# Exact results on cavity cooling in a system of a two-level atom and a cavity field*

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Using the algebraic dynamical method, this paper investigates the laser cooling of a moving two-level atom coupled to a cavity field. Analytical solutions of optical forces and the cooling temperatures are obtained. Considering Rb atoms as an example, it finds that the numerical results are relevant to the recent experimental laser cooling investigations.

**Keywords:** laser cooling, algebraic dynamical method, optical forces, cooling temperatures
**PACC:** 3280, 4250, 3280P, 4250V

## 1. Introduction

Cavity cooling has attracted significant attention recently.[1] Since dissipation takes place via cavity loss rather than by spontaneous emission, cavity cooling can avoid or reduce several problems of the conventional laser cooling schemes, such as photon reabsorption and recoil heating, which make it become a more efficient means to cool atoms and a more attractive approach for creating ultra-cold molecules than conventional methods.[2−6] Recently, further cavity cooling improvements by introducing a linear feedback,[7] by exploiting interference effects[8] and by keeping the Rabi frequency invariant but reducing the mode volume were also suggested.[9] Furthermore, application of such cooling mechanism would improve the performance of atom–cavity systems for quantum information processing[10−13] and cavity Bose–Einstein condensation.[14] The interaction of a two-level system with a quantized single mode laser field, named the Jaynes–Cummings model(JCM), is the simplest possible model focusing on the cavity cooling. We enjoy a vast number of theoretical developments, these include the density matrix method,[1,15−17] the dressed-state method,[18] the Monte Carlo wavefunction method[19,20] and the classical and quantum Langevin approach.[7,21,22] Unfortunately, a great part of them are not easy to handle and most of the interesting physics have to be extracted from heavy numerical solutions and calculations, especially when the atomic centre-of-mass motion is taken into account.

The objective of the present paper is to give an exact quantum mechanical solution for the cavity cooling of the improved two-level JCM with the atomic centre-of-mass motion included and quantized as indicated by its kinetic energy operator by using the algebraic dynamical method in an ideal cavity (The general analytical solution to cavity cooling including the cavity decay is extremely complicated and not very instructive by using the algebraic dynamical method. In this paper, we only discuss the simplest cavity system with a rather small loss or the cavity decay lifetime as infinite, hence the exact solution of cavity cooling can be obtained).[23] The important idea in this paper is to introduce a canonical transformation for the Hamiltonian and linearize it in terms of a set of Lie algebraic generators. Using the algebraic dynamical approach, we find that the linear algebraic dynamical systems are integrable and thus solvable, then the evolution matrices, as well as the eigenvalues and eigenstates of the Hamiltonian can be obtained easily. As a result, the analytical expressions for optical forces and cooling temperatures can be obtained.

In Section 2, we present the basic model of the atom–cavity system and introduce the reduced density operator of the considered system by using algebraic dynamical method. In Section 3, we derive the analytical solutions of the optical forces and the cooling temperatures. The numerical results that confirm the

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experimental investigations are also shown. A summary is included in Section 4.

## 2. The physical model and the density operator

We only consider a moving two-level atom interacting with a one-mode field in an ideal optical cavity. Of course, the cavity loss is neglected for simplicity, the model is thus ideal and only applicable to a cavity system with rather small loss. Inclusion of cavity loss will result in increased complexity of the model, and the dissipation due to the cavity loss is usually described by the non-Hermitian Hamiltonian which is difficult to deal with by using the algebraic dynamical method. We restrict the motion of the atom to one dimension, i.e., along the cavity axis. The Hamiltonian which describes the unitary dynamics of the whole system is

$$\hat{H} = \frac{\hat{p}_x^2}{2m} + \hbar\omega_0 S_z + \hbar\omega a^+ a + \hbar\Omega(a^+ S_- \mathrm{e}^{-\mathrm{i}\,kx} + aS_+ \mathrm{e}^{\mathrm{i}\,kx}), \quad (1)$$

where we take the propagating direction of the cavity field as the $x$-axis. The term $\hat{p}_x^2/2m$ is the kinetic energy of atomic centre-of-mass motion. The physical quantities $\omega_0$ and $\omega$ are the atomic transition frequency and the cavity field frequency, respectively. The last term is the interaction Hamiltonian. The operators $a$ and $a^+$ are the cavity annihilation and creation operators. The operators $S_z$, $S_-$ and $S_+$ are atomic pseudo-spin operators, $x$ is the coordinate of atomic centre-of-mass motion, $\Omega$ is the atom–field coupling coefficient, $k$ denotes wave number.

