Asymptotical p-moment stability of stochastic impulsive differential system and its application to chaos synchronization*

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In this paper, the asymptotical p-moment stability of stochastic impulsive differential equations is studied and a comparison theory to ensure the asymptotical p-moment stability of the trivial solution is established, which is important for studying the impulsive control and synchronization in stochastic systems. As an application of this theory, we study the problem of chaos synchronization in the Chen system excited by parameter white-noise excitation, by using the impulsive method. Numerical simulations verify the feasibility of this method.

Keywords: p-moment stability, impulsive, stochastic differential equations, chaos synchronization

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1. Introduction

The stability of systems with impulsive effects has been of ongoing interest[1−8] over the past decades due to the fact that impulses may have significant effects on system dynamics. Baniov *et al.*[1] presented some basic methods of studying impulsive differential equations. Yang *et al.*[2−5] applied the stability theory of impulsive differential equations first to designing impulsive control and synchronization methods, and they used the comparison method of impulsive differential equations to judge whether the system under consideration was stable, thereby determining whether the system under consideration could be controlled by using the impulsive method. But in the studies such as Refs. [1]–[8], most of them were devoted to the systems without stochastic factor. To the best of our knowledge, there exists the stochastic factor almost everywhere during the development of our world, so it is necessary to study the impulsive system with stochastic noise excitation. Yang *et al.*[9] studied the p-moment stability of stochastic impulsive differential equations with delays. What is a pity is that the condition for the theory was not easy to validate for its application to a real nonlinear chaotic system. Wu *et al.*[10,11] investigated the p-moment stability of stochastic differential equations with jumps, and a theory of this problem was constructed. Li *et al.*[12] tried to give a comparison theory in stochastic impulsive differential equations and apply it to chaos control and synchronization. In the present paper, we aim at gaining an insight into the asymptotical p-moment stability of trivial solutions in stochastic impulsive differential equations and attempting to study its application to chaos synchronization. We use the theory constructed in our paper to judge whether the error system of the stochastic Chen system is asymptotical p-moment stable, from which we can judge whether this system can be synchronized with the impulsive method in the sense of p-moment.

2. Basic definitions and lemma

To make the present work self-contained, we address some basic definitions and results in this section. These results can be found in Refs.[1], [2], and [13]. In the present paper, let R^n be the n-dimensional Euclidean space, $\|X\|$ be the norm of element $X \in \boldsymbol{R}^n$, $\boldsymbol{R}^{n\times m}$ be $n \times m$ matrix. $R_+ = \{x : x \in R, x \geq 0\}$, $R_{t_0} = \{t : t \in R_+, t \geq t_0\}$, $S_\rho = \{X : |X| \leq \rho, X \in R^n\}$. Let $C^{1,2}(R_{t_0} \times R^n, R_+)$ denote the family of all nonnegative functions from $R_{t_0} \times R^n$ to R_+ which are once continuously differentiable with respect to the first variable and twice continuously differentiable with respect to the second variable. Recall the following stochastic impulsive differential system:

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$$\begin{cases} \mathrm{d}X(t) = f(t, x(t)))\mathrm{d}t + g(t, X(t))\mathrm{d}w(t), & t \neq \tau_i, \\ \Delta X|_{t=\tau_i} = U(i, X) = X(\tau_i^+) - X(\tau_i^-), & t = \tau_i, \\ X(t_0^+) = X_0, & i = 0, 1, 2, \ldots, \end{cases} \tag{1}$$

