Improvement on generalised synchronisation of chaotic systems*

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In this paper, the problem of generalised synchronisation of two different chaotic systems is investigated. Some less conservative conditions are derived using linear matrix inequality other than existing results. Furthermore, a simple adaptive control scheme is proposed to achieve the generalised synchronisation of chaotic systems. The proposed method is simple and easy to implement in practice and can be applied to secure communications. Numerical simulations are also given to demonstrate the effectiveness and feasibility of the theoretical analysis.

Keywords: chaotic systems, generalised synchronisation, secure communication
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1. Introduction

Synchronisation phenomena has been of central importance in recent research in nonlinear dynamics due to their many potential applications to the analysis of physiology, physics, optimisation of nonlinear system performance, modeling brain activity, and pattern recognition phenomena, secure communication and control.[1−8] Different types of synchronisation have been found in interacting chaotic systems. Complete,[9−12] generalised,[13−26] phase,[27−30] lag,[31,32] noise-induced,[33] time scale,[34] and anticipating[35,36] synchronisations of chaotic oscillators have been described theoretically and observed experimentally. And several effective methods have been proposed for synchronisation of chaotic systems, such as, impulsive control,[37,38] adaptive control,[39−42] intermittent control,[43,44] feedback control,[11,12,45] and drive-response control.[46]

Among all kinds of synchronisation schemes, generalised synchronisation, which is defined as the presence of some functional relation between the states of response and drive systems, i.e., $y(t) = F(x(t))$, has been first reported by Rulkov *et al.*[13] in unidirectionally coupled chaotic systems and well studied and understood in Refs. [13]–[21]. In comparison with other types of synchronisation, generalised synchronisation is a phenomenon more widespread in nature and in technical applications as well as a scheme more practical in secure communication. So the research on the generalised synchronisation of chaotic systems has attracted many authors' attention. Chen *et al.*[14] investigated complete synchronisation and generalised synchronisation in the coupled unified chaotic systems dissatisfying a globally Lipschitz condition with noise perturbation, and several sufficient criteria were rigorously established in light of the LaSalle-type invariance principle for stochastic differential equations. Jia *et al.*[18] investigated the generalised projective synchronisation of a class of chaotic (or hyperchaotic) systems, in which certain parameters can be separated from uncertain parameters. Using the adaptive technique, the globally generalised projective synchronisation and the parameter identification of two identical chaotic (hyperchaotic) systems are achieved by designing a novel nonlinear controller. Hramov and Koronovskii[19] explained the arising of generalised synchronisation and gave the physical reasons why the generalised synchronisation appear in unidirectionally coupled chaotic systems. Recently, Meng and Wang[16] studied the generalised synchronisation in nonidentical chaotic systems via nonlinear control. However, the results obtained in Ref. [16] is more conservative, and is hard to implement in practice since the controller designed may be complex. In this paper, we first derive some suffi-

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cient conditions to keep the generalised synchronisation of chaotic systems based on linear matrix inequality, which are less conservative than existing results in Ref. [16], i.e., for some controller K obtained via linear matrix inequality, the conditions in Ref. [16] cannot determine whether the generalised synchronisation of chaotic systems could be achieved. Furthermore, in practice, however, the controllers via nonlinear control designed as the above-mentioned may be complex and difficult to implement. Then, one has to design a simple controller which is easy to implement in practice. Inspired by the seminal work in Ref. [47], we propose a simple adaptive control scheme to guarantee generalised synchronisation of two different chaotic systems. Finally, we also consider the problem of secure communication based on the adaptive generalised synchronisation.

The rest of the paper is organised as follows. In Section 2, the problem to be studied is formulated and some preliminaries are presented. In Section 3, sufficient conditions for generalised synchronisation of chaotic systems are obtained. In Section 4, numerical simulations are presented to demonstrate the effectiveness and feasibility of the theoretical analysis, and the applications to secure communication are investigated in Section 5. Conclusion is given in Section 6.

2. Problem formulation

Consider the following drive chaotic system

$$\dot{x}(t) = f(t, x(t)), \tag{1}$$

where $x \in R^n$ is the state variable, $f : R^+ \times R^n \to R^n$ is a continuous function.

In order to obtain the generalised synchronisation, suppose that the response system of system (1) is given by

$$\dot{y}(t) = \boldsymbol{A}y(t) + \boldsymbol{B}g(t, y(t)) + u(t), \tag{2}$$

where $y \in R^l$ is the state variable, $\boldsymbol{A}$ and $\boldsymbol{B}$ are constant matrixes with appropriate dimensions. And $g : R^+ \times R^l \to R^l$ is a continuously differentiable function (see Ref. [48]), $u(t)$ is the control input.

