Analysis of diffusive coefficient of electron heat pulse based on singular value decomposition in HT-7 tokamak

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In this paper, singular value decomposition (SVD) as a filter-noise method is applied to electron cyclotron emission (ECE) diagnostic signals. The decomposed vectors contain the information about sawtooth such as the temporal vectors that show the sawtooth period and the spatial vectors that indicate the inverse radius. The propagation of electron heat pulse is investigated from electron cyclotron emission signals by using the perturbation method in HT-7 tokamak. The heat diffusivities are obtained at different densities in ohmic plasmas. The special result is that the heat diffusivity becomes larger as the heat pulse propagates outwards from the outside of the inverse radius.

Keywords: electron cyclotron emission, ohmic plasma, heat diffusivity, tokamak
PACC: 5270, 5270G, 5225, 5225F

1. Introduction

Electron heat transport has been investigated in many tokamaks, e.g. HT-7,[1] TFTR,[2] HL-2A,[3] etc. The perturbation method has been used to analyse the sawtooth heat pulse so as to obtain the heat diffusivity. The main idea of this method is that a small perturbation is applied first to a steady-state plasma, then the heat diffusivity is determined by measuring the changes of fluxes due to variations of gradients. However, this method depends on the choice of different models. We solve the following perturbed equation:

$$\frac{3}{2}n_{\rm e}\frac{\partial\tilde{T}(r,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\,n_{\rm e}\chi_{\rm e}^{\rm hpp}\frac{\partial\tilde{T}(r,t)}{\partial r}\right], \quad (1)$$

where $n_{\rm e}$ is the electron density, $\tilde{T}(r,t)$ is the electron fluctuating temperature, and $\chi_{\rm e}^{\rm hpp}$ is the heat diffusivity. For HT-7 tokamak, the dipole method is more suitable than the monopole.[4] Another way to deal with heat pulse is usually called the power-balance method. Contrary to the perturbation method, one can obtain the heat diffusivity in a steady-state plasma by measuring the total heat flux and the gradients.

In Ref. [5], the differences between the power-balance and the perturbing method were discussed. In contrast to the power-balance, the perturbation analysis, in principle, does not require the diagnostic system to make an absolute or even relative calibration but needs a good temporal and spatial resolution. Considering the present-day situation of the ECE diagnostic system in HT-7, the perturbation method should be optimal way for investigating the problem of heat pulse propagating.

Even if the ECE device is not absolutely calibrated, it is the most important diagnostic tool to study electron heat transport because of its good temporal and spatial resolutions. However, usually the noise contaminates the signals seriously, thereby leading to the difficulty in determining the time for the heat pulse to reach its peak, $t_{\rm p}$. The poor signal–noise ratio can influence the accuracy of measuring $t_{\rm p}$.[6] So finding out an effective method to filter noise has become an extremely urgent task.

SVD is known as an effective technique to suppress the noise in digital signal processing domain. It was used to deal with the magnetohydrodynamic data of soft x-ray (SXR) first in JET,[7] then in HL-1M[8] and HT-7,[9,10] from which a similar conclusion is obtained, i.e. SVD is an effective technique to deal with multi-channels spatially correlated signals such as ECE, SXR, etc.

2. Application of singular value decomposition in ECE signals

The SVD is an important content of the subject of numerical liner algebra which is used to decompose

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the $M \times N$ matrix ($M \geq N$). Here we consider only the practical application and avoid the complicated deduction.

One can construct the matrix from 16 channels ECE signals such as

$$\boldsymbol{X}_{M\times 16} = \begin{bmatrix} a_{01}^{1} & a_{02}^{1} & \cdots & a_{16}^{1} \\ a_{01}^{2} & a_{02}^{2} & \cdots & a_{16}^{2} \\ \vdots & \vdots & \cdots & \vdots \\ a_{01}^{m-1} & a_{02}^{m-1} & \cdots & a_{16}^{m-1} \\ a_{01}^{m} & a_{02}^{m} & \cdots & a_{16}^{m} \end{bmatrix}. \quad (2)$$