where $f(\cdot,\cdot) \in C[R_{t_0} \times R^n, R^n]$, $g(\cdot,\cdot) \in C[R_{t_0} \times R^n, R^{n\times m}]$, $w(t)$ is a normal m-dimensional Wiener process. Denote $\Gamma = \{\tau_i : i = 1, 2, \ldots, t_0 = \tau_0 < \tau_1 < \tau_2 < \ldots < \tau_i < \tau_{i+1} < \ldots\}$, and assume that $\lim\limits_{i\to\infty} \tau_i = +\infty$, where τ_i means the moment when impulsive happens. Assume that $f(t, 0) = 0$, $g(t, 0) = 0$, and $U(t, 0) = 0$, which guarantee the existence of trivial solution in system (1). In the present paper, we always assume that there is only one stochastic process $X(t) = X(t, t_0, X_0)$ satisfying system (1), of which the left side is continuous, i.e. $X(\tau_i^-) = \lim\limits_{t\to\tau_i-0} X(t) = X(\tau_i)$, and the right side is limitable, i.e. $X(\tau_i^+) = \lim\limits_{t\to\tau_i+0} X(t)$.

Definition 1

Let $p > 0$, the trivial solution of system (1) is said to be

(i) p-moment stable(stable in the pth mean) if for any $\varepsilon > 0$, there exists $\delta > 0$ such that $E(\|X(t)\|^p) < \varepsilon$, when $t > t_0$ and $\|X_0\|^p < \delta$;

(ii) asymptotical p-moment stable (asymptotical stable in the pth mean) if the trivial solution of system (1) is p-moment stable and $\lim\limits_{t\to\infty} E(\|X(t)\|^p) = 0$ holds for any initial condition X_0 in the neighbourhood of $X = 0$.

Definition 2

For each $V(t, X) \in C^{1,2}(R_{t_0} \times R^n, R_+)$, define an operator L from $R_{t_0} \times R^n$ to R as

$$\begin{aligned} \mathrm{L}V(t, X) = {} & V_t(t, X) + V_X(t, X)f(t, X) \\ & + \frac{1}{2}\mathrm{trace}(g^T(t, X)V_{XX}g(t, X)) \end{aligned}$$

where

$$V_t(t, X) = \frac{\partial V(t, X)}{\partial t},\ V_{XX}(t, X) = \left(\frac{\partial^2 V(t, X)}{\partial x_i \partial x_j}\right)_{n\times n},$$

and

$$V_X(t, X) = \left(\frac{\partial V(t, X)}{\partial x_1}, \frac{\partial V(t, X)}{\partial x_2}, \ldots, \frac{\partial V(t, X)}{\partial x_n}\right).$$

Definition 3

Function $V : R_{t_0} \times R^n \to R_+$ is said to belong to class $\mathcal{V}_0$ if

(i) V is continuous in sets $(\tau_{k-1}, \tau_k] \times R^n$ for all $t \geq 0$ and $k, V(t, 0) = 0$, and

$$\lim_{(t,y)\to(\tau_k^+,x)} V(t, y) = V(\tau_k^+, x),$$

(ii) V is locally Lipschitzian in $x \in R^n$.

Definition 4

Continuous function $\phi : [0, r] \to R_+$ is said to belong to class $\mathcal{K}$, if $\phi(0) = 0$ and ϕ is strictly increasing on $[0, r]$.

Definition 5

The following stochastic differential equations is referred to as an adjoint model system of system (1)

$$\begin{cases} \mathrm{d}Y(t) = f(t, Y)\mathrm{d}t + g(t, Y)\mathrm{d}w(t), \\ Y(t_0) = Y_0, \end{cases} \quad t > t_0, \tag{2}$$

where f and g are the same as those in system (1).

Definition 6

Let $V(t, X) \in \mathcal{V}_0$ and assume that

$$\begin{aligned} & \mathrm{L}V(t, X) \leq H(t, V(t, X)), \quad t \neq \tau_k, \\ & V(t, X + U(k, X)) \leq \psi_k(V(t, X)), \quad t = \tau_k, \end{aligned}$$

where $H : R_{t_0} \times R^n \to R_+$ is continuous in $(\tau_{k-1}, \tau_k] \times R^n$. And for each $X \in R^n$, $k = 1, 2, \ldots$, $\lim\limits_{(t,y)\to(\tau_k^+,x)} V(t, y) = V(\tau_k^+, x)$ exists. $\psi_k : R_{t_0} \times R^n \to R_+$ is nondecreasing and concave. Then the following system is called the comparison system of (1)

$$\begin{cases} \dot{\omega} = H(t, \omega), & t \neq \tau_k, \\ \omega(\tau_k^+) = \psi_k(\omega(\tau_k)), & k = 1, 2, \ldots, \\ \omega(\tau_0^+) = \omega_0 \geq 0. \end{cases} \tag{3}$$

