One-step implementation of an N-qubit quantum phase gate through a double Raman passage*

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We propose a scheme for controllably implementing an N-qubit phase gate by one step within a ground-state subspace of N three-state atoms trapped in a cavity through a double Raman passage. We can extend our scheme to the realisation of an arbitrary N-qubit phase gate by appropriately adjusting coupling strengths and detunings between atoms and external driving fields. The advantage of this one-step scheme is its robustness against decoherence.

Keywords: Raman-coupled model, quantum computation, quantum phase gate
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1. Introduction

Since Shor's algorithm and Grover's algorithm were discovered, the tremendous speedup of the quantum computer compared with its classical counterpart has motivated a lot of studies in recent years in theory and practice of quantum computation.[1,2] It is well known that a quantum computing network can be partitioned into a sequence of one-qubit rotation and two-qubit universal gates.[3] Therefore the procedure becomes complicated as the number of qubits increases. Thus it is very useful to realise a one-step implementation of multiqubit gates in constructing the quantum-computing network. Zou *et al.*[4−8] proposed several theoretical protocols for the direct implementation of an N-qubit phase gate.

Cavity quantum electrodynamics (CQED) is an appropriate candidate for quantum information processing and quantum computation, and much progress has been made practically in the trapping of single atoms inside a small microwave or optical cavity in a regime of strong coupling,[9,10] and in the observing of atom-photon quantum entanglement.[11] However, the main obstacles for implementing quantum information processing in CQED are cavity decay and atomic spontaneous emission. In Ref. [7], a scheme for implementing an N-qubit phase gate was proposed by Zou *et al.* However, the decay of the cavity is a main source of decoherence. In Ref. [8], Xiao *et al.* proposed a scheme for implementing an N-qubit phase gate where the spontaneous emission of excited states is a dominant decoherence source. Also Wu *et al.*[12] and Cai *et al.*[13] proposed a robust scheme to construct a two-qubit phase gate which could not be directly extended to the implementation of an N-qubit phase gate.

In the present paper, we propose a scheme for the one-step implementation of a robust N-qubit phase gate with N atoms trapped in the cavity through a double Raman passage. The scheme is robust to the decay of the cavity field and the atomic spontaneous emission. The quantum states of the collective atomic mode in the symmetric Dicke subspace construct a decoherence-free subspace (DFS).

This paper is organised as follows. In Section 2, we give a brief description of the system. In Section 3, we show how to generate a symmetric entangled Dicke state and construct an arbitrary N-qubit phase gate by using the Dicke state. In Section 5, some conclusions drawn from the present study are presented.

2. System

The system is composed of N atoms, which are trapped in an optical cavity and coupled to fields with a double Λ Raman interaction. The energy level configuration of the atom is depicted in Fig .1.

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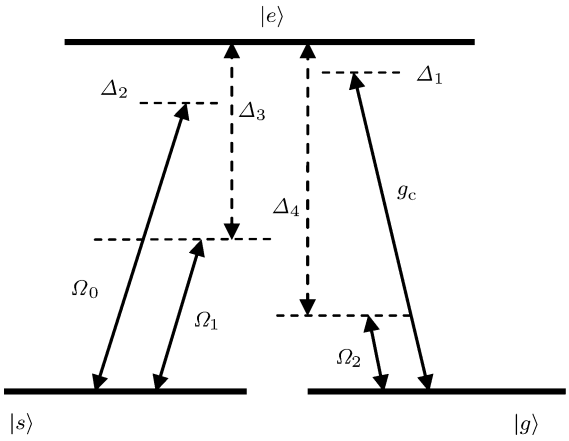


Fig. 1. Schematic diagram of atomic energy level configuration and corresponding atomic transitions.