To linearize the Hamiltonian by the canonical transformation, an operator is introduced as

$$U_\mathrm{g} = \exp\left[\frac{\theta}{N^{1/2}}(aS_+ \mathrm{e}^{\mathrm{i}\,kx} - a^+ S_- \mathrm{e}^{-\mathrm{i}\,kx})\right], \quad (2)$$

where

$$\theta = -\arctan[(\sqrt{\Delta^2/4 + \hbar^2\Omega^2 N} - \Delta/2)/\hbar\Omega N^{1/2}],$$

$$\Delta = \hbar k P_g/m + \hbar^2 k^2/2m + \hbar\Delta'.$$

$\Delta'$ is defined as $\omega_0 - \omega$, describing the detuning of the atomic resonance frequency $\omega_0$ from cavity field frequency $\omega$. $N = a^+a + S_z + 1/2$ and $P_\mathrm{g} = \hat{p}_x - \hbar k(S_z + 1/2)$ are two invariants of the system, which represent the excitation number operator and the momentum of atomic ground state, respectively.

Considering SU(2) algebra generators $\{S_z, N^{-1/2}aS_+\mathrm{e}^{\mathrm{i}\,kx}, N^{-1/2}a^+S_-\mathrm{e}^{-\mathrm{i}\,kx}\}$ and letting $S_z = J_0$, $N^{-1/2}aS_+\mathrm{e}^{\mathrm{i}\,kx} = J_+$ and $N^{-1/2}a^+S_-\mathrm{e}^{-\mathrm{i}\,kx} = J_-$, we find that $J_0$, $J_+$ and $J_-$ satisfy the following commutation relations

$$[J_0, J_+] = J_+, \quad [J_0, J_-] = -J_-, \quad [J_+, J_-] = 2J_0. \quad (3)$$

Then, $U_\mathrm{g}$ can be reduced to the following form

$$U_\mathrm{g} = \exp[\theta J_+ - \theta J_-]. \quad (4)$$

After the canonical transformation, the dressed Hamiltonian can be written as

$$\hat{H}' = U_\mathrm{g}^{-1}\hat{H}U_\mathrm{g} = E(N, P_\mathrm{g}) + \lambda S_z, \quad (5)$$

where $\lambda = \sqrt{\Delta^2 + 4\hbar^2\Omega^2 N}$, $E(N, P_\mathrm{g}) = (P_\mathrm{g}^2 + \hbar k P_\mathrm{g} + \hbar^2 k^2/2)/2m + \hbar\omega(N - 1/2)$.

To obtain the time evolution expressions of the optical forces and final temperatures of the cooled atom, the time evolution operator should be introduced. From Eqs. (4) and (5), the time evolution operator of the system is expressed by

$$U(t) = \mathrm{e}^{-\mathrm{i}\,\hat{H}t/\hbar} = U_\mathrm{g}\mathrm{e}^{-\mathrm{i}\,\hat{H}'t/\hbar}U_\mathrm{g}^{-1}, \quad (6)$$

where $\mathrm{e}^{-\mathrm{i}\,\hat{H}'t/\hbar} = \mathrm{e}^{-\mathrm{i}\,Et/\hbar}(\cos(\lambda t/2\hbar) - \mathrm{i}\,2S_z\sin(\lambda t/2\hbar))$, then,

$$U(t) = \mathrm{e}^{-\mathrm{i}\,Et/\hbar}\left[\cos\left(\frac{\lambda t}{2\hbar}\right) - \mathrm{i}\,2J_0\sin\left(\frac{\lambda t}{2\hbar}\right)\cos(2\theta) + \mathrm{i}(J_+ + J_-)\sin\left(\frac{\lambda t}{2\hbar}\right)\sin(2\theta)\right]. \quad (7)$$

