A new approach to detecting weak signal in strong noise based on chaos system control*

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In this paper, a chaos system and proportional differential control are both used to detect the frequency of an unknown signal. In traditional methods the useful signal is obtained through the Duffing equation or other chaotic oscillators. But these methods are too complex because of using a lot of chaos oscillators. In this paper a new method is presented that uses the Rössler equation and proportional differential control to detect a weak signal frequency. Substituting the detected signal frequency into the Rössler equation leads the Rössler phase state to be considerably changed. The chaos state can be controlled through the proportional differential method. Through its phase diagram and spectrum analysis, the unknown frequency is obtained. The simulation results verify that the presented method is feasible and that the detection accuracy is higher than those of other methods.

Keywords: chaos system, proportional differential control, spectrum analysis, signal detection
PACC: 0547, 0545

1. Introduction

In recent years, chaos and its applications for complex dynamics have become hot topics of research. Much work has focused on the generation of chaos. Chaos is widely used in a lot of fields because it is sensitively dependent on its initial value and disturbance. Recently we have paid more attention to the application of chaotic theory to signal detection. The previous researches show that most of the detection methods are to detect the amplitude after knowing the frequency of the detected signal and the frequency is not detected directly. Of course there are still some methods to detect the frequency of the unknown signal but they are very complex in manipulation. Meanwhile more chaotic oscillators are needed to detect the frequency of the unknown signal, thereby leading the detected method to be too complex.[1−4]

In the present paper we present a new approach to detecting the frequency of a sine signal in a strong noise environment based on Rössler chaos control. The biggest advantage of the method is that it is simpler than the conventional method. First, we control a chaos system from a chaotic state into a periodic state. Certainly in the chaotic state of the system does exist an input unknown signal and the system has an undetected signal. Now if we directly use spectrum analysis to detect the frequency then it cannot be useful. So the proportional differential control method is used to control the system into a periodic state. Then we detect the frequency of the unknown signal by using spectrum analysis. This method is not only simple, but also highly accurate.

2. Proportional differential control strategy

Consider an autonomous or a non-autonomous system of the third order, which is expressed as

$$\dot{x} = G(x, \mu, t), \tag{1}$$

where $x \in R^n, G = [g_1, g_2, g_3, \ldots, g_n]$ is a slider vector field, μ is the system parameter and when μ is equal to a certain value the system is in the chaotic state.

Now we apply the proportional differential control to the above x_i. Then we obtain

$$x_i' = k_1 x_i + k_2 \dot{x}_i. \tag{2}$$

In the above equation k_1 and k_2 are the adjusted parameters in this method. Regulating the two values

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we can realize the different control targets. Feedback variable x_i' are as follows:

$$\begin{aligned} \dot{x}_j &= g_j(x_1 x_2 \ldots x_i' x_n \mu), \\ x_j &= g_j(x_1 x_2 \ldots x_i x_n \mu), \\ x_i' &= k_1 x_i + k_2 x_i', \end{aligned} \tag{3}$$

where $j = i, j = 1, 2, \ldots, n$.

We know that the subsystem $g_i(\ )$ is restricted in feedback control. If subsystem $g_i(\ )$ has no state variable x_i and freely evolutes without being controlled, then this control method controls only a small part of the system, thereby reducing the difficulty in control and cost. Meanwhile it does not lose the features of the original system and its structure is much simpler. The control result is very useful. It can control the chaos state of the nonlinear system efficiently if we control the small parameter value and the control speed is faster. This is the reason why we use it in the present study.[5]

3. Research on the chaos system

Since the first chaos attractor was discovered by Lorenz in 1963, the research on chaos has developed in a lot of fields, such as biological engineering, chemical engineering, information engineering and so on. Shortly the German scholar Rössler pointed out that there were two methods to extract the simplest and dissymmetric attractor structure by using the Lorenz attractor in 1976: one was to study the equation when $r \gg 28$, and the other was to remodel the attractor. The two methods had the same topology structure with a nonlinear term xz. This nonlinear term can create chaos behaviour. That is Rössler system.[6]

