Effects of granular size on stress transmission in a granular column*

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We investigate the deflection and the fluctuation of stresses due to different sizes of granular material in a cylindrical column. It is experimentally observed that the saturation mass systemically increases with granule diameter . The results indicate the shielding of vertical stress in silo is varying. We show that the ratio between the horizontal and the vertical stresses, K, decreases with the increase in grainule diameter d. In addition, it has also been found that the presence of larger granules leads to stronger stress fluctuation on the bottom plate of silo.

Keywords: fluctuations, stress transmission granular materials
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1. Introduction

Description of static properties in dry granular systems has attracted much attention from both the scientific and the engineering communities, because of their varieties of applications.[1−7] One of the fundamental problems is to determine how forces are carried out in granular materials. The highly heterogeneous and anisotropic nature of the force network in static granular media hinders a rigorous macroscopic theory of granular mechanics from developing. Moreover, many geological processes are also closely related to mechanical properties of granular matter. For example, it is reasonable to assume that there is a strong connection between the deformation of force chains and the mechanics of earthquake and faulting. Now, the static granular media such as confining granules in cylindrical column have emerged as an important system to study propagation of force and also deformation of force chains in disordered materials. The pioneering work of Janssen has demonstrated in particular that unlike the hydrostatic case, the mass measured below a granular column does not increase indefinitely as the height increases, and it reaches a constant value independent of height instead; the apparent mass is only a part of total mass of the granules in that column and the rest is supported by side walls, which is attributed to the shielding effect leading most of weight to side walls rather than to the bottom of silo.[8] Vanel *et al.*[9] devised a measurement device aiming at accurately and reproducibly measuring the apparent mass as well as its fluctuation on the bottom of a granule column moving down at a slow velocity, moreover, in order to rectify some minor discrepancies found between their experiment and Janssen model, they also proposed oriented stresses linearity model. Although Janssen model has been extensively studied, however, till now, to the best of our knowledge, no or few experimental studies have been reported for the relation between the stress transmission and the grain size in a granular column.

In the present study, we report on experimental results of the influence of different diameters of granules on saturation mass, the deflection of vertical stresses into horizontal ones in a silo as well as fluctuations caused by different granule diameters

2. Experimental set-up

Our experimental set-up is shown in Fig. 1. Mass $M_{\rm f}$ is poured into a hollow vertical glass tube of length $L = 550$ mm and internal diameter $D = 43.6$ mm. A cylindrical piston is inserted at the bottom of cylinder. The piston is designed such that it neither touches the

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walls of silo during its motion nor allows the granules to leak out. The bottom end of piston lies on the weight sensor allowing one to measure the force exerted by the granule on the top surface of piston. The static pressure is entirely transmitted to a weight sensor, and a computer records data given by sensor throughout the process. The instantaneous pressure on the weight sensor is referred to as effective mass $M_{\rm t}$, of course its value is different from the total filled mass $M_{\rm f}$. The moving speed of the piston is controlled by high performance electric motor. For every experimental run, the piston is inserted inside the vertical cylinder to a certain height and then mass M of granules is poured uniformly into the vertical cylinder via a funnel placed at the top of cylinder. The ramming of grains results in a rather compact packing. A short relaxation time (normally 30 to 60 s) is allowed, after the granules are filled into the cylindrical column, elastic energy accumulated by weight sensor may be relaxed. When the cylinder is filled with granules the piston is allowed to descend down at a slow velocity (0.02 mm/s) over a total distance of 20 mm. The slow downward motion of the piston could fully mobilize the friction force between the granule column and the walls of cylinder. Simultaneously, the average packing fraction tends to reach a stable value. Throughout the experiments, the total height of granule column is measured when its motion ceases, so that the packing fraction can be calculated. In our case, the packing fraction is 0.677±0.023.

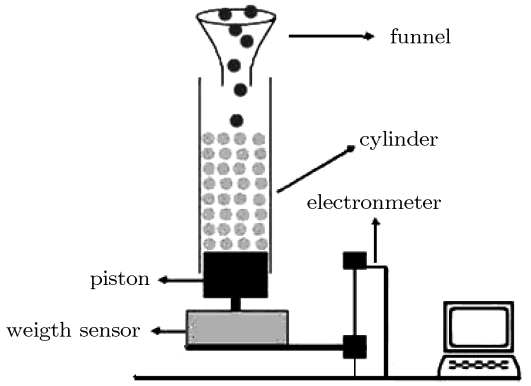


Fig. 1. Schematic diagram of the experimental apparatus and filling method.

