Distortion of optical feedback signals in microchip Nd:YAG lasers subjected to external multi-beam interference feedback*

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This paper proposes a theoretical analysis for the characteristics of an external cavity Nd:YAG laser with feedback of multiple-beam interference, which is induced by the multi-reentrance of the light from the external Fabry–Perot cavity. The theoretical model considers the multiple beam interference of the external Fabry–Perot cavity. It is found that the optical feedback signals are distorted to pulse waveforms instead of the sinusoidal ones in conventional feedback. The experimental results are in good agreement with the theoretical analysis. The obtained theoretical and experimental results can advance the development of a laser feedback interferometer.

Keywords: optical feedback, multi-beam interference

PACC: 4225H, 4255R, 4260D

1. Introduction

Self-mixing interference happens when a portion of the laser output is reinjected by a moving reflector back into the laser resonator, which is also called external optical feedback. The interference between the reinjected field and the stationary field can result in laser intensity modulation, from which the displacement or velocity of the external moving object can be recovered. Compared with a conventional interferometer, the self-mixing interference system has not only a similar phase sensitivity and modulation depth ratio, but also inherent simplicity, compactness, robustness, and economy. Thus, the self-mixing interference technique has attracted much attention in the application of measuring displacement,[1−6] velocity,[7−9] surface profile,[10,11] distance[12−14] and vibration.[15−17]

In order to explain the observed self-mixing interference phenomena, various theoretical models are presented.[1,4,18−24] Groot *et al.*[23] developed a theory based on the mode structure of a three-mirror Fabry–Perot cavity. This theory can be used to explain most basic modulation signals in the self-mixing laser cavity. However, in these works only the first round trip of beams in the external feedback cavity is considered and the multiple feedback effects within the external cavity are neglected. Some theoretical models take the front facet of the laser and the measurement target together as an equivalent mirror with a complex reflection coefficient.[18,23]

This paper considers the external Fabry–Perot cavity, from which the multiple beams transmitted can re-inject into a laser resonator. A theoretical analysis and a comparison with the experimental results on the self-mixing interference are presented. The described theoretical model considers the multi-beam interference of the external Fabry–Perot cavity. Compared with the sine-like waveforms in traditional optical feedback, the feedback signals are distorted to pulse waveforms in a feedback system with external Fabry–Perot cavity. Section 2 describes the experimental arrangement. The theoretical analysis and numerical simulations are presented in Section 3, and Section 4 gives the experimental results, which are in good agreement with the theoretical analysis in Section 3. Finally, conclusions are drawn in Section 5.

2. Experimental arrangement

The experimental setup is shown schematically in Fig. 1. The laser chip is a parallel-plane plate with thickness $d = 2$ mm and diameter $\Phi 5$ mm, made

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of YAG crystal containing 1.1 atomic % Nd^{3+} ions. Its input face is coated to be highly reflective (R_1 = 99.9%) at the lasing wavelength of 1064 nm, and antireflective at the pumping wavelength of 808 nm. The other surface is coated to be highly transmissive (T = 99.9%) at 1064 nm. The resonator with a cavity length l = 50 mm is composed of the input face of Nd:YAG crystal and a concave mirror M_2 (radius: 150 mm, reflectivity: R_2 = 98.8% at 1064 nm). It is longitudinally pumped by a fibre-coupled laser diode (LD), whose output is focused on the laser crystal by graded index (GRIN) lens (GL). The mirror M_3 is the external feedback mirror with a reflectivity R_3 = 90% and a curvature radius of 150 mm. The mirrors M_3 and M_2 together form the external Fabry–Perot cavity with length of about 76 mm. A piezoelectric transducer (PZT) drives the mirror M_3 to modify the length of external cavity by a few microns. The laser intensity emitted from M_3 is detected by photoelectric detector (PD).

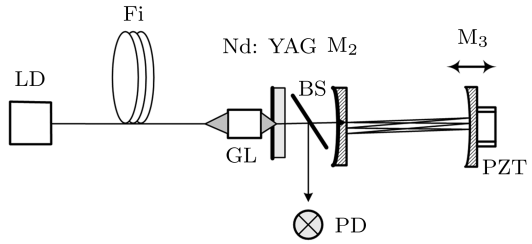


Fig. 1. Experimental setup. LD: laser diode; Fi: fibre; GL: GRIN lens; Nd:YAG: laser crystal; M_2: mirror; M_3: external feedback mirror; PZT: piezoelectric transducer; PD: photoelectric detector.