It includes two stable ground states $|s\rangle$ and $|g\rangle$, and one excited state $|e\rangle$. In this paper, we encode the states $|s\rangle$ and $|g\rangle$ as logic zero and one state, i.e. $|s\rangle \equiv |1\rangle$ and $|g\rangle \equiv |0\rangle$. The single cavity mode couples with the atomic transition $|g\rangle \leftrightarrow |e\rangle$ with the coupling strength g_c, and a classical field couples with the atomic transition $|s\rangle \leftrightarrow |e\rangle$ with Ω_0 as its coupling strength. Moreover, there are two weak classical fields driving the atomic transitions $|g\rangle \leftrightarrow |e\rangle$ and $|s\rangle \leftrightarrow |e\rangle$ with the coupling constants Ω_1 and Ω_2 (both are assumed to be real), respectively. Δ_i $(i = 1, 2, 3, 4)$ represent the corresponding one-photon detunings. Without taking into account the two weak classical fields, and with taking the rotating-wave approximation, we obtain the interaction Hamiltonian for describing the cavity–atom system

$$H = \sum_{j=1}^{N} (g_c a|e_j\rangle\langle g_j|e^{i\,\Delta_1 t} + g_c a^+|g_j\rangle\langle e_j|e^{-i\,\Delta_1 t} + \Omega_0|e_j\rangle\langle s_j|e^{i\,\Delta_2 t} + \Omega_0|s_j\rangle\langle e_j|e^{-i\,\Delta_2 t}), \quad (1)$$

where the subscript j represents the jth atom, and a^+ and a are the photon creation and annihilation operators of the quantized cavity mode respectively. The one-photon detunings are $\Delta_1 = \omega_e - \omega_s$ and $\Delta_2 = \omega_e - \omega_g$, where the state $|s\rangle$ and $|g\rangle$are both assumed to be degenerate and have zero energy. With the large-detuning conditions $|\Delta_1| \gg |g_c|$ and $|\Delta_2| \gg |\Omega_0|$, i.e. the one-photon detunings $\Delta_i (i = 1, 2)$ are much larger than the associated coupling strengths, we can adiabatically eliminate the excited state $|e\rangle$ and obtain the effective Hamiltonian[12]

$$H_{\text{eff}} = \lambda' \sum_{j=1}^{N} (a^+|g_j\rangle\langle s_j|e^{-i(\Delta_2-\Delta_1)t} + a|s_j\rangle\langle g_j|e^{i(\Delta_2-\Delta_1)t}), \quad (2)$$

where

$$\lambda' = \frac{g_c\Omega_0}{2}\left(\frac{1}{\Delta_1} + \frac{1}{\Delta_2}\right).$$

The effective Hamiltonian (2) describes the dynamical transition between the two long-living atomic levels $|g\rangle$ and $|s\rangle$.

In the case of $|\Delta_2 - \Delta_1| \gg \lambda'$, i.e. the detuning of the Raman transition is sufficiently large, and if the cavity is initially prepared in the vacuum state $|0\rangle_c$, then the effective interaction Hamiltonian can be transformed into

$$H_{\text{eff}} = \lambda \left(\sum_{j=1}^{N} |s_j\rangle\langle s_j| + \sum_{\substack{j,k=1 \\ j\neq k}}^{N} |s_j\rangle\langle g_j| \otimes |g_k\rangle\langle s_k| \right), \quad (3)$$

where $\lambda = {\lambda'}^2/|\Delta_2 - \Delta_1|$. This Hamiltonian describes a collection of effective two-level atoms coupling through an effective dipole–dipole interaction mediated by the cavity mode. Because the cavity is virtually excited, the decay of the cavity is largely suppressed.

As a result, the dynamics of the whole system reduces to a decoherence-free subspace (DFS) consisting of only two ground states. Compared with the proposal presented in Ref. [8], in which the two-level atoms are used to implement a quantum logic gate, our protocol utilises the two ground states of the three-level atoms, so the atomic spontaneous emission can be ignored.

3. Generating a symmetric entangled Dicke state and Constructing an arbitrary N-qubit phase gate

Here we introduce the collective operators

$$S^+ = \sum_{j=1}^{N} |s_j\rangle\langle g_j|, \quad S^- = \sum_{j=1}^{N} |g_j\rangle\langle s_j|. \quad (4)$$