The density operator for the atom–field is $\rho(t)$, which evolves according to the formula

$$\rho(t) = U(t)\rho(0)U^+(t). \quad (8)$$

Assume that the internal state of the atom is in the ground state $|1\rangle$, and the initial density operator for the system is composed of three parts,

$$\rho(0) = |\psi_F(0)\rangle\langle\psi_F(0)| \otimes |\psi_{p'_x}(0)\rangle\langle\psi_{p'_x}(0)| \otimes |1\rangle\langle 1|, \quad (9)$$

where cavity field, $|\psi_F(0)\rangle = \sum_{n'} F_{n'}|n'\rangle$, is the superposition of Fock states, the external state of the atom, $|\psi_{p'_x}(0)\rangle = \int \mathrm{d}p'_x C(p'_x)|p'_x\rangle$, is the superposition of plane waves. In order to explore the dynamics of the atom, the reduced density operator for atomic internal state, $\rho^\mathrm{A}(t)$, is introduced.

$$\rho^A(t) = \int \mathrm{d}p_x \sum_n \langle n, p_x|\rho(t)|p_x, n\rangle. \quad (10)$$

Substituting Eqs. (6), (7), (8) and (9) into Eq.(10), we can write the expression of $\rho^\mathrm{A}(t)$ as

$$\rho^{A}(t)=\int \mathrm{d}p_x \sum_n |C(p_x)|^2|F_n|^2\left[\left(\cos\left(\frac{\lambda t}{2\hbar}\right)+\mathrm{i}\sin\left(\frac{\lambda t}{2\hbar}\right)\cos(2\theta)\right)|1\rangle+\mathrm{i}\sin\left(\frac{\lambda t}{2\hbar}\right)\sin(2\theta)|2\rangle\right]$$
$$\times\left[\langle 1|\left(\cos\left(\frac{\lambda t}{2\hbar}\right)-\mathrm{i}\sin\left(\frac{\lambda t}{2\hbar}\right)\cos(2\theta)\right)-\langle 2|\,\mathrm{i}\sin\left(\frac{\lambda t}{2\hbar}\right)\sin(2\theta)\right]. \tag{11}$$

In the following, Eq. (11) will be used to obtain the exact solutions of cavity cooling.

## 3. Analytical solutions of cavity cooling

In this section we will present the analytical solutions of cavity cooling, the optical forces and the final cooling temperatures are both investigated. The expectation value for the force on the atom can be obtained straightforwardly by the following relation:

$$f=\langle F(t)\rangle=-\langle\nabla\hat{H}_{\mathrm{int}}\rangle=-\,\mathrm{Tr}(\rho^{\mathrm{A}}(t)\nabla\hat{H}_{\mathrm{int}}). \tag{12}$$

From the expressions of $\nabla\hat{H}_{\mathrm{int}}$ and $\rho^{\mathrm{A}}(t)$ and by using Eq. (12), we find that the force has the following form:

$$f=-\int \mathrm{d}p_x \sum_n |C(p_x)|^2|F_n|^2\left[\left(\cos\left(\frac{\lambda t}{2\hbar}\right)+\mathrm{i}\sin\left(\frac{\lambda t}{2\hbar}\right)\cos(2\theta)\right)a\mathrm{e}^{\mathrm{i}\,kx}\right.$$
$$\left.+\left(\cos\left(\frac{\lambda t}{2\hbar}\right)-\mathrm{i}\sin\left(\frac{\lambda t}{2\hbar}\right)\cos(2\theta)\right)a^{+}\mathrm{e}^{-\mathrm{i}\,kx}\right]\hbar k\Omega\sin\left(\frac{\lambda t}{2\hbar}\right)\sin(2\theta). \tag{13}$$

Using the Einstein relation, we can obtain the cavity cooling temperature

$$T=\frac{D_{\mathrm{p}}}{\beta k_{\mathrm{B}}}, \tag{14}$$

where $D_{\mathrm{p}}$ is the momentum diffusion coefficient, $\beta$ is friction coefficient, $k_{\mathrm{B}}$ is Boltzmann constant. $D_{\mathrm{p}}$ and $\beta$ are determined by

$$D_{\mathrm{p}}=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(\Delta\hat{\mathrm{p}}_x)^2, \tag{15}$$

$$\beta=-\frac{\mathrm{d}}{\mathrm{d}\upsilon}f(\upsilon). \tag{16}$$

To obtain the temperatures, the mean values of $\hat{p}_x$ and $\hat{p}_x^2$ should be considered, which can be obtained from the following relation

$$\langle\hat{p}_x\rangle=\left\langle P_{\mathrm{g}}+\hbar k\left(S_z+\frac{1}{2}\right)\right\rangle. \tag{17}$$