Rössler equation is nonlinear and simple in structure, and it is defined as

$$\begin{aligned} x' &= -(y + z), \\ y' &= x + ly, \\ z' &= m + (x - n)z. \end{aligned} \tag{4}$$

The Rössler system is simpler than the Lorenz system. The Rössler system is a third order autonomous system that has a nonlinear term xz. This term creates bifurcation and chaos dynamic behaviour. The shape of the Rössler attractor is shown in Fig. 1 when $l = m = 0.2$ and $n = 5$.

Actually, any equation of the Rössler system is sensitive to a weak periodic signal. The input weak periodic signal can make Rössler system behaviour changed greatly.[7,8]

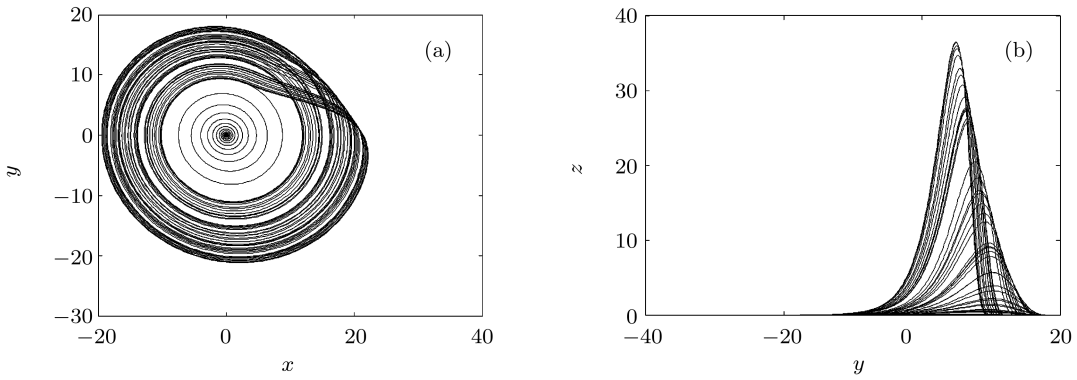


Fig. 1. Phase diagram of Rössler attractor. (a) y versus x. (b) z versus y.

The stability of the Rössler system is as follows when $l = 0.1,\ \ m = 0.2,\ n = 5$. Here

$$\begin{aligned} x' &= -(y + z) = 0, \\ y' &= x + ly = 0, \\ z' &= m + (x - n)z = 0. \end{aligned} \tag{5}$$

So there are two balance points in this system: $s_1 = (0, 0, 0)$, and $s_2 = (-lm + n, (lm - n)/l, -(lm - n)/l)$. That is, $s_1 = (0, 0, 0)$ and $s_2 = (4.98, -49.8, 49.8)$.

We discuss the stability of the two balance points as follows.

For $s_1 = (0, 0, 0)$, the Jacobian matrix $\boldsymbol{M}$ is

$$\boldsymbol{M} = \begin{bmatrix} n & -1 & -1 \\ 1 & l & 0 \\ m & 0 & n \end{bmatrix}. \tag{6}$$

Its characteristic equation is $a^3 + 4.7a^2 - 0.3a + 4.98 = 0$. Then the characteristic values are $a_1 = 0.13 + 0.99\mathrm{i}$, $a_2 = 0.13 - 0.99\mathrm{i}$, and $a_3 = -4.96$. So the point $s_1 = (0, 0, 0)$ is an unstable focus point. Then for the balance point $s_2 = (4.98, -49.8, 49.8)$, its Jacobian matrix $\boldsymbol{M}$ is

$$\boldsymbol{M} = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0.3 & 0 \\ 49.8 & 0 & -0.06 \end{bmatrix}. \tag{7}$$

The characteristic values are $a_1 = -0.02 + 4.20\mathrm{i}$, $a_2 = -0.02 - 4.20\mathrm{i}$ and $a_3 = 0.28$. So s_2 is an unstable saddle point.[9,10]