3. Results and discussion

First we study effective mass as a function of filling mass poured in a silo by using different sizes of granules. The granules used in the present experiment are mono-disperse glass beads. The values of granule diameter d used are 2, 4, 6, 7 and 8 mm, separately. The filling mass is increased in steps of 50 g. Figure 2 displays the time variation of effective mass $M_{\rm t}$ for the filling mass of 600 g, for five kinds of granules. It can be seen that the static pressure measured by the weight sensor exhibits comparatively large fluctuations in the beginning of the course of the recording, and eventually a relatively steady state is reached with the descent displacement step. Here, we take the statistical average over 500 points in the sequence of time series after the steady state has been reached, as a measured value of $M_{\rm e}$. To assure the reproducibility and also reduce the error, we do the experiments 5 times, and take the average value of these measured values as a data point $M_{\rm e}$ shown in Fig. 3.

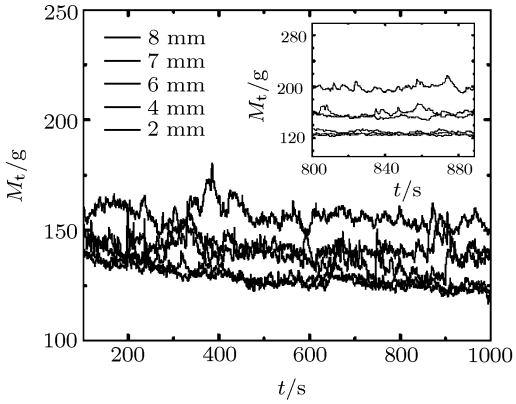


Fig. 2. Time variation of the effective mass with the granule diameters of 2, 4, 6, 7 and 8 mm, respectively.

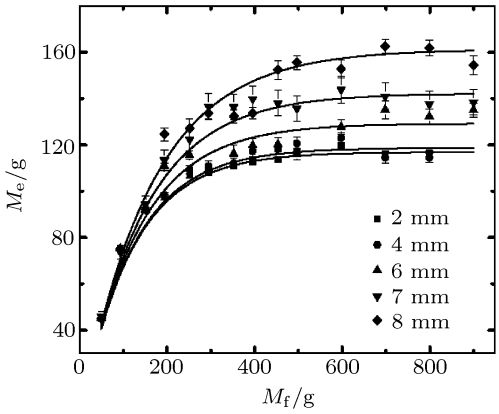


Fig. 3. Curves for effective mass versus filling mass for different bead diameters.

Figure 3 depicts the dependence of effective mass on the filling mass. The scatters each show the measurement error in experiment, and the lines each represent a fitting to all data for different diameters of granules. It can be observed that for low values of filling mass, $M_{\rm e}$ is approximately equal to $M_{\rm f}$ which

suggests the hydrostatic behaviour of granular material in a silo. As filling mass is increased, M_e seems to display a saturate state, implying that screening effect occurs and the state has some fluctuations. Note that the trend exhibited by each line is almost the same; however, they branch off and increase with the granule diameter increasing. Obviously, this phenomenon inspires us to make its further investigation since the stress transmission is affected by the bead size in the granular column.

In order to reveal the cause of the separating of data lines, we use the Janssen model which is still used in designing of silos despite of its simplicity. The model is based on the assumption that in a silo horizontal stress is proportional to vertical stress, that frictional force is in yield criterion and the medium is considered as continuous. Janssen model describes that the mean pressure on the bottom does not increase linearly with the height of filling, on the contrary, it saturates to a value depending on silo radius and friction between the wall and grains. The relationship between the filling mass and effective mass is given as

$$M_e = M_s(1 - e^{-M_f/M_s}), \quad (1)$$

where saturation mass is

$$M_s = \rho\pi(D/2)^3/(2\mu K), \quad (2)$$

M_f is the filling mass, ρ represents the material density, μ is the coefficient of friction between granules and walls, which could be considered as a constant, and K is the ratio between horizontal and vertical stresses. Liu *et al.*[10] have experimentally verified that saturation mass obeys the three-power law of the diameter of silo in accordance with Eq. (1). Note that Eq. (1) does not give any relation between granule diameter and saturation mass. However, we can use Eq. (1) to fit the data points and then the values of M_s can be determined. The fitting lines by this procedure are not shown in Fig. 3. The values of M_s obtained by the fitting procedure for different values of d are further fitted by a smooth function, so that the variation of M_s with d could be ascertained as shown in the inset of Fig. 4. It is found that M_s is an increasing function of d. This variation may also be attributed to the reduced shielding of vertical stresses from walls of cylinder. Furthermore, considering that all the obtained values of M_s should change with granule diameter according to a uniform law, we fit them so as to obtain the modified M_s', and from the fitting curve we obtain the value of M_s'. Substituting all the modified values of M_s' into Eq. (1) again, the modified M_e can be calculated and the fitting lines could be easily obtained as shown in Fig. 3, which are more accurate than those obtained by separately fitting with Eq. (1) for each value of M_s.

