γ_1
0	0	0	0	γ_0

Here each input (u_{i-1}, u_i, u_{i+1}) is identified as a vertex of a cube centred at the origin. Each of these eight vertices has one of the integers $0, 1, 2, \cdots, 7$, by reverting back to its corresponding three-bit binary word. The colour of each vertex means that: grey when y_i is '–1', or black when y_i is '1' in row 'n' of the truth table. It is important to know that the Boolean cube contains exactly the same information as the truth table. Given the Boolean cube shown in Fig. 1, it is easy to identify its rule number by simply adding the decimal number 2^k for each integer k associated with a black vertex. Also, the rule number associated with the four black 1,3,4,6 vertices is equal to $2^1 + 2^3 + 2^4 + 2^6 = 90$, which is equal to what we obtained from the truth table.

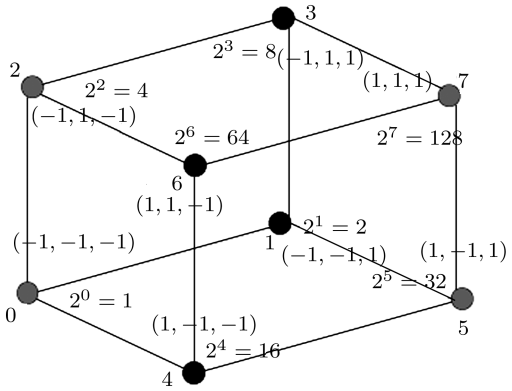


Fig. 1. A Boolean cube with coloured vertices.

3. The model

CA have been used to transit messages in chips[18–22] with millions of CMOS transistors. In this subsection, we try to use CA rules to ascertain the decision from the limited memory of humans.

In 2000, Sznajd-Weron proposed a very simple model based on the Ising spin system, which has already been applied to many fields, such as politics, marketing and finance.[23] The model was aimed at describing global social phenomena (sociology) by local social interactions. All the vertices have the answers 'yes' or 'no'. We notice that the idea of binary choices consists with the principle of the truth table in a sense. This connection provides an intriguing and valuable bridge for researching the complexity of small world networks under the CA rules.

In our model, small world networks[24,25] are adopted in order to represent real social networks. The one-dimensional network is made of $N = 100$ identical cells with a periodic boundary condition.

Each cell 'i' is coupled only to its left two neighbours and right two neighbours $(i-2, i+2)$ at first. The final $\overline{k}$ for the networks is 8. Assume all the vertices have the answers 'yes' or 'no', the same as Sznajd's model. 1 and -1 are used to replace two opposite opinions, respectively. Initially, we will assign a random value to each vertex, which will keep both opinions half at the beginning, and it can be easily proved that the initial states of the networks will not influence the result.

Considering the influence from one's neighbours, every vertex in the small world networks is allocated a memory of length $L = 3$ to store information from the mass in three periods, respectively. Thus, the stored information is 1 if and only if the majority's opinion around it has the value 1 and vice versa, and this message will be taken to its memory storeroom 1. This is the same situation as a tracking survey in opinion diffusion.

Once the new information in the following period is successfully processed, the fresh message will be stored in memory storeroom 1, while the data that was previously stored in 1 would be delivered to memory storeroom 2 (see the state of 1 in Table 2 during 3 time steps). After the next piece of data is processed, the latest massage will be always stored in memory storeroom 1, and the data previously stored in memory storeroom 1 and 2 will delivered to memory storeroom 2 and 3, respectively. Since then, there is no vacant storeroom. An evaluation for the question could be made by the most recent three feedbacks, that is, everyone would give his eventual decision by using a CA rule based on the data stored in the three memory storeroom. After that, a new round of gathering information starts.

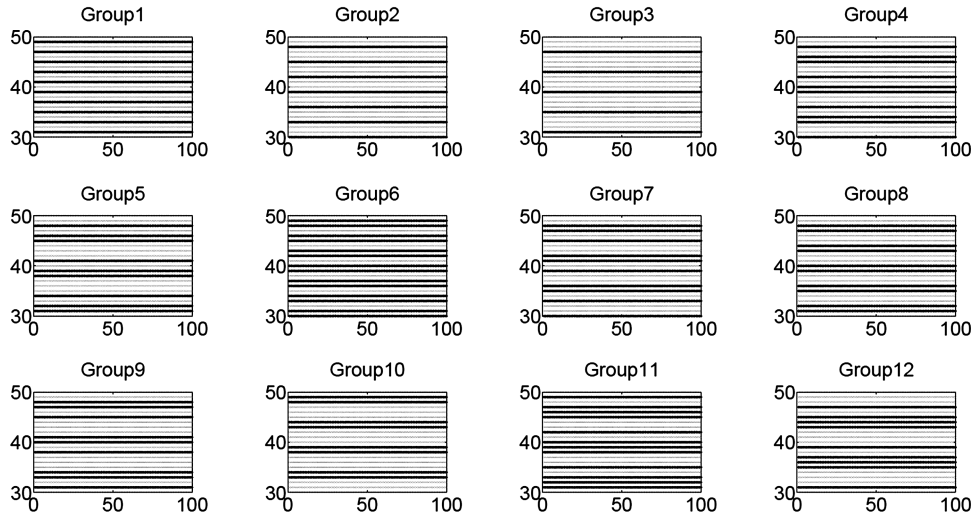
Table 2. Evolution of opinion in memory.

