Scheduling trains on railway network using random walk method*

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According to random walk, in this paper, we propose a new traffic model for scheduling trains on a railway network. In the proposed method, using some iteration rules for walkers, the departure and the arrival times of trains at each station are determined. We test the proposed method on an assumed railway network. The numerical simulations and the analytical results demonstrate that the proposed method provides an effective tool for scheduling trains. Some characteristic behaviours of train movement can be reproduced, such as train delay.

Keywords: train schedule, random walk model, passenger transportation plan

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1. Introduction

With the development of modern railway transportation, new issues and challenges are arising. One of the important issues is to design a new passenger transportation plan. Usually, the design of a new plan roughly includes three steps.[1−3] First, according to the passenger demand, workers and planners determine the departure and the arrival stations of trains on a given network. Then, they determine the routes of trains. Finally, according to the routes of trains, the departure and the arrival times of trains at each station are determined. The third step is complicated and difficult for scheduling trains.

The method of determining the departure and the arrival times of trains can be divided into two groups: computer-aided simulation and mathematical programming methodologies. In the practical design process of a passenger transportation plan, the mathematical programming methodologies are not widely employed. Instead, computer-aided simulation methodologies are mostly used.[2,4−6] For every computer-aided simulation methodology, a sophisticated and effective simulation model is necessary. This simulation model needsb to provide an account of system behaviours through simulations. In addition, since practical railway networks have large sizes, this kind of simulation model needs to have a high computation efficiency. The random walk model is a good choice for solving such a problem.

Here a new model for determining the departure and the arrival times of trains is proposed, which is based on the random walk model. The proposed method consists of some iteration rules which can be used to simulate the complex dynamic behaviours of train movement. Since the time is discrete, the numerical implementation of the proposed method is relatively simple. The remainder of the present paper is organized as follows. We introduce the random walk model in Section 2. In Section 3, the principle of the train control system is introduced. In Section 4, we outline the proposed model. The numerical and the analytical results are presented in Sections 5. Finally, conclusions of this approach are presented in Section 6.

2. Random walk model

The theory of random walk has a long history and has been applied to solving numerous theoretical and practical problems.[7,8] In general, the random walk represents a link between the microscopic dynamics and macroscopic observation. It provides a powerful tool which can be used to explore the connection between the network topology and the functional property of the network.[8] Usually, random walk models are divided into two classes, i.e. the oriented model

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and un-oriented model. In the oriented model, the direction of each step is chosen relative to a fixed compass direction. However, in the un-oriented model, the direction is chosen relative to the preceding step.[9]

The random walk is a fundamental dynamic process,[10] which has been used to study many systems in reality, such as biological movements.[11,12] The basic rule for random walk is as follows. A random walk is defined as a walking manner that a walker moves along the edges of a given network. At each time step, the walker is restricted to move to one of its neighbour nodes. In the random walk model, random walk can be described by a master equation. The master equation is usually expressed by a probability forma.[13,14] When we use a random walk model to approximate a dynamic process, the master equation reflects the dynamic behaviour of system.

3. Moving block signaling system

Two types of the train control systems have been developed, i.e. the fixed block system and moving block system. In the fixed block system, each track section is divided into several blocks. The safety distance between two successive trains is at least larger than the length of each block. With the moving block system, the safety distance between two successive trains is larger than the safety stopping distance $d_{\rm s}$, where $d_{\rm s} = v_{\max}^2/(2b) + \tau v_{\max}$. τ is called the reaction time of drivers.

In a moving block system, the continuous two-way digital communication between each controlled train and a wayside control centre are adopted. In this system, the line is usually divided into areas or regions, each area is under the control of a computer and has own radio transmission system. Each train transmits its identity, location, direction and speed to the area computer. The radio link between each train and the area computer is continuous so that the computer knows the locations of all the trains in its area all the time. It transmits to each train the location of the train in front and gives it a braking curve to enable it to stop before it reaches that train.