So we can rewrite Eq. (3) as follows:

$$H_{\text{eff}} = \lambda S^+ S^-. \quad (5)$$

Then we consider the two weak external driving fields with frequencies ω_1 and ω_2, respectively. In the interaction picture and the rotating-wave approxima-

tion, we can obtain the following interaction Hamiltonian:

$$H_1 = \sum_{j=1}^{N}(\Omega_1|e_j\rangle\langle s_j|\mathrm{e}^{\mathrm{i}\,\Delta_3 t} + \Omega_1|s_j\rangle\langle e_j|\mathrm{e}^{-\,\mathrm{i}\,\Delta_3 t} + \Omega_2|e_j\rangle\langle g_j|\mathrm{e}^{\mathrm{i}\,\Delta_4 t} + \Omega_2|g_j\rangle\langle e_j|\mathrm{e}^{-\,\mathrm{i}\,\Delta_4 t}), \quad (6)$$

where $\Delta_3 = \omega_\mathrm{e} - \omega_\mathrm{s}$ and $\Delta_4 = \omega_\mathrm{e} - \omega_\mathrm{g}$ are the associated one-photon detunings and $\Delta_3 - \Delta_4 = \omega_2 - \omega_1$ the two-photon detuning. Assume large detuning conditions $|\Delta_3| \gg |\Omega_1|$ and $|\Delta_4| \gg |\Omega_2|$, then the effective Hamiltonian will take[14,15]

$$H_2 = \Omega\sum_{j=1}^{N}(\mathrm{e}^{\mathrm{i}(\omega_2-\omega_1)t}|g_j\rangle\langle s_j| + \mathrm{e}^{-\,\mathrm{i}(\omega_2-\omega_1)t}|s_j\rangle\langle g_j|) = \Omega(\mathrm{e}^{\mathrm{i}(\omega_2-\omega_1)t}S^- + \mathrm{e}^{-i(\omega_2-\omega_1)t}S^+), \quad (7)$$

where

$$\Omega = \frac{\Omega_1\Omega_2}{2}\left(\frac{1}{\Delta_3} + \frac{1}{\Delta_4}\right).$$

Now the full effective Hamiltonian can be rewritten as

$$H_\mathrm{full} = \lambda S^+S^- + \Omega(\mathrm{e}^{\mathrm{i}(\omega_2-\omega_1)t}S^- + \mathrm{e}^{-\,\mathrm{i}(\omega_2-\omega_1)t}S^+), \quad (8)$$

where $\omega_2 - \omega_1$ and Ω can be tuned by external driving field.

If the system is initially in a symmetric Dicke state $|J, -J+k\rangle$,[16] its evolution will be within the symmetric Dicke subspace spanned by $\{|J, -J\rangle,\ |J, -J+1\rangle, \ldots |J, J\rangle\}$ $(J = N/2)$. The state $|J, -J+k\rangle$ is a state with k atoms in the state $|s\rangle$ and other atoms in the state $|g\rangle$, i.e.

$$|J, -J+k\rangle = \binom{2J}{k}^{-1/2}\sum_j P_j(|s_1, s_2, \ldots, s_k, g_{k+1}, g_{k+2}, \ldots, g_N\rangle), \quad (9)$$

where P_j denotes the set of all distinct permutations of the qubits. We have

$$S^+|J, -J+k\rangle = \sqrt{(2J-k)(k+1)}|J, -J+k+1\rangle,$$
$$S^-|J, -J+k\rangle = \sqrt{k(2J-k+1)}|J, -J+k-1\rangle. \quad (10)$$

Suppose $\Omega \ll \lambda$ and $\omega_2 - \omega_1 = 2J\lambda$(these conditions can be fulfilled by adjusting the amplitudes and frequencies of the external driving fields), the transition $|J, -J\rangle \to |J, -J+1\rangle$ is resonant with the pair of external classical fields, while the other transitions remain far off-resonance and can be neglected.[17] Therefore, the symmetric Dicke subspace can be reduced to $\{|J, -J\rangle, |J, -J+1\rangle\}$ and the Hamiltonian (8) reduces to

$$H_\mathrm{full} = \alpha_1|J, -J+1\rangle\langle J, -J+1| + \eta_0(\mathrm{e}^{-\,\mathrm{i}\,\alpha_1 t}|J, -J+1\rangle\langle J, -J| + \mathrm{e}^{\mathrm{i}\,\alpha_1 t}|J, -J\rangle\langle J, -J+1|), \quad (11)$$

where $\alpha_1 = 2J\lambda$, $\eta_0 = \sqrt{2J}\Omega$.