3. Theoretical analysis

Figure 2(a) is the schematic diagram of three-cavity-mirror model of the laser external feedback, and Figure 2(b) is the equivalent Fabry–Perot compound cavity. Correspondingly, the $r_{2\mathrm{eff}}$ is the equivalent reflectivity of the compound cavity.

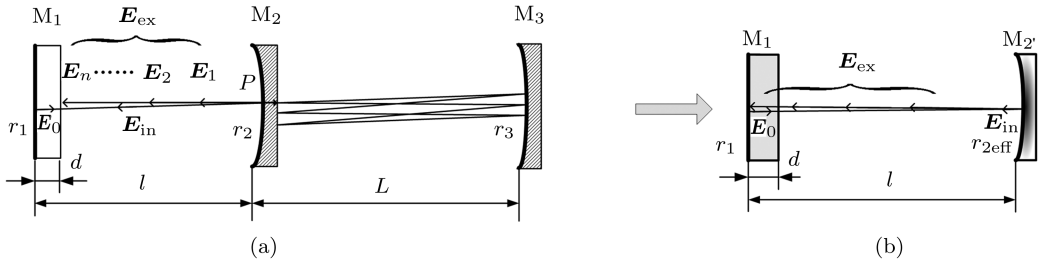


Fig. 2. Schematic diagram of optical feedback: (a) multiple-beam interference of the light fed back from the external Fabry–Perot cavity; (b) equivalent Fabry–Perot compound cavity.

The initial electric vector $\boldsymbol{E}_0(t)$ emitting from the outputted surface of Nd:YAG impinges on the point P of the mirror M_2, and then transmits into the external Fabry–Perot cavity, which is formed by mirrors M_2 and M_3. After one round trip, it emits from the same point P, and changes to $\boldsymbol{E}_1(t)$:

$$\boldsymbol{E}_1(t) = r_1 t_2 (r_3 r_2)^2 r_3 t_2 \exp(\mathrm{i}\omega\tau_l + 2gd) \times \exp[\mathrm{i}(\omega\tau_{\mathrm{L}} + \delta)]\boldsymbol{E}_0(t), \quad (1)$$

where r_1, r_2 and r_3 are the reflectivity of the laser mirrors and the external feedback mirror, respectively. The $\tau_l = 2l/c$ is the internal cavity round-trip time; $\tau_{\mathrm{L}} = 2NL/c$ is the round-trip time of the external feedback cavity, and N is the number of reflections on the mirror M_3; δ is the extra phase variation induced by the difference between the practical optical path and external cavity length. Notice that according to the model established by Huang *et al.*,[25,26] selecting proper cavity parameters, we can let mirrors M_2 and M_3 form a multi-reentrant nonplanar ring Fabry–Perot cavity (number of points:3, number of circulations:1), which is also proved by the outputted points 3 of the bidirectional beam propagation in the M_3 during the experiments. Thus, here, for each round trip, the inputted electric field will be reflected 3 times (N = 3) by mirror M_2 or M_3.

After two round trips, the emitting electric field $\boldsymbol{E}_2(t)$ from the M_2 back into laser resonator can be written as:

$$\boldsymbol{E}_2(t) = r_1 t_2 (r_3 r_2)^5 r_3 t_2 \exp(\mathrm{i}\omega\tau_l + 2gd) \times \exp[\mathrm{i}(\omega 2\tau_{\mathrm{L}} + 2\delta)]\boldsymbol{E}_0(t). \quad (2)$$

After n round trips, the emitting electronic field $\boldsymbol{E}_n(t)$ from the M_2 back into laser resonator can be written as:

$$\boldsymbol{E}_n(t) = r_1 t_2 (r_3 r_2)^{3n-1} r_3 t_2 \exp(\mathrm{i}\omega\tau_l + 2gd) \times \exp[\mathrm{i}(\omega n \tau_\mathrm{L} + n\delta)]\boldsymbol{E}_0(t). \quad (3)$$