4. The proposed method

In this paper, we use the oriented random walk model to schedule trains on railway network. To assume that the routes of trains are given in advance, our task is to determine the departure and the arrival times of trains at each station. Here the railway network is regarded as a network graph which is initialized with N nodes. The nodes and the edges of the network graph respectively represent the stations and track sections. In general, one route of trains consists of many linked edges. The direction of the walk movement is determined according to these linked edges.

According to passenger transportation plan, walkers are created at their departure nodes, and leave at their arrival nodes. The routes that walkers moving along are outlined in passenger transportation plan. At each node, there is one platform or more where passengers may board and alight from trains. Several walkers can occupy a node at the same time. If they are on the same platform, they must queue according to their arrival times. When we consider the moving-block control system, the safety stopping distance between two successive walkers must be kept. On the basis of the above considerations, we design an algorithm model for simulating the walker movements on network graph. The master equation (or dynamical behaviour) of the walker movement is described by some update rules. Our algorithm consists of the following steps.

1) According to the route of train, walker starts from its departure node, and moves with the speed $v_{\max}$ along the edges that constitute such a route.

2) At each time step, walker moves v (current speed) units toward its arrival node.

3) As a walker arrives at a middle node, it directly passes through the node, or stops at the node for a time interval $T_{\rm d}$ (called station dwell time).

4) On the same edge, as the distance between two successive walkers is smaller than the safety stopping distance $d_{\rm s}$, the follower must decelerate to stop.

5) At the same node, walker can select different platforms. If several walkers need stop on the same platform, they need queue. Newcomer, walker, should stay at the end of the queue. So the walker that is at the head of the queue will depart first.

6) As walker arrives at its arrival node, it simply moves out of the network graph.

At the same time, two or more walkers possibly use the same edge. In this case, the attrition of the walkers' paths would occur. In order to resolve this attrition problem, we propose the following control strategies. 1) When two walkers are at two different nodes, the fast walker moves first, and the slow walker waits there. For example, as shown in Fig. 1(a), two

walkers move in the opposite directions, one is faster, and the other is slower. 2) On the same edge, the walkers are not allowed to move in opposite directions at the same time. The walker on edge moves first, and the walker on node waits. For example, as shown in Fig. 1(b), one walker moves on the edge, and the other walker is at a node.

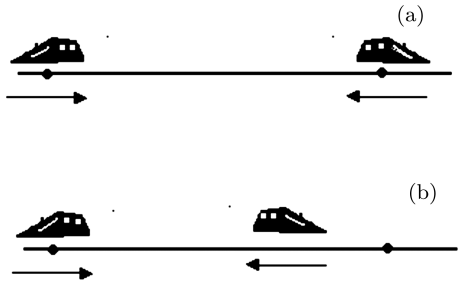


Fig. 1. Attrition of two walkers' path.

In the design of practical transportation plan, we usually consider many constraints. Some of these constraints are about the traffic environments, and some of them are about the traffic demand. However, most of these constraints can be considered in the proposed method. For example, from departure station to an arrival station, the travel time of train must be smaller than a give value. In the proposed method, we can change the train number and the departure time of trains such that the travel time of train is smaller than a give value.

5. Numerical computations

We carry out a computer simulation for the train schedule. The computation approach is to iterate the rules in the proposed method. The main program is that at each time step, for all trains, we use the current speeds and positions of trains to calculate the speeds and sites of these trains at the next time step. Assume the railway network is considered, which is depicted in Fig. 2. This assumed railway network includes 7 track sections and 8 stations. The lengths of all track sections are illustrated in Table 1. The nodes A, C, E, G, H are called terminal nodes where trains may enter and leave the network. The terminal nodes correspond to stations or train yards in real railway traffic. For the sake of convenience, in the following figures, the stations A, B, C, D, E, F, G and H are respectively represented by numbers 1, 2, 3, 4, 5, 6, 7 and 8. In order to compare computation results with field measurements, one iteration roughly corresponds to 1 s, and the length of a unit is about 5 m. This means, for example, that $v_{\max} = 10$ units/update corresponds to $v_{\max} = 180$ km/h.