Definition[16] Given map $\phi : R^n \to R^l$, systems (1) and (2) under the controller $u(t)$ satisfy the following equation

$$\lim_{t\to\infty} ||y(t) - \phi(x)|| = 0,$$

where $\phi(x)$ is a continuously differentiable function.

The map ϕ is continuously differentiable at point x on the attractor. Note that this defines differentiability that requires the evaluation of ϕ only at points on the drive attractor (see Ref. [48]). So, with this definition, our study about generalised synchronisation has more extended meaning than before.

Define the error function between the drive system (1) and response system (2): $e = y - \phi(x)$. Then one obtains an error dynamical system

$$\begin{aligned} \dot{e} &= \dot{y} - D\phi(x) \cdot f(x) \\ &= Ay + Bg(y) - D\phi(x) \cdot f(x) + u, \end{aligned} \tag{3}$$

where $D\phi(x)$ is the Jacobian matrix of the $\phi(x)$.

Before giving our results, the following assumption is needed.

Assumption[16] Let function $g(\cdot)$ be globally Lipschitz, i.e., there exists a positive constant L such that

$$||g(x) - g(y)|| \leq L||x - y||, \quad \forall x, y \in R^n. \tag{4}$$

3. Main results

In this section, some less conservative conditions are derived for guaranteeing the asymptotically stable of error system (3) under the controller $u(t)$.

Theorem 1 Under the above Assumption, the controller $u(t)$ is designed as follows:

$$\begin{aligned} u(t) = {} & D\phi(x) \cdot f(x) - Bg(\phi(x)) \\ & -A\phi(x) - \boldsymbol{K}(y - \phi(x)), \end{aligned} \tag{5}$$

where $\boldsymbol{K} = (k_{ij})_{m\times m}$ is a constant matrix to be determined. If there exist symmetric matrices $\boldsymbol{P} > 0$, $\boldsymbol{Q} > 0$, $\boldsymbol{M} > 0$ and $\boldsymbol{X} > 0$ with appropriate dimensions such that the following matrix inequality holds, then the error system (3) is asymptotically stable, that is, the generalised synchronisation between systems (1) and (2) can be achieved.

$$\Delta = \begin{bmatrix} \Xi & PB \\ B^{\mathrm{T}}P & Q - L^{-2}M \end{bmatrix} < 0 \tag{6}$$

with $\Xi = A^{\mathrm{T}}P + PA - X - X^{\mathrm{T}} + M$ and $K = P^{-1}X$.

Proof Substituting Eq. (5) into system (3), one obtains

$$\dot{e}(t) = (\boldsymbol{A} - \boldsymbol{K})e(t) + \boldsymbol{B}G(e(t)), \tag{7}$$

where $G(e(t)) = g(y(t)) - g(\phi(x(t)))$.

Construct the Lyapunov function

$$V(t) = e^{\mathrm{T}}(t)Pe(t) + \int_0^t G^{\mathrm{T}}(e(s))QG(e(s))\mathrm{d}s, \quad (8)$$

where $P = P^{\mathrm{T}} > 0$, $Q = Q^{\mathrm{T}} > 0$; it is easy to see that the Lyapunov function is positive definite. Then, the derivative of $V(t)$ along the solution of system (7) is

$$\begin{aligned}\dot{V}(t) &= e^{\mathrm{T}}(t)[(A^{\mathrm{T}} - K^{\mathrm{T}})P + P(A-K)]e(t) \\ &\quad +2e^{\mathrm{T}}(t)PBG(e(t)) + G^{\mathrm{T}}(e(t))QG(e(t)) \\ &= e^{\mathrm{T}}(t)[(A^{\mathrm{T}} - K^{\mathrm{T}})P + P(A-K) + M]e(t) \\ &\quad +2e^{\mathrm{T}}(t)PBG(e(t)) \\ &\quad +G^{\mathrm{T}}(e(t))(Q - L^{-2}M)G(e(t)) \\ &= \begin{bmatrix} e^{\mathrm{T}}(t) & G^{\mathrm{T}}(e(t)) \end{bmatrix} \cdot \Delta \cdot \begin{bmatrix} e(t) \\ G(e(t)) \end{bmatrix} < 0. \quad (9)\end{aligned}$$

From Lyapunov stability theory, one obtains that the system (3) with the controller $u(t)$ is asymptotically stable. This completes the proof.