time	memory store1	memory store2	memory store3
$T-1$	1	-1	-1
T	1	1	-1
$T+1$	-1	1	1

When the new date collection is completed but the database is filled, the investigator deletes the data in memory storeroom 3, and these three memory storerooms will welcome the new tenants in turn. More precisely, the information of memory storerooms $3, 2, 1$ will be replaced by that previously stored in memory storerooms 2, 1 and the new data, respectively, then a fresh evaluation happens.

It is important to note that everyone in the model would use a rule to fix the output, which would be his current opinion. Corresponding to distinct ideology in diverse societies and times, all the 256 rules would be taken into account, and a different rule means a different scheme for the final decision. While under rule 90, if someone's current memory is $(-1, 1, -1)$, using Fig. 1, we obtain the output $\gamma_2 = -1$, which is the opinion he or she holds.

Though there are more than two hundred rules, they show periodic states after several steps with random values that keep the opposite initial opinions equally. We classify them to 20 groups based on the outcomes (see Table 3 and Fig. 2).



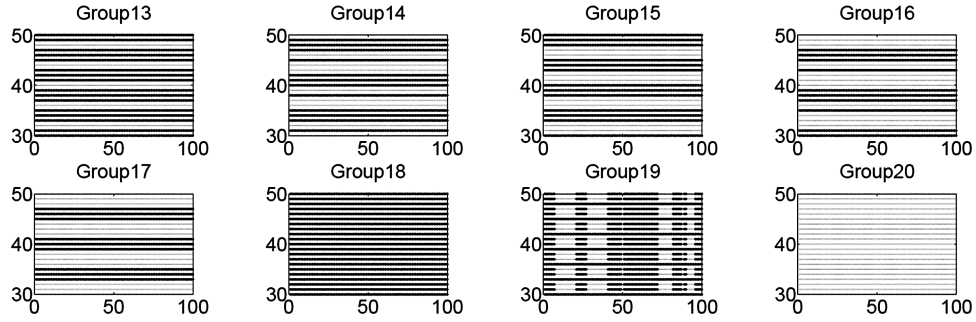


Fig. 2. The final periods of Groups No.1–20 are shown. In each figure, the X-axis shows all the 100 vertices and the Y-axis displays the upwardly moving steps. Areas coloured grey represent 1 and those coloured black represent -1.

Table 3. In this table, the final period means the rules in that group for repeated treatment. As for Group No.1, it would show '–1', '1', '–1', '1', '–1', '1', ...

group No.	final period	local rule
1	–1,1	5,6,7,13,14,15,28,29,30,31,69,70,71,76,77,78,79,92,93,94, 95,133,134,135,140,141,142,143,156,157,158,159,197,198, 199,204,205,206,207,220,221,222,223
2	–1,1,1	41,42,43,44,45,46,47,57,58,59,60,61,62,63,168,169, 170,171,172,173,174,175,184,185,186,187,188,189,190,191
3	–1,1,1,1	104,105,106,107,108,109,110,111,120,121, 122,123,124,125,126, 127
4	–1,–1,1,–1,1,1	38,39,166,167
5	–1,–1,1,–1,1,1,1	102,103
6	–1,–1,1	2,3,10,11,12,34,35,66,67,74,75,98,99,130,131,138,139,162, 163,194,195
7	–1,–1,1,1,–1,1	26,27,154,155
8	–1,–1,1,1	18,19,22,23,50,51,54,55,146,147,150,151,178,179,182,183
9	–1,–1,1,1,1,–1,1	90,91
10	–1,–1,1,1,1	82,83,86,87,114,115,118,119
11	–1,–1,–1,1,–1,1,1	37,165
12	–1,–1,–1,1,–1,1,1,1	101
13	–1,–1,–1,1	1,9,33,65,73,81,97, 129,137,161,193,201, 225
14	–1,–1,–1,1,1,–1,1	25,153
15	–1,–1,–1,1,1	17,21,49,53,145,149,177, 181
16	–1,–1,–1,1,1,1,–1,1	89
17	–1,–1,–1,1,1,1	85,113,117
18	all –1	0,4,8,16,20,24,32,36,48,52,64,68,72,80,84,88,96,100,112, 116,128,132,136,144,148,152,162,164,176,180,192,196, 200,208,212,216,224,228,240,244
19	disunion	40,56,226,227
20	all 1	202,203,209,210,211,213,214,215,217,218,219,229,230, 231,232,233,234,235,236,237,238,239,241,242,243,245,246,247,248,249, 250,251,252,253,254, 255