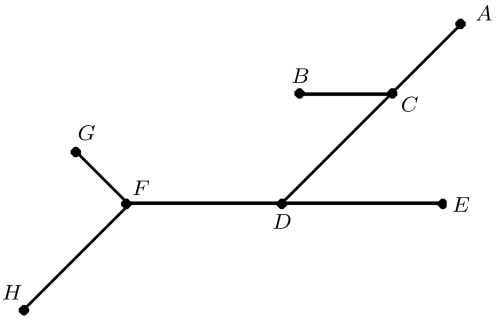


Fig. 2. Example of railway network.

Table 1. Length of track section.

section	length
AB	480
BC	460
BD	780
DE	780
DF	740
FG	380
FH	740

The traffic situations are as follows. 1) Railway lines consist of single-line track sections. 2) Two types of trains are considered, i.e. long distance trains and local distance trains. In general, long distance trains have a higher maximum speed and local distance trains have a lower maximum speed. We assume that all trains need stop at station for passengers to board and alight from trains. At the same station, the train randomly selects a platform. The train that first arrives at the station will depart first. In step 4 of the proposed model above, train deceleration is from $v_{\max}$ to zero.

In simulations, the maximum speed of long distance trains is set to be $v_{\max}^{\text{long}} = 15$, and the maximum speed of local distance trains is set to be $v_{\max}^{\text{local}} = 10$. The computing time T is taken as $T = 1440$ time steps. The parameter τ is set to be $\tau = 1$. The routes of trains and daily train numbers are given in Table 2. For example, the train number '3' shown in column 3 means that there are 3 trains traveling from A to H per day. In Table 3, the number of platform tracks and planned station dwell time are provided.

Table 2. Train route and train number per day.

service	route	train number
long distance	*A-B-D-F-H*	3
long distance	*G-F-D-E*	3
long distance	*C-B-D-E*	3
local distance	*D-F-G*	7
local distance	*E-D-F*	7
local distance	*B-D*	7
local distance	*F-H*	7

Table 3. Number of platform tracks and planned station dwell time.

station	platform track	occupation time
A	1	5
B	2	5
C	1	5
D	3	5
E	1	5
F	2	5
G	1	5
H	1	5

Using the method proposed in this paper, walker actually represents train. The process of walker moving represents the process of train traveling. Figure 3 displays the train schedule. The horizontal axis represents time, and the vertical axis denotes station. In Fig. 3, every diagonal line denotes that trains start from their departure stations and end at their arrival stations. Among these diagonal lines, some lines which have positive gradients are for a set of trains traveling in up direction, and others which have negative gradient are for the set of trains traveling in down direction. The diagonal lines which have higher gradients correspond to long distance trains, and other lines which have lower gradients correspond to local distance trains.

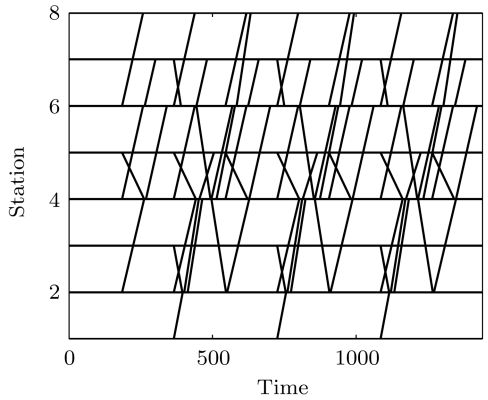


Fig. 3. Schedule of trains traveling on an assumed railway network.

Figure 4 shows an example of train schedule in which one line starts from the station D and end at the station G. In Fig. 4, the time of train arrival at the station F is 258, and the departure time from the station F is 264. In the time interval from the time 258 to the time 264, the train stops for passengers to board and alight. The results mean that the proposed model can basically capture the main character of the train scheduling on railway network.