Remark 1 In Ref. [16], Meng and Wang investigated generalised synchronisation problems in chaotic systems and proposed a new generalised synchronisation scheme via nonlinear control, which can widen the application range of the generalised synchronisation methods. Compared with the results in Ref. [16], in our work the condition for guaranteeing generalised synchronisation in Theorem 1 is based on linear matrix inequality, which is easy to obtain and extend the existing results; while in Ref. [16], the generalised synchronisation conditions obtained via nonlinear control are more conservative than our results, i.e., for some controller K obtained via linear matrix inequality, the conditions in Ref. [16] cannot ensure that the generalised synchronisation of chaotic systems should be achieved, which will be illustrated by numerical simulations.

Form Theorem 1, it is easy to see that although a sufficient condition obtained could guarantee the generalised synchronisation of two different chaotic systems, the added controller $u(t)$ may be too big to be physically practical due to the complexity of the controller $u(t) = D\phi(x)\cdot f(x) - Bg(\phi(x)) - A\phi(x) - K(y - \phi(x))$. Hence, for practical application, we have designed a simple controller, and proposed an adaptive-feedback scheme in the following to achieve the generalised synchronisation of two chaotic systems. Unlike the usual nonlinear control, the adaptive feedback gain is automatically adapted to synchronise the chaotic systems. This scheme is analytical, and easy to implement in practice.

Consider the following error system between systems (1) and (2):

$$\begin{aligned}\dot{e}(t) &= Ay(t) + g(y(t)) + u(t) - D\phi(x)\cdot f(x(t)) \\ &= Ae(t) + g(e(t)+\phi(x)) + A\phi(x) \\ &\quad -D\phi(x)\cdot f(x(t)) + u(t) \\ &= Ae(t) + g(e,x) + \psi(x(t)) + u(t), \quad (10)\end{aligned}$$

where $g(e,x) = g(e(t)+\phi(x))$, $\psi(x) = A\phi(x) - D\phi(x)\cdot f(x(t))$. We assume that $||g_i(e,x)|| \le h_i||e||$, $i = 1,2,\ldots,n$, with unknown Lipschitz constants h_i. And also assume the nonlinear function $|\psi(x)| \le \psi_M \le \infty$, where ψ_M is an unknown positive constant.

For function $g(e,x)$, it is often required that the Lipschitz constants h_i should be known in order to design the controller. However, it is very difficult to find the precise value of h_i in practice, so h_i are selected to be larger than the actual value for producing a higher control gain, and the results obtained should be restrictive. For the function $\psi(\cdot)$, it is noted that $\psi(x)$ is bounded due to the boundness of the chaotic attractor. Then, the controller $u(t)$ is designed as

$$u_i = k_i e_i, \quad \dot{k}_i = -\boldsymbol{r}_i e_i^2, \quad (11)$$

where $i = 1,2,\ldots,n$; $\boldsymbol{r} = (r_1,\ldots,r_n)$ is an arbitrary positive constant vector.

Choose the following Lyapunov function as a candidate

$$V(t) = e^{\mathrm{T}}(t)e(t) + \sum_{i=1}^{n}\frac{1}{r_i}(k_i + l)^2, \quad (12)$$

where l is a positive constant to be determined. Differentiating $V(t)$ with respect to time along the solution of Eq. (10), one obtains

$$\begin{aligned}\dot{V}(t) &= e^{\mathrm{T}}(t)[A^{\mathrm{T}} + A]e(t) + 2e^{\mathrm{T}}(t)g(e,x) \\ &\quad +2e^{\mathrm{T}}(t)\psi(x) + 2e^{\mathrm{T}}(t)Ke(t) \\ &\quad -2\sum_{i=1}^{n}(k_i + l)e_i^2 \\ &\le e^{\mathrm{T}}(t)[A^{\mathrm{T}} + A - 2lI]e(t) \\ &\quad +2||e^{\mathrm{T}}(t)||h||e(t)|| + 2||e(t)||\psi_M \\ &\le e^{\mathrm{T}}(t)[A^{\mathrm{T}} + A + 2hI - 2lI]e(t) \\ &\quad +\frac{\psi_M^2}{\varepsilon}||e(t)||^2 + \varepsilon \\ &\le e^{\mathrm{T}}(t)\left[A^{\mathrm{T}} + A + 2hI + \frac{\psi_M^2}{\varepsilon}I - 2lI\right]e(t) + \varepsilon \\ &\le -e^{\mathrm{T}}(t)e(t) + \varepsilon, \quad (13)\end{aligned}$$

where ε is a suitable positive constant, $h = \max_i\{h_i\}$, $(i = 1,2,\ldots,n)$, $l = [\lambda_{\max}(A^{\mathrm{T}}+A) + 2h + \psi_M^2/\varepsilon + 1]/2$. Therefore, the adaptive control error decays exponen-

tially and is ultimately bounded. Since the parameter ε can be freely adjusted, the error accuracy is guaranteed. The parameter ε should be made as small as possible in order to achieve generalised synchronisation via adaptive control.