The column vectors of the matrix represent the sampling data of different channels during one shot while the row vectors are corresponding to different sampling time-slices. Equation (2) means m-fold sampling in one discharge. Then the matrix $\boldsymbol{X}_{M\times 16}$ can be decomposed by SVD as follows:

$$\boldsymbol{X}_{M\times 16} = \boldsymbol{U}_{M\times 16}\boldsymbol{S}_{16\times 16}\boldsymbol{V}_{16\times 16}^{\mathrm{T}}. \quad (3)$$

One knows that the vectors $\boldsymbol{U}$ and $\boldsymbol{V}$ are all the orthonormal matrixes, i.e.

$$\boldsymbol{U}\boldsymbol{U}^{\mathrm{T}} = \boldsymbol{V}\boldsymbol{V}^{\mathrm{T}} = \boldsymbol{I}, \quad (4)$$

where the vector $\boldsymbol{I}$ is the unit matrix; the vector $\boldsymbol{S}$ is the diagonal matrix, of which the diagonal values represent the energy of signals, while the vectors of $\boldsymbol{U}$ and $\boldsymbol{V}$ describe the evolutions in temporal domain and spatial domain respectively. In fact, only the first several ones of s_{ii} almost occupy 100% of the whole signal energy, i.e.

$$\frac{\sum_{1}^{j} s_{ii}^{2}}{\sum_{1}^{16} s_{ii}^{2}} \xrightarrow{j\to 16} 1. \quad (5)$$

In general, $j = 3$ is enough to satisfy the requirement for accuracy. Then we reconstruct the matrix $\boldsymbol{X}_{M\times 16}$ by using the vectors $\boldsymbol{U}_{M\times j}$, $\boldsymbol{S}_{j\times j}$ and $\boldsymbol{V}_{16\times j}$ approximately as follows:

$$\boldsymbol{X}_{M\times 16} \approx \boldsymbol{U}_{M\times j}\boldsymbol{S}_{j\times j}\boldsymbol{V}_{16\times j}^{\mathrm{T}}. \quad (6)$$

What one should keep in mind is that the noise is nearly distributed in all vectors due to its uncorrelated characteristics. When only the first several vectors are chosen for reconstruction, a great many of noises are removed without loss of signals because of its spatially correlated behaviour as indicated above. This is the reason why the SVD is well known as a noise-filtering technique.

3. Analysis of the electron heat pulse with perturbing method

The diagnostic system of ECE has been used to determine the electron heat diffusivity in HT-7 (a medium superconductive tokamak, its major radius is 122 cm and minor radius is 30 cm). A typical ohmic discharge for observing sawtooth is the shot 92838 as shown in Fig. 1. It can be seen that SVD is effective to filter noise compared with the two panels of Fig. 1.

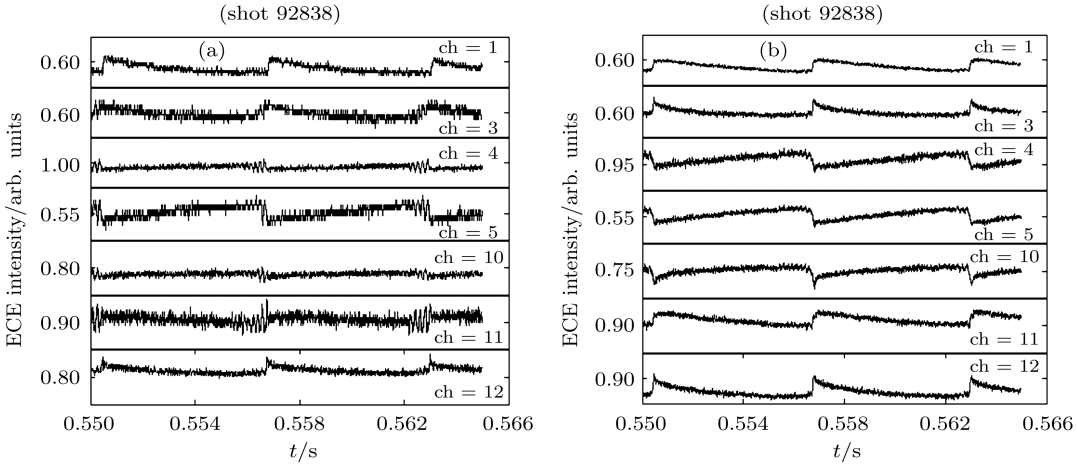


Fig. 1. Time trace of intensity of ECE signals before (a) and after (b) SVD filtering. The parameters of shot 92838 are the plasma current I_{p} 160 kA, the toroidal magnetic field B_t 1.95 Tesla, and the line average density n_{e} 3×10^{19} m^{-3}.