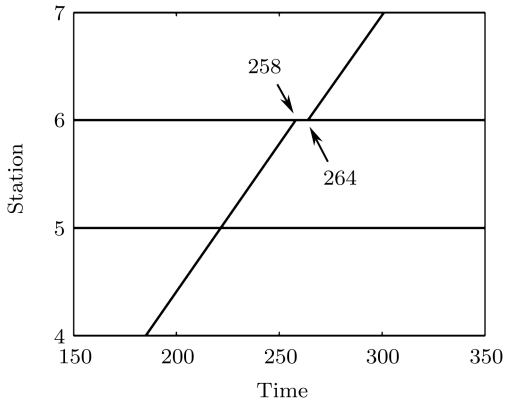


Fig. 4. Departure and arrival time of train.

The railway network considered in this paper has single-line track sections. Under this condition, trains are not allowed to travel in the opposite directions on the same track section at the same time. In order to test the proposed method, we investigate the space–time diagram of the traffic flow. Figure 5 shows the local space–time diagram for $v^{\text{long}} = 15$ and $v^{\text{local}} = 10$. In Fig. 5, the numbers represent the serial numbers of trains traveling on the track section CD. For example, one train whose serial number is 12, departs from the station C at the time 550, and arrives at the station D at the time 628.

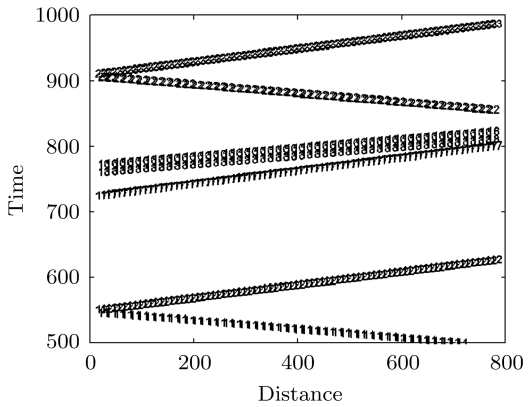


Fig. 5. Local space–time diagram of railway traffic flow.

From Fig. 3, we can see that some trains travel in up direction from the station C to the station D,

and some trains travel in down direction from the station D to the station C. On the same track section at the same time, no trains travel in the opposite directions. On the same track section, between two tracked trains, the safety stopping distance is maintained between them. This is the characteristic behaviour of the train movement under the moving-block condition.

In simulations, at each station, the number of trains varies with simulation time. In order to evaluate the proposed model, we record the number of trains at the station D. Figure 6 presents the result. From Fig. 6, we can clearly see that the maximum number of trains is 3. Since the number of platforms at the station D is 3, there is not train delay occurring at the station D. All trains can run through the station D within the planned station dwell time. We change the train number from the station C to the station E. Figure 7 shows the distribution of trains at the time $t = 1000$. Here we increase the train number from 3 to 23. In Fig. 7, it is clearly seen that the number of trains at station E is larger than that at other stations. The result indicates that there could be a delay formed at the station E. At station E, the practical occupation time of trains is larger than the planned station dwell time. In this case, we need further adjust the train schedule and ensure that there occurs no or smaller train delay. This means that the proposed model provides a tool for designing the train schedule on railway network.

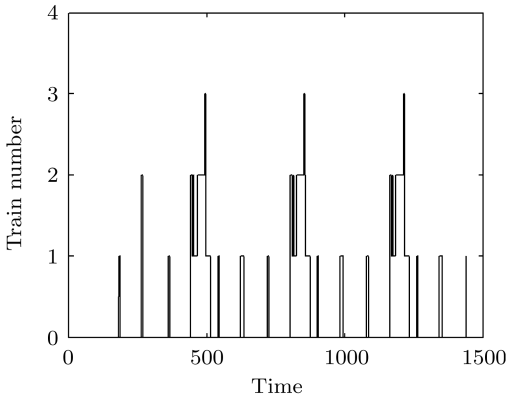


Fig. 6. The number of trains as a function of time.