After calculation, the expectation value of $S_z$ is

$$\langle S_z\rangle=-\frac{1}{2}\int \mathrm{d}p_x \sum_n |C(p_x)|^2|F_n|^2\left(\sin^2\left(\frac{\lambda t}{2\hbar}\right)\cos(4\theta)+\cos^2\left(\frac{\lambda t}{2\hbar}\right)\right). \tag{18}$$

Thus,

$$\langle\hat{p}_x\rangle=-\frac{\hbar k}{2}\int \mathrm{d}p_x \sum_n |C(p_x)|^2|F_n|^2\left(\sin^2\left(\frac{\lambda t}{2\hbar}\right)\cos(4\theta)+\cos^2\left(\frac{\lambda t}{2\hbar}\right)\right)+P_{\mathrm{g}}+\frac{1}{2}\hbar k. \tag{19}$$

Similarly,

$$\langle\hat{p}_x^2\rangle=-\frac{1}{2}(\hbar^2k^2+2P_{\mathrm{g}}\hbar k)\int \mathrm{d}p_x \sum_n |C(p_x)|^2|F_n|^2\left(\sin^2\left(\frac{\lambda t}{2\hbar}\right)\cos(4\theta)+\cos^2\left(\frac{\lambda t}{2\hbar}\right)\right)+P_{\mathrm{g}}^2+\frac{1}{2}\hbar^2k^2+\hbar kP_{\mathrm{g}}. \tag{20}$$

Using Eqs. (13), (15), (16), (19) and (20), we get the momentum diffusion coefficient and the friction coefficient

$$D_{\mathrm{p}}=\frac{k}{8}(2P_{\mathrm{g}}+\hbar k)\int \mathrm{d}p_x \sum_n |C(p_x)|^2|F_n|^2\lambda\sin\left(\frac{\lambda t}{\hbar}\right)(1-\cos(4\theta)), \tag{21}$$

$$\beta = \frac{2m\hbar k\Omega \int \mathrm{d}p_x \sum\limits_n |C(p_x)|^2 |F_n|^2 f(\theta,\lambda,t) \sin\left(\dfrac{\lambda t}{2\hbar}\right) \sin(2\theta)}{\hbar k \int \mathrm{d}p_x \sum\limits_n |C(p_x)|^2 |F_n|^2 \left( -\sin^2\left(\dfrac{\lambda t}{2\hbar}\right)\cos(4\theta) - \cos^2\left(\dfrac{\lambda t}{2\hbar}\right)\right) + 2P_\mathrm{g} + \hbar k}, \tag{22}$$

where

$$f(\theta,\lambda,t) \equiv \left[\left(\cos\left(\frac{\lambda t}{2\hbar}\right) + \mathrm{i}\sin\left(\frac{\lambda t}{2\hbar}\right)\cos(2\theta)\right) a\mathrm{e}^{\mathrm{i}\,kx} + \left(\cos\left(\frac{\lambda t}{2\hbar}\right) - \mathrm{i}\sin\left(\frac{\lambda t}{2\hbar}\right)\cos(2\theta)\right) a^{+}\mathrm{e}^{-\mathrm{i}\,kx}\right].$$

Substituting Eqs. (21) and (22) into Eq. (14) leads to the cavity cooling temperature

$$\begin{aligned} T = {} & \left[\hbar k \int \mathrm{d}p_x \sum_n |C(p_x)|^2 |F_n|^2 \left( -\sin^2\left(\frac{\lambda t}{2\hbar}\right)\cos(4\theta) - \cos^2\left(\frac{\lambda t}{2\hbar}\right)\right) + 2P_\mathrm{g} + \hbar k\right] \\ & \times \frac{(2P_\mathrm{g} + \hbar k)\left[\int \mathrm{d}p_x \sum\limits_n |C(p_x)|^2 |F_n|^2 \lambda \sin\left(\dfrac{\lambda t}{\hbar}\right)(1 - \cos(4\theta))\right]}{16\hbar k_\mathrm{B} m\Omega \int \mathrm{d}p_x \sum\limits_n |C(p_x)|^2 |F_n|^2 f(\theta,\lambda,t) \sin\left(\dfrac{\lambda t}{2\hbar}\right)\sin(2\theta)}. \end{aligned} \tag{23}$$

In order to analyse the results, we consider the general solutions by using a numerical method. To discuss them, we treat the laser field classically[24]

$$a \to \alpha \mathrm{e}^{-\mathrm{i}\,\omega t}, \quad a^{+} \to \alpha^{*}\mathrm{e}^{i\omega t}. \tag{24}$$