4. Illustrative example

In this section, to illustrate the effectiveness of the proposed methods based on Theorem 1, two examples are considered in the following.

Example 1 Consider the following three-dimensional Rösler system[16]

$$\begin{aligned}\dot{x}_1 &= -x_2 - x_3,\\ \dot{x}_2 &= x_1 - 0.2x_2,\\ \dot{x}_3 &= 0.2 + x_3(x_1 - 5.7).\end{aligned} \tag{14}$$

The response system of system (14) is described as follows:

$$\begin{bmatrix}\dot{y}_1\\ \dot{y}_2\\ \dot{y}_3\end{bmatrix} = \begin{bmatrix}0 & -1 & -1\\ 1 & 0.2 & 0\\ 0 & 0 & -5.7\end{bmatrix}\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix} + \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\ 0\\ y_1y_3 + 0.2\end{bmatrix} + \begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix}. \tag{15}$$

Define the map ϕ[16] as follows:

$$\phi(x) = [-2x_1^2 + 1, x_2^2, 2x_3]^{\mathrm{T}},$$

then $D\phi(x) = \mathrm{diag}\{-4x_1, 2x_2, 2\}$.

From Theorem 1, we solve the matrix inequality (6) to obtain

$$P = \mathrm{diag}\{15.5073, 20.9804, 901827\},$$
$$X = \mathrm{diag}\{32.1396, 36.3357, -20.2020\},$$
$$\boldsymbol{Q} = \begin{bmatrix}18.3655 & -1.0946 & 3.1015\\ -1.0946 & 18.3655 & 0\\ 3.1015 & 0 & 18.3655\end{bmatrix},$$
$$\boldsymbol{M} = \begin{bmatrix}36.7310 & -2.1893 & 6.2029\\ -2.1893 & 36.7310 & 0\\ 6.2029 & 0 & 36.7310\end{bmatrix},$$

and $K = \mathrm{diag}\{2.0725, 1.7319, -2.2000\}$. We can obtain $\min(k_i) = -2.2000$, and it is easy to find that Eq. (6) in Ref. [16] is not satisfied, that is, according to Theorem 1 in Ref. [16], the results cannot ensure that the generalised synchronisation between systems (14) and (15) could be achieved. But using our method, it is easily verified that the error system between systems (14) and (15) is asymptotically stable. So, our result is less conservative than the results in Ref. [16]. Therefore, according to Theorem 1 in this paper, the generalised synchronisation between the systems (14) and (15) could be achieved in a physical sense. This is shown in Fig. 1, where each trajectory $(x_i(t), y_i(t))$ converges to the generalised synchronisation manifold $y = \phi(x)$: $y_1(t) = 1 - 2x_1(t)^2$, $y_2(t) = x_2(t)^2$, $y_3(t) = 2x_3(t)$. The synchronisation errors between systems (14) and (15) are depicted in Fig. 2, from which we can see that the synchronisation errors have been asymptotically stabilised to zero. Namely, systems (14) and (15) achieve the generalised synchronisation.

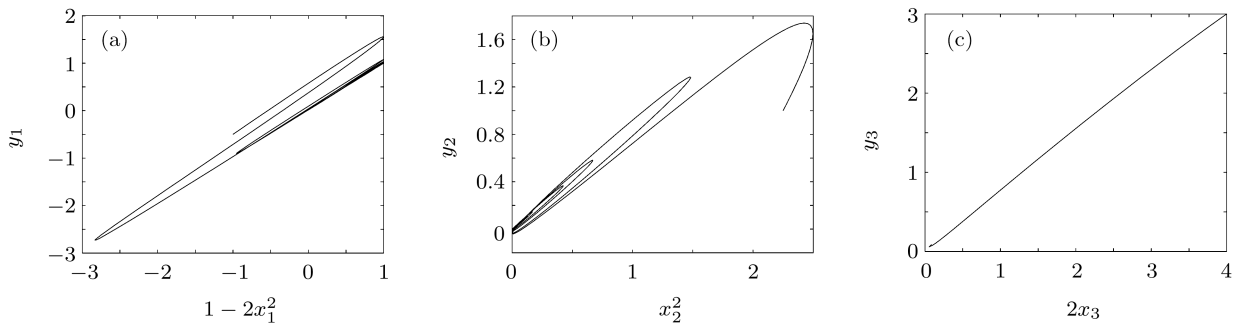


Fig. 1. Generalised synchronisation between systems (14) and (15) under control law (5). (a) $1 - 2x_1^2$ vs y_1; (b) x_2^2 vs y_2; (c) $2x_3$ vs y_3.