What is the mechanism that results in the variation of M_s with d? According to Eq. (2), M_s changes only due to the fact that the values of K are taken to be different values because the other parameters are constant. So it may be suggested that the increase in granule diameter reduces the diffusion of vertical stresses into horizontal ones. Using the data points of M_s and Eq. (2), we can further calculate the value of μK, so as to obtain the changing trend of K with d as depicted in Fig. 4. Note that μ is a constant, so it does not affect the trend exhibited by K. It can be observed that the value of μK decreases as d increases. The particle size effect disappears when the granule diameter is small enough compared with the internal diameter of the container, as illustrated in Fig. 3 for the 2–4 mm diameter beads, where the saturation lines are nearly the same. It implies that redirection parameter K works well in converting vertical stresses into horizontal ones in the case of smaller diameter granules compared with in the case of the larger diameter ones. This may be due to the fact that the number of contact points with side wall of silo for the case of larger granules are less than that for the case of smaller ones, so larger diameter granules transfer a smaller fraction of stress to the walls of silo, allowing a larger fraction of stress to be transmitted to the bottom plate of granule column, hence the shielding effect is weakened in the presence of larger beads and we obtain a higher value of saturation mass.

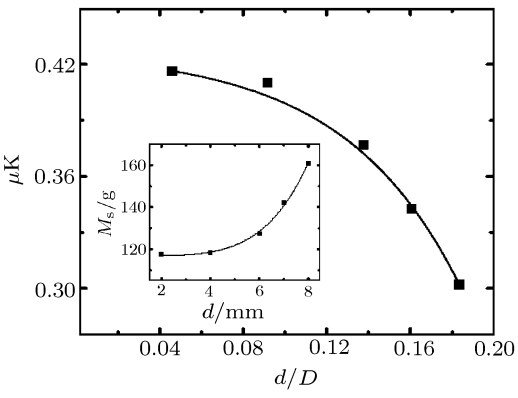


Fig. 4. Variation of μK with scaled d. The inset shows the variation of M_s with d.

As mentioned earlier, the stress transmission is inhomogeneous in granular material, which also gives rise to strong stress fluctuations as shown in Fig. 2. The dependence of fluctuation on bead diameter is quantitatively summarized and plotted in Fig. 5. Here every data point is obtained as a standard deviation of 500 points in the final phase of graphical recording of effective mass where relatively steady state is attained. Every data point is the average over five experimental runs. It was observed that the strong fluctuations appeared when the effective mass started to saturate and that the same behaviour was reported in the previous experimental studies.[11] So we choose the data in the saturation region to illustrate the behaviour of fluctuations. The data points depicted in Fig. 5 are obtained for the filling mass of 600 g. It is revealed from the figure that the fluctuation varies somewhat linearly with the value of d. The results indicate that fluctuations increase as granule diameter is increased. The result demonstrates that fluctuations along with other parameters, are also very sensitive to bead diameter. This dependency is likely to correspond to that for larger diameter of beads the number of force chain is smaller, but the strength is stronger than the case of smaller granule diameter which give rise to larger and stronger changes at the bottom plate of granule column when the formation and the rupture of arches and force chain deformation take place.

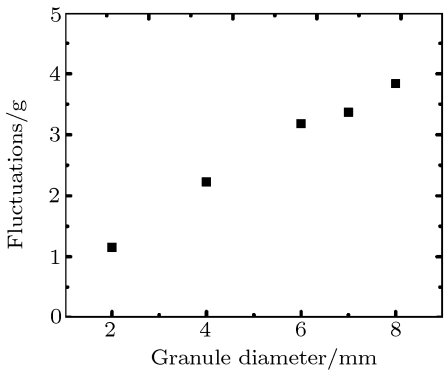


Fig. 5. Quantitative representation of fluctuations as a function of granule diameter.

4. Conclusion

We experimentally explore the effects of the variation of granule diameter on the force screening effect as well as on stress fluctuation in a silo. The results show that saturation mass systemically increases with the increase in granule diameter. Moreover, the unusual shielding effect exhibited by granular material in silos weakens in the presence of larger size granules. Furthermore, the fluctuations are found to exhibit a strong correlation with granule diameter. Our experimental set-up mimics the vibration caused by the variety of stratum. It is expected that our experimental study will be helpful for better understanding the complex force network in silo.

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