Thus, all feedback electric fields $\boldsymbol{E}_1(t)$, $\boldsymbol{E}_2(t)$, ..., $\boldsymbol{E}_n(t)$ will result in multi-beam interference, and form the final feedback electric field $\boldsymbol{E}_\mathrm{ex}(t)$ as

$$\begin{aligned}
\boldsymbol{E}_\mathrm{ex}(t) &= \boldsymbol{E}_1(t) + \boldsymbol{E}_2(t) + \ldots + \boldsymbol{E}_n(t) \\
&= r_1 t_2 (r_3 r_2)^2 r_3 t_2 \exp(\mathrm{i}\omega\tau_l + 2gd) \exp[\mathrm{i}(\omega\tau_\mathrm{L} + \delta)]\boldsymbol{E}_0(t) \\
&\quad + r_1 t_2 (r_3 r_2)^5 r_3 t_2 \exp(\mathrm{i}\omega\tau_l + 2gd) \exp[\mathrm{i}(\omega 2\tau_\mathrm{L} + 2\delta)]\boldsymbol{E}_0(t) \\
&\quad + \ldots r_1 t_2 (r_3 r_2)^{3n-1} r_3 t_2 \exp(\mathrm{i}\omega\tau_l + 2gd) \exp[\mathrm{i}(\omega n \tau_\mathrm{L} + n\delta)]\boldsymbol{E}_0(t) \\
&= r_1 r_3 t_2^2 (r_3 r_2)^2 \exp(\mathrm{i}\omega\tau_l + 2gd)\, \mathrm{e}^{\mathrm{i}\phi}[1 + (r_3 r_2)^3 \mathrm{e}^{\mathrm{i}\phi} + (r_3 r_2)^6 \mathrm{e}^{\mathrm{i}2\phi} + \ldots + (r_3 r_2)^{3n} \mathrm{e}^{\mathrm{i}n\phi}]\boldsymbol{E}_0(t) \\
&= r_1 r_3 t_2^2 (r_3 r_2)^2 \frac{\mathrm{e}^{\mathrm{i}\phi}}{1 - (r_2 r_3)^3 \mathrm{e}^{\mathrm{i}\phi}} \exp(\mathrm{i}\omega\tau_l + 2gd)\boldsymbol{E}_0(t), \quad (4)
\end{aligned}$$

where $\phi = \omega\tau_\mathrm{L} + \delta$ is the external phase that light undergoes a roundtrip in the external cavity.

In presence of optical feedback, the initial electric field $\boldsymbol{E}_0(t)$ can be divided into two parts. The first one $\boldsymbol{E}_\mathrm{in}(t)$ travels within the internal cavity, while the second one $\boldsymbol{E}_\mathrm{ex}(t)$ travels in the external cavity and then couples into the internal cavity. These two parts of electric fields superpose in the internal cavity and construct the self-mixing interference. The final electric field $\boldsymbol{E}_\mathrm{f}(t)$ can be expressed as

$$\begin{aligned}
\boldsymbol{E}_\mathrm{f}(t) &= \boldsymbol{E}_\mathrm{in}(t) + \boldsymbol{E}_\mathrm{ex}(t) \\
&= r_1 r_2 \exp(\mathrm{i}\omega\tau_l + 2gd)\boldsymbol{E}_0(t) + r_1 r_3 t_2^2 (r_3 r_2)^2 \frac{\mathrm{e}^{\mathrm{i}\phi}}{1 - (r_2 r_3)^3 \mathrm{e}^{\mathrm{i}\phi}} \exp(\mathrm{i}\omega\tau_l + 2gd)\boldsymbol{E}_0(t) \\
&= r_1 r_2 \left[1 + \frac{r_3}{r_2} t_2^2 (r_3 r_2)^2 \frac{\mathrm{e}^{\mathrm{i}\phi}}{1 - (r_2 r_3)^3 \mathrm{e}^{\mathrm{i}\phi}}\right] \exp(\mathrm{i}\omega\tau_l + 2gd)\boldsymbol{E}_0(t) \\
&= r_1 r_2 \left[1 + \eta \frac{1}{\mathrm{e}^{-\mathrm{i}\phi} - (r_2 r_3)^3}\right] \exp(\mathrm{i}\omega\tau_l + 2gd)\boldsymbol{E}_0(t), \quad (5)
\end{aligned}$$

where $\eta = r_3 / r_2 t_2^2 (r_3 r_2)^2$.

Comparing Fig. 2(a) with Fig. 2(b), in accordance with Eq. (5), we find that the equivalent reflectivity of the compound cavity mirror M_2 is $r_{2\mathrm{eff}} = r_2 \left[1 + \eta / (\mathrm{e}^{-\mathrm{i}\phi} - (r_2 r_3)^3)\right]$.