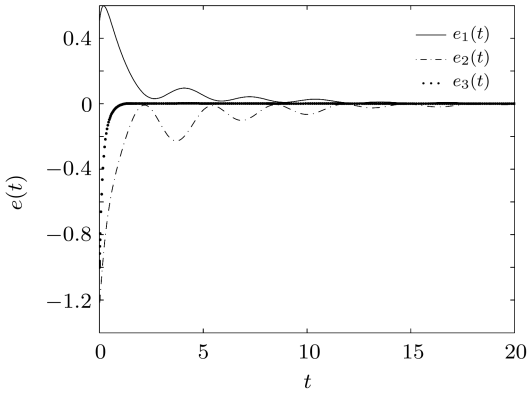


Fig. 2. Generalised synchronisation errors between systems (14) and (15), based on Theorem 1.

Example 2 Consider the following Hyper Rösler system[16]

$$\begin{aligned}\dot{x}_1 &= -x_2 - x_4,\\ \dot{x}_2 &= x_1 - 0.25x_2 + x_3,\\ \dot{x}_3 &= 0.05x_3 - 0.5x_4,\\ \dot{x}_4 &= 3 + x_1x_4. \end{aligned} \tag{16}$$

The response system of system (16) is described as follows:

$$\begin{bmatrix}\dot{y}_1\\ \dot{y}_2\\ \dot{y}_3\end{bmatrix} = \begin{bmatrix}-35 & 35 & 0\\ -7 & 28 & 0\\ 0 & 0 & -3\end{bmatrix}\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix} + \begin{bmatrix}0 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\ y_1y_3\\ y_1y_2\end{bmatrix} + \begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix}. \tag{17}$$

Define the map ϕ[16] as follows:

$$\phi(x) = [-1.5x_1 + 1, x_2 + 2x_4, 2x_3]^{\mathrm{T}},$$

then

$$D\phi(x) = \begin{bmatrix}-1.5 & 0 & 0 & 0\\ 0 & 1 & 0 & 2\\ 0 & 0 & 2 & 0\end{bmatrix}.$$

Based on Theorem 1, we solve the matrix inequality (6) to obtain

$$P = \mathrm{diag}\{0.0556, 0.1612, 0.9781\},$$
$$X = \mathrm{diag}\{2.8528, 8.1906, 2.2136\},$$
$$\boldsymbol{Q} = \begin{bmatrix}2.7641 & -0.1653 & 0\\ -0.1653 & 2.2912 & 0\\ 0 & 0 & 2.9389\end{bmatrix},$$
$$\boldsymbol{M} = \begin{bmatrix}5.5282 & -0.3308 & 0\\ -0.3308 & 4.5864 & 0\\ 0 & 0 & 5.7004\end{bmatrix},$$

and $K = \mathrm{diag}\{51.3522, 50.8234, 2.2631\}$. We can obtain $\min(k_i) = 2.2631$, and it is easy to find that Eq. (6) in Ref. [16] is not satisfied, that is, the results in Ref. [16] cannot ensure that the generalised synchronisation between systems (16) and (17) could be achieved. But using our method, it is easily verified that the error system between systems (16) and (17) is asymptotically stable. So, our result is less conservative than that in Ref. [16]. Then, according to Theorem 1, the generalised synchronisation between the systems (16) and (17) could be achieved in the physical sense. This is clearly shown in Fig. 3, where each trajectory $(x_i(t), y_i(t))$ immediately converges to the generalised synchronisation manifold $y = \phi(x)$: $y_1(t) = 1 - 1.5x_1(t)^2$, $y_2(t) = x_2(t) + 2x_4(t)$, $y_3(t) = 2x_3(t)$. The synchronisation errors between systems (16) and (17) are shown in Fig. 4, from which we can see the synchronisation errors have been asymptotically stabilised to zero. Namely, systems (16) and (17) have achieved the generalised synchronisation.

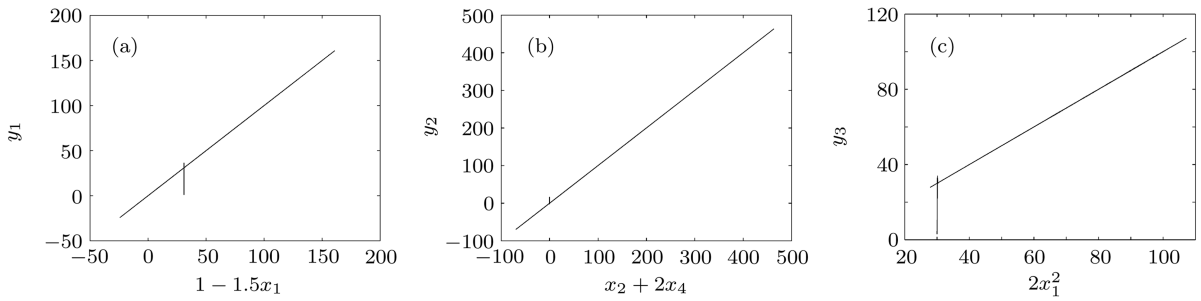


Fig. 3. Generalised synchronisation between systems (16) and (17) under control law (5). (a) $1 - 1.5x_1$ vs y_1; (b) $x_2 + 2x_4$ vs y_2; (c) $2x_3$ vs y_3.