Transforming to the frame rotating with α_1, we obtain the effective interaction Hamiltonian

$$H_\mathrm{int} = \eta_0(|J, -J+1\rangle\langle J, -J| + |J, -J\rangle\langle J, -J+1|). \quad (12)$$

Assume that the whole N-qubit system is initial in the state $|J, -J\rangle$, i.e. all the atoms are in the state $|g\rangle$, then the time evolution of the system can be written as

$$|J, -J\rangle \to \cos(\eta_0 t)|J, -J\rangle - \mathrm{i}\sin(\eta_0 t)|J, -J+1\rangle. \quad (13)$$

If we choose the evolution time $t = \pi/\eta_0$, then we will obtain the transforms $|J, -J\rangle \to -|J, -J\rangle$ and $|J, -J+k\rangle \to |J, -J+k\rangle (k \neq 0)$, which is a phase gate operation. Therefore, through the selective atom-cavity-classical field interaction (8), we can acquire an N-qubit conditional phase gate in a DFS with one-step operation through a double Raman passage.

We choose the suitable parameters[18] as follows: $g_\mathrm{c} = 2\pi \times 110$ MHz, $\Omega_0 = 1.05g_\mathrm{c}$, $\Delta_1 = 20g_\mathrm{c}$, $\Delta_2 = 21g_\mathrm{c}$, $\Omega_1 = g_\mathrm{c}/100$, $\Omega_2 = 1.05\Omega_1$, $\Delta_3 = 10g$ and $\Delta_4 = \Delta_3 - \sqrt{3}\lambda$. Therefore, the system evolution can be effectively described by Hamiltonian (11). The required effective time for an N-qubit phase gate is $t = \pi/\eta_0 = \pi/\sqrt{N}\Omega \approx 4.54 \times 10^{-6}$ s for only three atoms trapped in the cavity.

Under the conditions of weak coupling and large detuning, we can extend our scheme to an arbitrarily marked specific state in the symmetric Dicke space with a π phase flip by using two external classical

fields which are resonant with the prechosen transition but off-resonant with other transitions.[17] Here we simply show the operation: we suppose that initially the cavity mode is in the vacuum state and the collective atomic state is $|J, -J+k\rangle$, and that the conditions $\Omega \ll \lambda$ and $\omega_2 - \omega_1 = 2(J-K)\lambda$ are fulfilled. When we choose the interaction time as $t = \pi/\sqrt{(2J-k)(k+1)}\Omega$, then the state $|J, -J+k\rangle$ will acquire an $e^{i\,\pi}$ phase factor while the other states remain unchanged. To initialise our system in the state $|J, -J+k\rangle$, we can use the scheme proposed by Zheng.[17] This allows us to mark an arbitrary collective state in the symmetric Dicke space, which is very useful in quantum search algorithm.

Finally, we show that our interaction model can be extended to the preparation of the Dicke state $|J, -J+k\rangle$ of N trapped neutral atoms. To prepare the Dicke state $|J, -J+k\rangle$, we choose the parameters to satisfy conditions $\Omega \ll \lambda$ and $\omega_2 - \omega_1 = k(2J+1-k)\lambda$. In this case, the state $|J, -J\rangle$ resonantly couples to the state $|J, -J+k\rangle$, while other transitions are dispersive. With a determined interaction time, the initial state $|J, -J\rangle$ is transferred to the final state $|J, -J+k\rangle$ which is desired. Now we generate the state $|J, -J+k\rangle$ by one step, which is unlike the previous step-by-step schemes.[17,19,20]

4. Conclusions

We propose a scheme for the implementation of a robust N-qubit phase gate by one step with N atoms trapped in a cavity through a double Raman passage. Via the extension, we can realise an arbitrary N-qubit phase gate by appropriately adjusting the coupling strengths and detunings between atoms and external driving fields, and this can be realised by changing the amplitudes and frequencies of the external fields. Also we generate the Dicke state by one step. The advantages of the scheme are its robustness against decoherence due to the spontaneous emission of the excited states and decay of the cavity. What is required is only commonly to address the atoms inside an optical cavity. In addition, this scheme reduces the steps for implementing the N-qubit phase gate operation.

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