4. Frequency detection method based on chaos control

We present a method of detecting an unknown frequency by using the Rössler chaos system. We put the detected signal into any one of the equations of the Rössler system and then the Rössler system becomes a new system which has an unknown signal. If we detect the unknown frequency through spectrum analysis now, we cannot know the detected signal frequency because the amplitude and the frequency are all different. However when the system is controlled into a big periodic state the amplitude of the detected signal is largest. So we can use the proportional differential control theory to control the chaos system into a big periodic state of chaos system. We can detect the frequency easily through the spectrum diagram.

The presented detection method is just as follows. First, the parameter of Rössler chaos system is regulated into a chaotic initial state. Then the detected signal is put into the system. At that time the system is chaotic. Second, the above proportional differential control is used to control the system parameter. Through its control, the chaos state is easily converted into a periodic state. Finally, analyzing spectrum is performed to detect the unknown frequency.

In this paper we choose variable x to be controlled and assume $k_1 = 1$. Then the controlled equation is as follows:

$$\begin{aligned} x' &= -k_2 x - y + z, \\ y' &= x + 0.2y - 0.2k_2 y, \\ z' &= z(x - 5) + 0.2. \end{aligned} \tag{8}$$

For the fixed-point $s_2 = (4.98, -49.8, 49.8)$ we perform the linearization and the Jacobian matrix $\boldsymbol{M}$ is

$$\boldsymbol{M} = \begin{vmatrix} -k_2 & -(1 + 0.2k_2) & 1 \\ 1 & 0.2 & 0 \\ z & 0 & x - 5 \end{vmatrix}. \tag{9}$$

We insert $\boldsymbol{M}$ into the characteristic formula and obtain the k_2 value from Routh–Hurwitz criterion. When $0.018 < k_2 < 287640.5$, the system fixed-point is stable.

For the detected signal we can put it into any equation of the Rössler system and the dynamic behaviour can be changed. In this paper we put the unknown signal into the first equation of the Rössler system.

5. Numerical simulation

In order to verify the balance point of the presented system we carry out a simulation. From Fig. 2 we can see the control method is feasible and after a long time the system can reach a balance point.

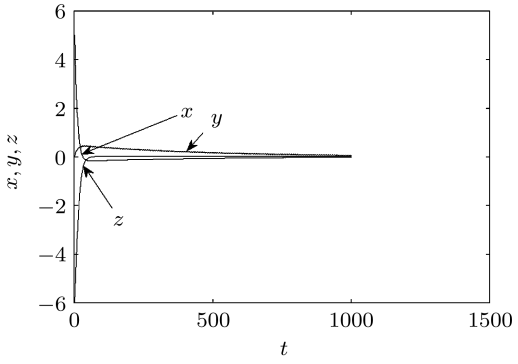


Fig. 2. Time-domain diagram of x, y, z.

Assuming that the unknown signal is $f(t) = 0.005 \times \cos(80t) + n(t)$, where the unknown angular frequency is 60 rad/s and the amplitude is 0.005 V, and the white noise is $n(t)$. The mean value is zero and the variance is 0.1. The spectrum analysis is shown in Fig. 3, with the input signal put into the second equation of Rössler system.