4. Results and comparison

The classification shown in Section 2 might explain the period of simplicity and complexity of fashion in various countries and times, mass communication and some political elections. In our model, some groups, such as Groups No. 18 and No. 20, could behave as in the Sznajd model.[26,27] Nevertheless, with interference from other rules, the state would alter immediately (see Figs. 3 and 4). We consider the average

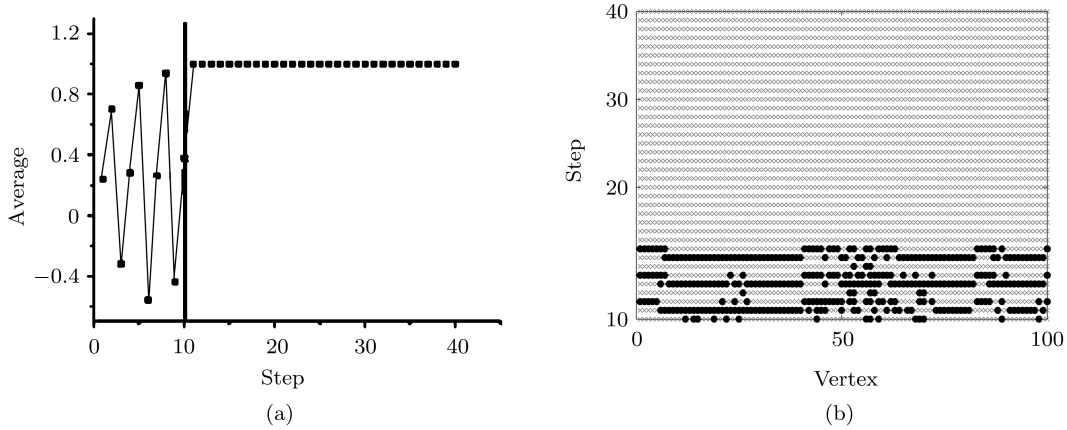


Fig. 3. Group No. 19 is interfered by No. 20. In panel (a) the vertical black line shows the interruption induced at step 10. Panel (b) displays the development of all vertices. Grey areas represent 1 and black areas represent −1.

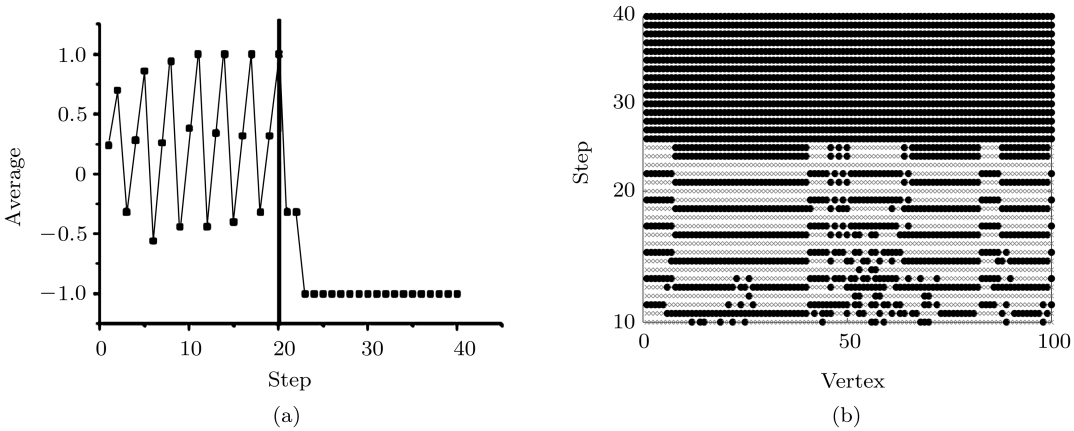


Fig. 4. Group No.19 is interfered by No.18. In panel (a) the vertical black line shows that the interruption comes at step 20. Panel (b) displays the development of all vertices. Grey areas represent 1 and those coloured black areas represent −1.

value of all vertices in our network, which is abbreviated to 'Ave.' in Figs. 3 and 4. It is obvious that once interference from other rules has happened, the state of the networks would change within 5 steps. A comment is that the original values in this network have a very small influence on the subsequent outcomes.