Assuming $\alpha^{*} = \alpha$, we find that the external state of the atom is in the momentum eigenstate $|p\rangle$ and the cavity field is still in the superposition of the number states, the force can be written as

$$f = -2\sum_n |F_n|^2 \left[\cos\left(\frac{\lambda t}{2\hbar}\right)\cos(kx - \omega t) - \sin\left(\frac{\lambda t}{2\hbar}\right)\cos(2\theta)\sin(kx - \omega t)\right] \hbar k\Omega\alpha \sin\left(\frac{\lambda t}{2\hbar}\right)\sin(2\theta), \tag{25}$$

where $|F_n|^2 = \mathrm{e}^{-\overline{n}}/(\overline{n}^n n!)$. From Eq. (25), it can be found that the radiation force depends on the coupling strength, the detuning, atomic coordinate and the trapping time, etc. To obtain the numerical results, we adopt the values of the parameters in Ref. [3]. The quantity $k = 2\pi/\lambda_\mathrm{c}$ is the wave vector of the cavity mode, where $\lambda_\mathrm{c}$ is the length of the micro-cavity, in general, the value of $\lambda_\mathrm{c}$ ranges from tens of microns to hundreds of micrometers. The initial velocity of the atomic centre-of-mass motion is of the order of 1 cm/s. The mass of Rb atom is $m = 1.41922 \times 10^{-25}$ kg, $\omega$ is about $1 \times 10^{15}$ Hz, the transition is $5^2S_{1/2}F = 3 \leftrightarrow 5^2P_{3/2}F = 4$ and the average photon number is $\overline{n} = 5$. In Fig. 1, the force is shown as the function of time and position of the atom in three dimensions. In order to visualize the influence of the coupling strength and the detuning, we plot the force as the function of the coordinate with different coupling strengths and detunings in two dimensions in Fig. 2, one can see that as the value of the coupling strength increases, the amplitude of the force increases too. As the coupling strength is proportional to the intensity of radiation field and inversely proportional to the volume of the micro-cavity according to cavity cooling theory, we can get the results that both increasing the intensity of the cavity field and reducing the volume of the cavity will enlarge the force, otherwise the force will be diminished.[3,22] Considering the actual fact that an atom will be localized in a small region, from Fig. 2 we can also show that, the force generated by the cavity field is

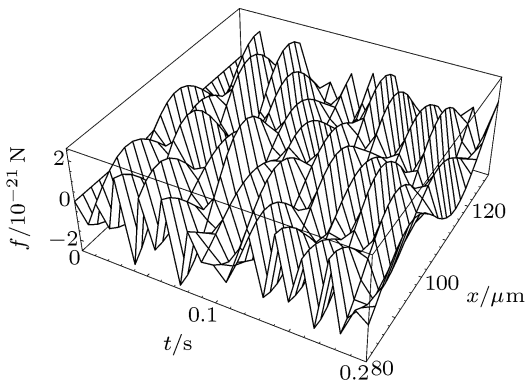



**Fig. 1.** Radiation force as a function of $t$ and $x$. The parameters are $\Delta' = 2\pi \times 32$ MHz, $\Omega = 2\pi \times 16$ MHz, $\bar{n} = 5$ and $\omega = 1 \times 10^{15}$ Hz.

stronger than the one in free-space. In this region, the interaction between the atom and the intracavity photon can dramatically influence their dynamical states, which is consistent with the experimental results.[3,15,25] Figure 3 shows the influence of the detuning. As the detuning becomes large, the cavity field will trap the atoms instead of cooling them, as a consequent result, the cooling force will decrease.

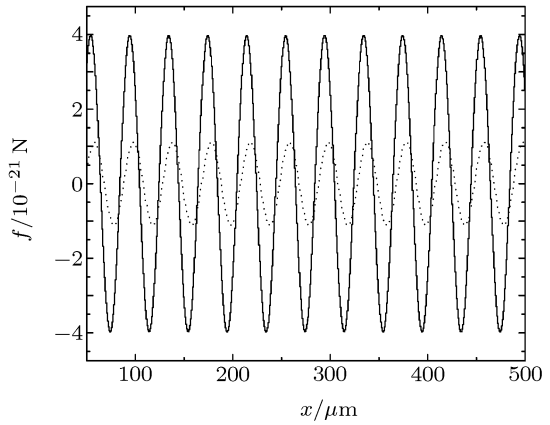



**Fig. 2.** Radiation force as a function of $x$ for different coupling strengths $\Omega = 2\pi \times 20$ MHz (dotted line), $\Omega = 2\pi \times 32$ MHz (solid line), $\Delta' = 2\pi \times 12$ MHz, $t = 1$ s, $\bar{n} = 5$ and $\omega = 1 \times 10^{15}$ Hz.