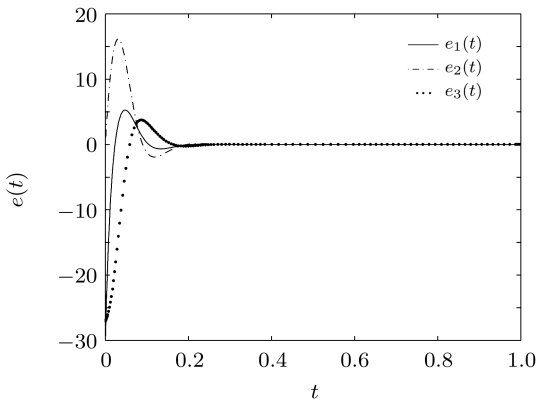


Fig. 4. Generalised synchronisation errors between systems (16) and (17), based on Theorem 1.

5. Applications to secure communication

An important application of chaotic synchronisation is that the technique can be applied to secure communication easily.[49−56] So, in what follows, we will consider the secure communication based on generalised synchronisation. One of the advantages of generalised synchronisation in communication is its more effective secrecy: for the intruder who wants to decipher the information transmitted by the drive system, apart from knowing the mechanism of the drive system and the way the driving signal operates, he also must know another 'code', i.e., the functional relation $\phi(\cdot)$ between the drive-response systems. Hence, it is noted that $\phi(\cdot)$ provides a hidden function that can be used as the secret key, which enhances the difficulty of decryption and the security in secure communication.

In what follows, we take the chaotic masking method into account, where one state of chaotic systems is added into the plaintext message $m(t)$ for masking the signal. Consider the following drive system as a transmitter

$$\begin{aligned}\dot{x}_1 &= -x_2 - x_4,\\ \dot{x}_2 &= x_1 - 0.25x_2 + x_3,\\ \dot{x}_3 &= 0.05x_3 - 0.5x_4 + m(t),\\ \dot{x}_4 &= 3 + x_1x_4,\end{aligned} \tag{18}$$

where $m(t)$ is the information signal. In the transmitter, the useful information $m(t)$ to be transmitted is blended to the state variable x_3 of drive system before transmission. The signal transmitted in the channel is $s(t) = m(t) + 2x_3(t)$. Since the transmitter system is chaotic and the function relation $\phi(\cdot)$ is freely selected, it is hard to extract the useful information from the signals in the channel for the intruder. In order to recover the original information, the generalised synchronisation between the transmitter system and receiver system needs to be achieved. Then we can recover the useful information by the hidden function. Thus the most important problem is to design a simple adaptive controller to make the states of the two chaotic systems synchronised.

Suppose function relation $\phi(x) = [-1.5x_1+1, x_2+2x_4, 2x_3]^{\mathrm{T}}$, then we can take $s(t) = m(t) + 2x_3(t)$ as the signal to transmit. The response system is described by

$$\begin{aligned}\dot{y}_1 &= -35(y_1 - y_2) + u_1,\\ \dot{y}_2 &= -7y_1 + 28y_2 - y_1y_3 + u_2,\\ \dot{y}_3 &= -3y_3 + y_1y_2 + 2x_3 - y_3 + m(t) + u_3.\end{aligned} \tag{19}$$

The controllers u_i, $(i = 1, 2, 3)$ are designed in Eq. (11). Thus the recovered signal in the receiver should be $m'(t) = s(t) - y_3(t)$.

First, the emitted signal is chosen as $m(t) = 5\sin(5t)$, and the dynamic behaviour of the drive system (18) is shown in Fig. 5, from which we can see that it is still chaotic. Therefore, one cannot obtain the useful information from the signals x_1, x_2 and x_3 transmitted in the channel. Figure 6 shows the generalised synchronisation errors $e(t)$ between systems (18) and (19) with adaptive controller, from which one has the trajectories of error $e(t)$ tending to zero, that is to say, the driving system (18) synchronises with the response system (19). The information signal $m(t)$ and the recovered signal $m'(t) = s(t) - y_3(t)$ are depicted in Fig. 7. It is easy to find that the information signal $m(t)$ is recovered exactly.