Figure 2 is obtained from the SVD which is used to reconstruct the ECE signals (i.e. Fig. 1(b)). Form Fig. 2(c), one can see that the first three singular values occupy more than 99.9% of the total energy which is the reason why only the first three vectors of the matrixes $\boldsymbol{U}, \boldsymbol{S}, \boldsymbol{V}$ are chosen to reconstruct the ECE signal matrix $\boldsymbol{X}_{M\times 16}$ and why the SVD is as an effective technique of filtering noise.

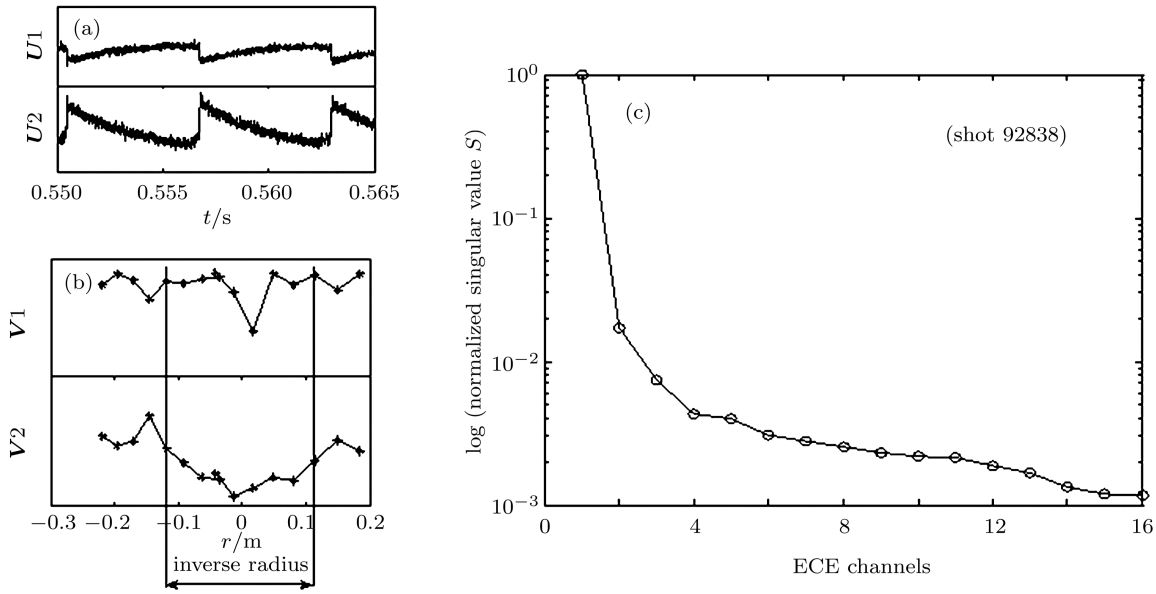


Fig. 2. Results of SVD. Panel (a) shows time traces of the first two vectors of $\boldsymbol{U}$. Panel (b) exhibits corresponding spatial distributions of the first two vectors of $\boldsymbol{V}$. Panel(c) indicates singular values $\boldsymbol{S}$. The 'inverse radius' shown in panel (b) will be used in the following to distinguish it from the mixing radius.

As pointed out in the Introduction, the dipole model should be used to determine the heat diffusivity for HT-7. An important condition one must care for is that the model is applicable only outside the mixing radius where the temperature is perturbed during the sawtooth crash.[11] In Ref. [12], the plasma was divided into three regions. The first region is inside inversion radius, which is called central-core region suffering internal disruption as a result of the safety factor dropping below unity. As seen in ECE signals (from ch = 1 to ch = 10 in Fig. 1.), the curve experiences a slow increase then a sharp dropping. The reverse process is shown outside the inversion radius. The second region is outside the inversion radius, which is a large region and called confined region. The inverse sawtooth implies a fast redistribution of energy from core-region to confined zone without immediate loss by rapid mixing.[13] The fact that the time of arrival at peak is delayed with the radius illustrates that the propagation of heat pulse is a diffusive process. The heat diffusivity is resolved as follows:[14−17]

$$\chi_{\rm e}^{\rm hpp} = \frac{r^2 - r_{\rm mix}^2}{8t_{\rm p}}, \tag{7}$$

where $r_{\rm mix}$ is the mixing radius which is at the outside of inverse radius a little and $t_{\rm p}$ is the time of arrival at peak of heat pulse.