According to self-consistent condition, the laser comes to equilibrium, when $\boldsymbol{E}_\mathrm{f}(t) = \boldsymbol{E}_0(t)$. The following equation can be obtained:

$$\begin{aligned}
&r_1 r_2 \left[1 + \eta \frac{1}{\mathrm{e}^{-\mathrm{i}\phi} - (r_2 r_3)^3}\right] \exp(\mathrm{i}\omega\tau_l + 2gd) = 1, \\
&r_1 r_2 \left[1 + \eta \frac{\cos\phi - (r_2 r_3)^3 + \mathrm{i}\sin\phi}{(\cos\phi - (r_2 r_3)^3)^2 + (\sin\phi)^2}\right] \\
&\times \exp(\mathrm{i}\omega\tau_l + 2gd) = 1. \quad (6)
\end{aligned}$$

Equation (6) can be solved as

$$\begin{aligned}
&r_1 r_2 \Big\{ \left[1 + \xi\cos\phi - \xi(r_2 r_3)^3\right]^2 \\
&+ [\xi\sin\phi]^2 \Big\}^{1/2} \exp(\mathrm{i}\omega\tau_l + 2gd) \exp(\mathrm{i}\theta) = 1, \quad (7)
\end{aligned}$$

where

$$\begin{aligned}
\xi &= \frac{\eta}{(\cos\phi - (r_2 r_3)^3)^2 + (\sin\phi)^2}, \\
\tan(\theta) &= \frac{\xi\sin\phi}{1 + \xi\cos\phi - \xi(r_2 r_3)^3}. \quad (8)
\end{aligned}$$

Equation (7) can be solved to get

$$\begin{aligned}
g = -\frac{1}{2d} \Bigg[&\ln(r_1 r_2) + \frac{1}{2} \ln \Big\{ \left[1 + \xi\cos\phi - \xi(r_2 r_3)^3\right]^2 \\
&+ [\xi\sin\phi]^2 \Big\} \Bigg], \quad (9)
\end{aligned}$$

$$\omega\tau_l + \theta = 2\Lambda\pi, \quad (10)$$

where Λ is positive integer.

In the absence of external feedback mirror M_3, there is no optical feedback. Thus $r_3 = 0$, and the linear gain per unit length due to the stimulated emission

inside the laser cavity in the absence of optical feedback is

$$g_0 = -\frac{1}{2d}\ln(r_1 r_2). \tag{11}$$

Thus the excess required gain Δg can be expressed as

$$\begin{aligned}\Delta g &= g - g_0 \\ &= -\frac{1}{4d}\ln\left\{\left[1+\xi\cos\phi-\xi(r_2r_3)^3\right]^2\right. \\ &\quad \left.+\left[\xi\sin\phi\right]^2\right\}. \end{aligned}\tag{12}$$

The laser intensity with optical feedback can be expressed as[27,28]

$$I = I_0(1 - K\Delta g), \tag{13}$$

where K is a constant and I_0 is the initial laser intensity without optical feedback.

Substituting Eq. (12) into Eq. (13), we can express the laser output intensity with optical feedback as

$$\begin{aligned} I &= I_0\left\{1+\frac{K}{4d}\ln\left[(1+\xi\cos\phi\right.\right. \\ &\quad \left.\left.-\xi(r_2r_3)^3)^2+(\xi\sin\phi)^2\right]\right\}, \\ \xi &= \frac{\eta}{(\cos\phi-(r_2r_3)^3)^2+(\sin\phi)^2}, \\ \eta &= \frac{r_3}{r_2}t_2^2(r_3r_2)^2. \end{aligned}\tag{14}$$

According to the previous works Refs. [1], [3] and [4], the outputed intensity of laser is proportional to cosine function of the external phase. While, from Eq. (14) we can see that due to the multi-beam interference of external Fabry–Perot cavity, the laser intensity is no longer the cosine function of the external phase ϕ, but a complex function as shown in Eq. (14).

From Fig. 3, we can see that due to the multi-beam interference in the external Fabry–Perot cavity, the laser intensity curve performs pulse waveform instead of the cosine one in conventional optical feedback system. At the same time, the peak values of these pulses are not equal as shown in the insert of Fig. 3, which is caused by the phase variations δ of the multi-feedback light of different round trips from the external cavity.