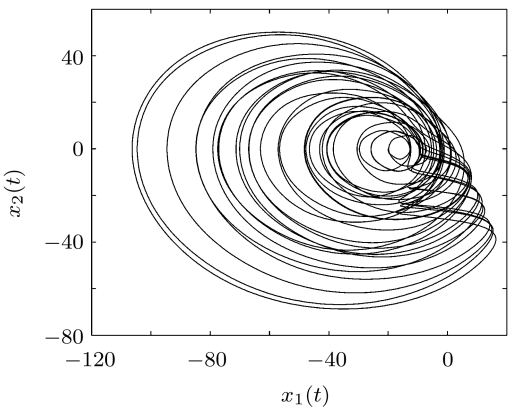


Fig. 5. The dynamic behaviour of chaotic system (18).

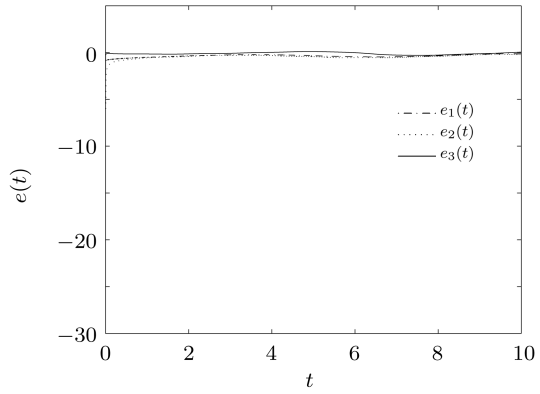


Fig. 6. Generalised synchronisation errors between systems (18) and (19) under adaptive control.

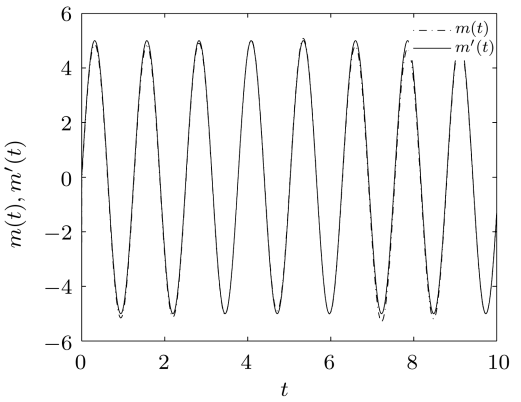


Fig. 7. The information signal $m(t)$ and recovered signal $m'(t)$.

Now we choose the binary signal as the information signal $m(t)$. Figure 8 depicts the trajectories of the drive system (18), which is still chaotic. The generalised synchronisation errors between systems (18) and (19) under an adaptive controller are displayed in Fig. 9. The information signal $m(t)$ and the recovered signal $m'(t) = s(t) - y_3(t)$ are illustrated in Fig. 10. It is certain that the information message $m(t)$ can be recovered by $m'(t)$ since the generalised synchronisation between systems (18) and (19) has been achieved.

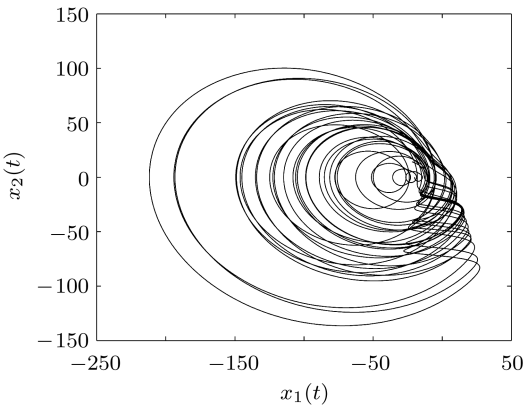


Fig. 8. The dynamic behaviour of chaotic system (18).

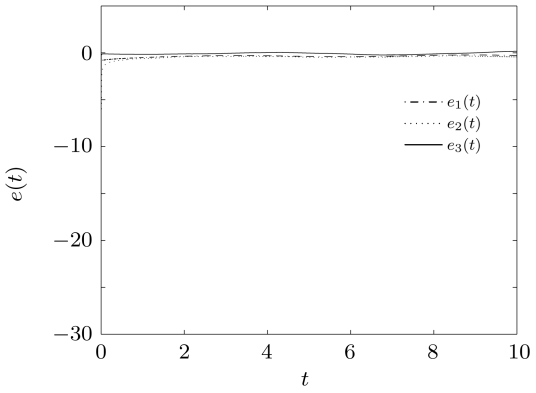


Fig. 9. Generalised synchronisation errors between systems 18) and (19) under adaptive control.