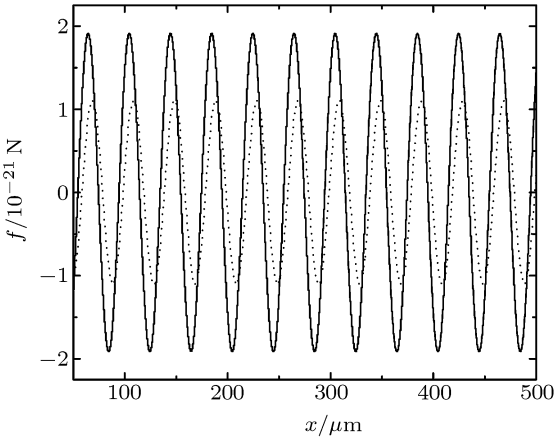



**Fig. 3.** Radiation force as a function of $x$ with different detunings, $\Delta' = 2\pi \times 40$ MHz (dotted line), $\Delta' = 2\pi \times 12$ MHz (solid line), $\Omega = 2\pi \times 16$ MHz, $t = 1$ s, $\bar{n} = 5$ and $\omega = 1 \times 10^{15}$ Hz.

In order to investigate the final temperature of the cooling, the same conditions used in discussing the force are taken into account. For simplicity, we assume $\Delta = 0$, then $\lambda = 2\hbar\Omega\sqrt{\alpha}$. In the region of cooling, we get the following rather simple expression for the final temperature

$$T = \frac{(2P_{\rm g} + \hbar k)\lambda\left[\hbar k \cos\left(\dfrac{\lambda t}{\hbar}\right) - 2P_{\rm g} - \hbar k\right]}{8m\hbar k_{\rm B}\Omega\alpha\cos(kx - \omega t)}. \quad (26)$$

From Eq. (26), it can be found that the final temperature is dependent on the coupling strength, the detuning, atomic coordinate, and trapping time, etc. We note that $\lambda$ is proportional to $\Omega$, so we can obtain the relation between $T$ and $\Omega$ by using the relation between $T$ and $\lambda$. Figure 4 displays the fact that the cooling temperature will be diminished as the coupling strength becomes large and the final temperature is about 100 nK, which agrees well with the theoretical and experimental results.[3,25−27]

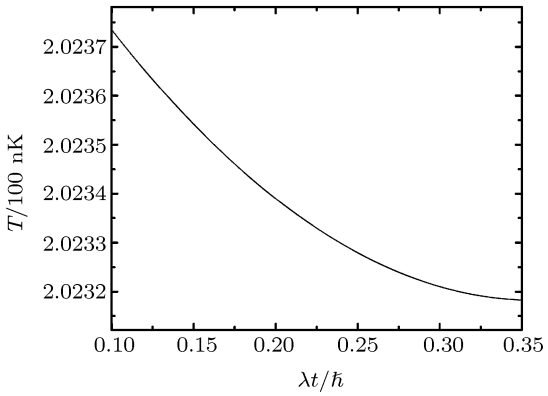



**Fig. 4.** The temperature of the atom as the function of $\lambda$. The parameters are $m = 1.41922 \times 10^{-25}$ kg, $\bar{n} = 5$, $x = 80$ $\mu$m and $\omega = 1 \times 10^{15}$ Hz.

# 4. Conclusion

We have described the dynamics of a coupling atom–cavity system by considering the atomic centre-of-mass motion. Using the algebraic dynamics method, we linearize the Hamiltonian for the system by introducing a canonical transformation, and then derive the reduced density operator of the considered system. Then analytical solutions and numerical results of cavity cooling are obtained. We find that the results are consistent with the experimental investigations. The explicit analytical solution Eq. (11) forms a good starting point for further seeking simpler solutions such as entanglement and decoherence etc. so that more physical insight can be sought. The related theoretical researches are meaningful and they may promote the development of the technology of laser cooling.

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