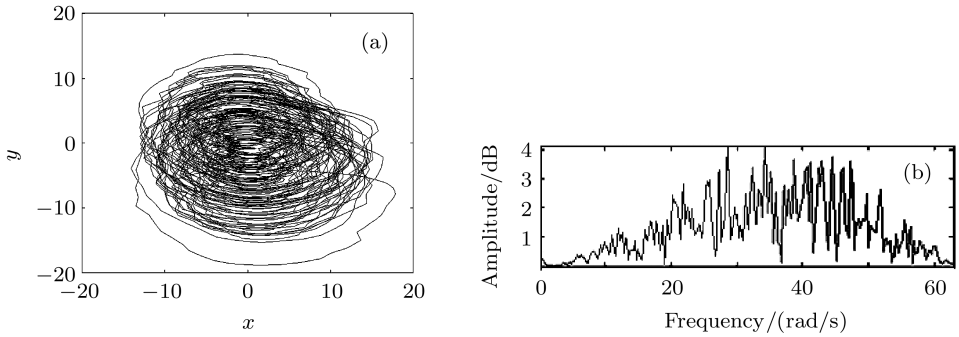


Fig. 3. Phase diagram (a) and spectrum analysis diagram (b) of the Rössler uncontrolled system which has an input signal.

From Fig. 3, we cannot identify the unknown signal. In Fig. 4, we use the proportional differential control to change Rössler chaotic system into a periodic state in order to detect the unknown frequency.

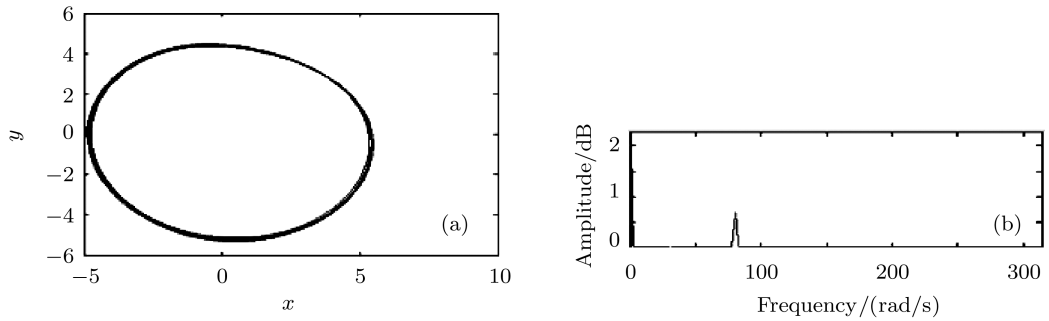


Fig. 4. Phase diagram when the unknown frequency is 60 rad/s (a) and spectrum diagram (b) of the controlled system.

We can directly see that the frequency of the periodic signal is 80 rad/s. When the unknown frequency is 300 rad/s, the spectrum of the system is just like that shown in Fig. 5.

Through a lot of simulation results we compare the presented method with the conventional chaos method under the same input signal-noise ratio. From Table 1 we can see that the method presented in this paper has more advantages than the other method and a higher detection accuracy.

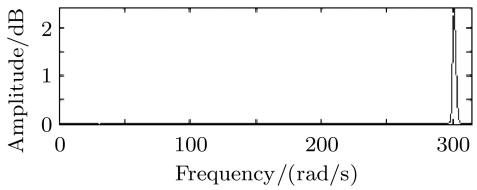


Fig. 5. Spectrum diagram of the controlled system when the unknown frequency is 300 rad/s.

Table 1. Comparison of result between two methods.

input signal–noise ratio/(dB)	frequency/(rad/s)	the presented method		the conventional chaotic method	
		the detected value/(rad/s)	error/(%)	the detected value/(rad/s)	error/(%)
–90	80	80.05	0.06	80.076	0.095
	300	300.06	0.02	300.12	0.04
–55	80	80.026	0.03	80.033	0.04
	300	300.04	0.01	300.058	0.02
–30	80	80	0	80.01	0.01
	300	300	0	300.03	0.01

6. Conclusion

For the case where a periodic signal with a strong noise is put into the chaotic system, we use both proportional differential control and the chaos system to detect the useful signal. We can regulate the control parameter from a chaotic state into a periodic state very well. We analyse the spectrum for the output signal of the mixed system. In the spectrum diagram the frequency corresponding to the largest amplitude is the unknown frequency. The noise seems to be filtered, and the periodic signal frequency is detected. The method has more advantages than other detection methods. The simulation results verify that the presented method is valuable. And the detection accuracy is higher. The presented method is feasible in theory.

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