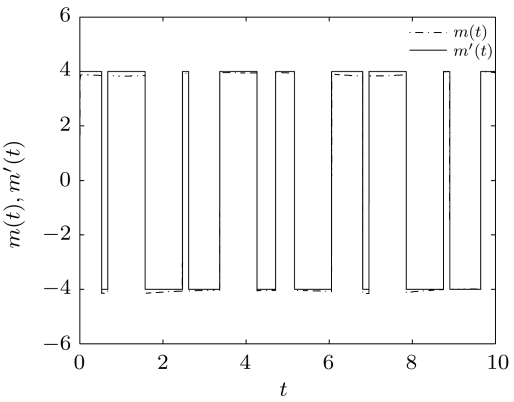


Fig. 10. The information signal $m(t)$ and recovered signal $m'(t)$.

Finally, we show that it is possible to choose the emitted signal $m(t)$ as a chaotic signal coming from a different chaotic system. We choose $m(t)$ as a chaotic signal of Lü system:

$$\begin{aligned} \dot{z}_1 &= 36(z_1 - z_2), \\ \dot{z}_2 &= 20z_2 - z_1 z_3, \\ \dot{z}_3 &= -3z_3 + z_1 z_2, \end{aligned} \tag{20}$$

i.e., $m(t) = 0.1z_2(t)$. The drive system (18) is still chaotic illustrated in Fig. 11. So the useful information from the signals x_1, x_2 and x_3 transmitted in the channel cannot be detected. Figure 12 shows the generalised synchronisation errors between systems (18) and (19), from which one has the trajectories of error $e(t)$ tending to zero, that is to say, the generalised synchronisation between driving system (18) and response system (19) is achieved. The information signal $m(t)$ and the recovered signal $m'(t)$ are illustrated in Fig. 13. One can see that the signal $m'(t)$ can succeed in recovering the former emitted signal $m(t)$.

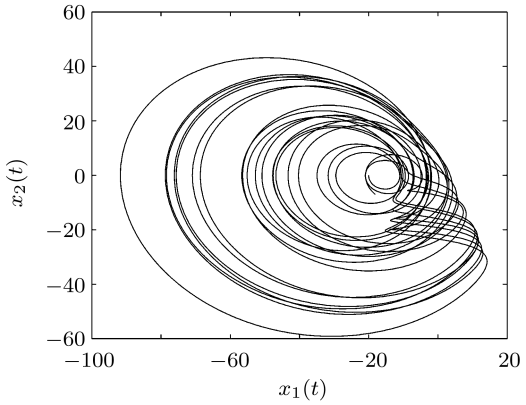


Fig. 11. The dynamic behaviour of chaotic system (18).

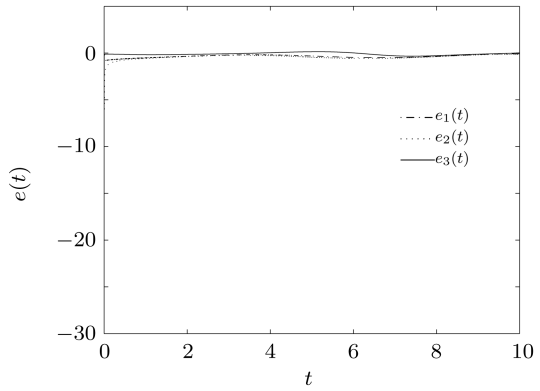


Fig. 12. Generalised synchronisation errors between systems (18) and (19) under adaptive control.

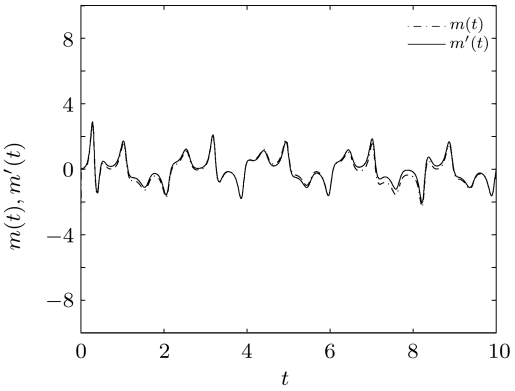


Fig. 13. The information signal $m(t)$ and recovered signal $m'(t)$.

6. Conclusion

We have studied the generalised synchronisation of chaotic systems. Using tools based on Lyapunov stability and linear matrix inequality, we obtain some less conservative conditions for guaranteeing synchronisation than those of the previous works. Furthermore, we present a simple adaptive control scheme for the generalised synchronisation of chaotic systems, which is easy to implement in practice. The proposed new approach could be applied to secure communication. Finally, our findings are illustrated through numerical simulations.

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