The proposed method is based on the random walk model. It is one of the computer-based simulation methods. Like other computer-based simulation methods, the proposed method considers some dynamic behaviours of train movement. The difference between the proposed method and other methods is that the proposed method adopts some advantages of random walk model. In addition, the proposed method uses some iteration rules which are suitable for executing by computer. Since the time is discrete, the numerical implementation of the proposed model is relatively simple. Moreover these iteration rules can be used to reproduce complex phenomena of train schedule. It can determine the departure times and arrival times of trains, and train delays as well.

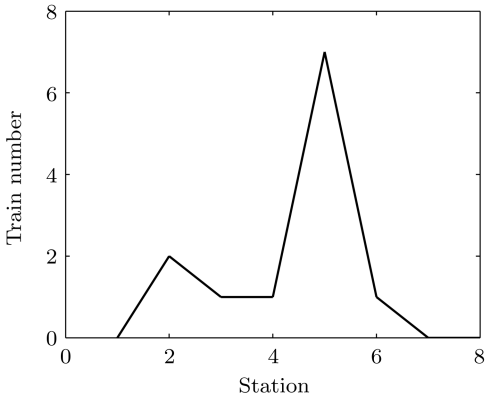


Fig. 7. Distribution of trains at the time $t = 1000$.

6. Conclusions

A simulation analysis method is proposed for scheduling trains on a railway network. Using the proposed method, it is possible to study and perform the experiment on the complex internal interactions in a railway system. When new changes arise in this system, we can predict what major problems will occur. For rail companies, they usually develop a kind of software, and use this software to produce the train schedule. In the process of the software design, part of the software is about how to determine the departure and arrival times of trains at each station. This part can be used by the proposed method.

It should be pointed out that the proposed method includes many approximations. Station is considered in a rather simple way, on the assumption that every train can use every platform track, and all trains have the same dwell time. The process of train movement is also simplified. Nevertheless, we think that the proposed method will open new perspectives for the design of the train schedule on railway network.

In our future work, we will extend the proposed model in order to consider some realistic traffic situ-

ations. We will use some dynamical rules or mathematical formula to restrict the train movement. The acceleration and the deceleration of train movement will be included. For example, in step 4 of the proposed model, we use some iteration rules in the cellular automata model (see Ref. [5]), or use the car-following model (see Ref. [6]). As the train moves into the station, it needs to consider a complex train route. Moreover, we will further discuss both of the fixed block and moving-block signaling control systems.

References

[1] Assad A A 1980 *Transport Res.* B **14** 101

[2] Bussieck M R, Winter and Zimmermann U T 1997 *Programming* **79** 415

[3] Ghoseiri K, Szidarovszky F and Asgharpour M J 2004 *Transport Res.* B **38** 927

[4] Kleinberg J M 2000 *Nature* **406** 845

[5] Li K P and Gao Z Y 2007 *Chin. Phys.* **16** 359

[6] Li K P and Guan L J 2009 *Chin. Phys.* B **18** 2200

[7] Spitzer F 1976 *Principles of Random Walk* (New York: Springer-Verlag)

[8] Jespersen S, Sokolov I M and Blumen A 2000 *Phys. Rev.* E **62** 4405

[9] Marsh L M and Jones R E 1988 *J. Theor. Biol.* **133** 113

[10] Hughes B D 1995 *Random Walks: Random Walks and Random Environments* (Oxford: Clarendon)

[11] Levin S A *Random Walk Models of Movement and Their Implication.* In: 1986 Hallam T and Levin S A (editors) *Mathematical Ecology: An Introduction* (Berlin: Springer-Verlag) pp.143–154

[12] Li B L, Loehle C and Malon D 1996 *Ecol. Model* **85** 285

[13] Noh J D and Rieger H 2003 *Phys. Rev. Lett.* **92** 088102

[14] Yamamura K, Moriya S and Tanaka D K 2003 *Ecol. Model* **161** 151