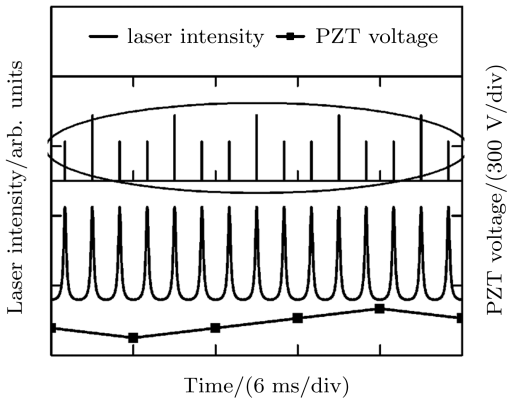


Fig. 3. Simulation of the laser feedback signal based on Eq. (14). The insert displays an amplitude zoom of the intensity signal.

4. Experimental results

We have analysed the effects of multi-beam interference feedback theoretically and simulated the laser feedback signals of pulse waveform in Section 3. To certify the correctness of the theoretical model and prove that the simulated results do exist, a series of experiments has been done.

First, the external feedback mirror was perfectly aligned. Then we connected the feedback loop and drove the PZT with a saw-tooth wave to change the optical phase of the external feedback light. Meanwhile, a precision motorized stage is used to modify the length of the external cavity with a step of 50 microns around 76 mm. It was easy to observe the laser feedback signals of pulse waveforms as shown in Fig. 4(a), and there are three pulses with un-equal amplitude in every period, which agrees well with the simulation result in Fig. 3. The different peak values of the pulses are induced by the extra phase variation of the feedback light of the different round-trips.[29]

Then we inserted an attenuator into the external cavity to control the feedback level being about 30%. So, the light of the second roundtrip in the external cavity would be only 0.07% of that of the first round-trip. Consequently, the multi-beam reflected from the external cavity can not construct the multi-beam interference due to the large amplitude difference between them, although the external cavity is a Fabry–Perot one. Thus, only the light $\boldsymbol{E}_1$ of the first roundtrip in Fig. 2 should be considered for low feedback level. Then the traditional sine-like feedback signals can be observed as shown in Fig. 4(b).

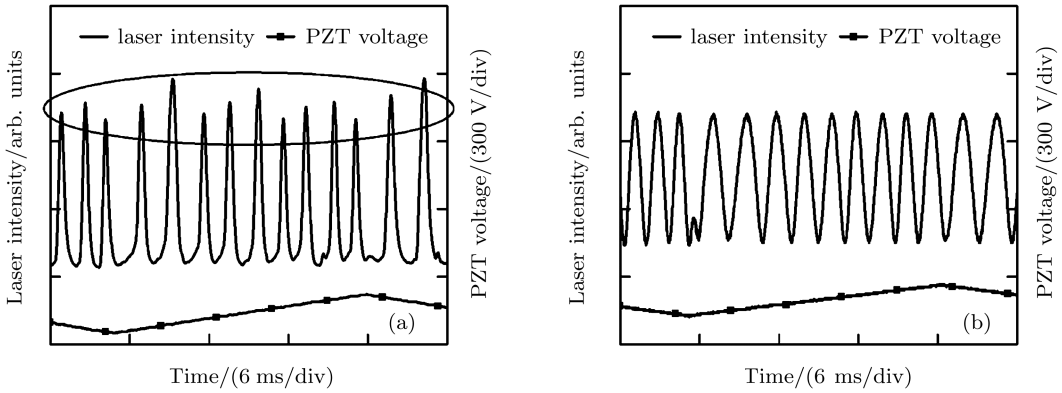


Fig. 4. Different optical feedback signals: (a) pulse waveforms; (b) sinusoidal waveforms.

5. Conclusions

A theoretical analysis together with experimental results has been carried out on the characteristics of the laser feedback signals with the external feedback of multi-beam interference. Under a high feedback level, the laser feedback signals present pulse waveforms due to the multi-beam interference of the aligned external cavity. Compared with the traditional sine-like feedback signals, we cannot use these pulse signals to discriminate the movement direction of the external feedback mirror, and are hardly subdivided to obtain higher resolution than half laser wavelength. Thus, appropriate feedback level and external cavity are of importance for the laser feedback system. Our research can advance the development of the laser feedback interferometer.

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