Figure 3(a) shows the electron heat diffusivity of the shot in the Fig. 1 after the sawtooth collapsing at 556.7 ms. It is easily found that the heat diffusivity on the low field side is larger than that on the high field side, indicating that the confinement is better on the high field side due to stronger magnetic field. The interested result is that the diffusivity is increasing as the heat pulse propagates outwards other than constant, which is experimental evidence of the K Itoh model.[18−22] In Fig. 3.(b), the heat diffusivity $\chi_{\rm e}^{\rm hpp}$ decreases as density goes up in a range from 2 to 3.0×10^{19} m^{-3}. But for the density $n_{\rm e} = 1.6 \times 10^{19}$ m^{-3}, the heat diffusivity $\chi_{\rm e}^{\rm hpp}$ is smaller than that for the higher density at the same location on the low field side. The mystery is probably attributed to the runaway electrons which can be generated when the torodial electric field is above a critical value. They usually exist on the low field side due to bad curvature of magnetic field just as thermal electrons. When magnetic field reconnects, they can be accumulated in magnetic islands due to scattering.[23,24] The electrons can be continually ac-

celerated by the electric field induced due to magnetic reconnection,[25] and the radiation can be detected by ECE first. Just as the well-known case where the accelerated process needs some time, on the order of a few milliseconds. This is probably the reason why the time of arrival at peak is delayed. A similar situation is also shown in Ref. [26] in which $t_{\rm p}$ was shown to vary from 1 to 5 ms.

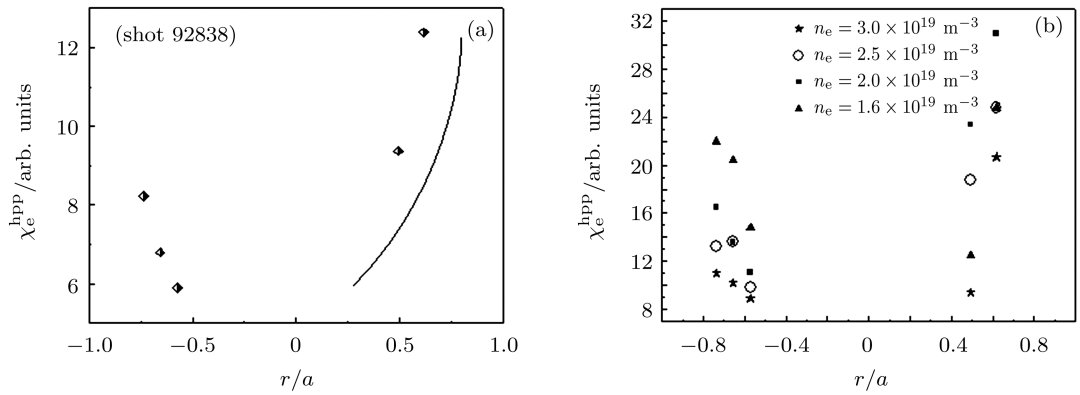


Fig. 3. Electron diffusivity versus normalizing radius (the mixing radius is 0.12 m, −0.15 m on the low field side and high field side respectively), with the solid line indicating the K. Itoh model (a); heat diffusivities of radial location at different densities (b).

4. Conclusions

SVD as an effective instrument of noise-filter is applied to analysing the data of ECE. The temporal vectors $\boldsymbol{U}$ of SVD can indicate the period of sawtooth, and the vectors $\boldsymbol{V}$ can give spacial properties of sawtooth e.g. the inverse radius.

The electron heat diffusivity $\chi_{\rm e}^{\rm hpp}$ is obtained from the filtered ECE signals with the perturbation method. Experimental results show that the heat diffusivity $\chi_{\rm e}^{\rm hpp}$ depends strongly on the density in a range from 2 to 3.0×10^{19} m^{-3}. It has a good consistence with the previous conclusions obtained from HT-7 within an uncertainty.

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