The tracking survey introduced in our model is always used in elections, with which newspapers or candidates get the updated voting state. The questions for the respondents are always the same, such as 'Which candidate do you support now?'. So it is interesting to simulate the American presidential election during the last 150 years through our model. The time gaps for each input opinion in our model would be one year. During those years the presidents are from either the Republican Party (abbreviated to 'R' in Fig. 5) or the Democratic Party (abbreviated to 'D' in Fig. 5). Suppose the Republican Party is set as 1 the and Democratic Party as −1 in our simulation.

30 presidents in 155 years are studied in Fig. 5. Those presidents are separated to 6 segments according to diverse local rules. The first segment is fitted to rule group 14. The second fits the local rule group 1, the same as the sixth segment. We notice that the fourth segment shares the last '1' with the next segment. It is strange that these six segments are separated by only four rule groups. Deep analysis reveals that special rule groups correspond to particular ages. Either internal (segment 1) or external (segment 4) conflict should lead to rule group 14. But a booming era (for both segments 2 and 6) would encourage humans to obey rule group 1.

There have been multiple assassination attempts on presidents of the United States. Four attempts on sitting presidents have succeeded: Abraham Lincoln (the 16th president), James A. Garfield (the 20th president), William McKinley (the 25th president) and John F. Kennedy (the 35th president). The last three

presidents before these four are all at the borders of segments in Fig. 5. Another president was injured in attempted assassinations: sitting President Ronald Reagan, who is also at the border. President Warren G. Harding (the 29th president) died of either a heart attack or a stroke. Above all, we discover that each new segment or new history was started with a severe contingency for the sitting president.

	Name	Party	Party Code	Local Rule Group Number
14	Franklin Pierce	D	1	
15	James Buchanan	D	1	No. 14
16	Abraham Lincoln	R	−1	Sectional Conflict
17	Andrew Johnson	D	1	1853 – 1881
18	Ulysses Simpson Grant	R	−1	
19	Rutherford B. Hays	R	−1	
20	James Garfield	R	−1	
21	Chester A. Arthur	R	−1	No. 1
22	Stephen Grover Cleveland	D	1	Gilded Age
23	Benjemin Harrison	R	−1	1881 – 1897
24	Stephen Grover Cleveland	D	1	
25	William McKinley	R	−1	No. 13
26	Theodore Roosevelt	R	−1	Progressive Era
27	William Howard Taft	R	−1	1897 – 1921
28	Woodrow Wilson	D	1	
29	Warren G. Harding	R	−1	No. 14
30	Calvin Coolidge	R	−1	Depression
31	Herbert Hoover	R	−1	& World Conflict
32	Franklin Delano Roosevelt	D	1	1921 – 1961
33	Harry S. Truman	D	1	
34	Dwight D. Eisenhower	R	−1	
35	John F. Kennedy	D	1	No. 4
36	Lyndon Johnson	D	1	Social Change
37	Richard Nixon	R	−1	& Soviet Relations
38	Gerald Ford	D	−1	1961 – 1989
39	Jimmy Carter	R	1	
40	Ronald Reagan	D	−1	
41	George H.W. Bush	D	−1	No. 1
42	Bill Clinton	R	1	Globalization
43	George W. Bush	D	−1	1989 –
44	Barack Hussein Obama Jr.	R	1	

Fig. 5. 'Party' shows the abbreviation for both parties, while 'Party code' shows settings of parties in simulation. With a different grey scale, areas point out which group of local rules these states of presidents are fitted to. The summary below the local rules is from 'American President: An Online Reference Resource' by Miller Center Public Affairs, University of Virginia [http://millercenter.org/academic/americanpresident].

Although the number of voting people in America is larger than that in our model and the influence is more complex, our simulation can be in accordance with historical classification. Furthermore, among 20 groups, only 4 groups are included. The result in this paper suggests that with the transformation of economic growth, the notions for voting in America have some common ground. In other words, each group would represent a majority axiology in ages. It is universally accepted that the assassination of Kennedy is an important moment in U.S. history because of its impact on the nation and the ensuing political repercussions. We conclude that abnormal deaths are critical turning points for American history.

5. Summary

We have introduced and analysed a model under CA rules, involving binary choices, where the decisions are calculated by limited memories. Our model focuses on information transmission in small world networks. With all attitudinal tracking surveys, outside factors can influence one's choice. Our model endows each vertice with a limited memory, used for storing the information from the mass in three periods. The CA rule, as a personal mechanism, gives the final ruling in one time period based on the most recent three feedbacks. All the rules are classified into 20 groups. The rules in different groups will lead to different results during a few steps. In particular, we concentrated our attention on elections in America. Remarkably, our simulation confirmed the observation concerning the turning points for America. We still do not know the exact meaning of these groups, for which so far a comparison with presidential voting is not enough. We hope that future research will help candidates in voting.

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