Lemma 1[13] If there exists $V(t, Y) \in C^{1,2}(R_{t_0} \times R^n, R_+)$ for adjoint system (2) such that

$$(H_1)\mathrm{L}V(t, Y) \leq H(t, V(t, Y)),$$

$(H_2)Y_{\max}(t) = Y_{\max}(t, t_0, Y_0)$ is the maximal solution of Eq. (2)

$$(H_3)V_0 = V(t_0, Y_0) \leq Y_0.$$

Then $E(V(t, Y)) \leq Y_{\max}(t)$ holds.

3. Asymptotical p-moment stability of stochastic impulsive differential system

We consider the asymptotical p-moment stability of the stochastic impulsive differential system in the following. In Ref. [12], Li *et al.* gave a theory about the asymptotical p-moment stability of the stochastic impulsive differential equation, but the solution in their work was assumed to be continuous on the right side and limitable on the left side, which is contrary to the traditional assumption about the solution of impulsive differential equations. In the present paper, we give a theory similarly under the traditional assumption about the solution of impulsive differential equations.

Theory 1

Let $\omega_{\max}(t)$ be the maximal solution of Eq. (3) on $[t_0,\infty)$, for any solution $X(t)=X(t,t_0,X_0)$ of Eq. (1) with $t\in[t_0,\infty)$, $V(t_0,X_0)\leq\omega_0$ implies $E(V(t,X(t)))\leq\omega_{\max}(t)$, $t\geq t_0$.

Proof

It is easy to find that $E(V(t,X(t)))\leq\omega_{\max}(t)$ holds for $t\in[t_0,\tau_1]$ from Lemma 1. Since $E(V(\tau_1,X(\tau_1)))\leq\omega_{\max}(\tau_1)$ and $\psi(\cdot)$ is nondecreasing and concave, from Jensen's inequality, we have

$$E(V(\tau_1^+,X(\tau_1^+)))\leq E(\psi_1(V(\tau_1,X(\tau_1))))$$
$$\leq\psi_1(E(V(\tau_1,X(\tau_1))))\leq\psi_1(\omega_{\max}(\tau_1))\leq\omega_{\max}(t)$$

for $t\in(\tau_1,\tau_2]$, $E(V(t,X(t)))\leq\omega_{\max}(t)$ holds from Lemma 1. Since $\psi(\cdot)$ is concave and nondecreasing, $E(V(\tau_2,X(\tau_2)))\leq\omega_{\max}(\tau_2)$, from Jensen's inequality, we have

$$E(V(\tau_2^+,X(\tau_2^+)))\leq E(\psi_2(V(\tau_2,X(\tau_2))))$$
$$\leq\psi_2(E(V(\tau_2,X(\tau_2))))\leq\psi_2(\omega_{\max}(\tau_2))\leq\omega_{\max}(t).$$

By repeating the same process for each $(\tau_{k-1},\tau_k]$ and time instants τ_k, $k=3,4,\ldots$, we can finish the proof.

Theory 2

Assume that there exists a function $V(t,X)\in C^{1,2}(R_{t_0}\times R^n,R_+)$ satisfying

(H_1) $\beta(\|X(t)\|^p)\leq V(t,X)\leq\alpha(\|X(t)\|^p)$, where $\alpha(\cdot)$, $\beta(\cdot)\in\mathcal{K}$ and $\beta(\cdot)$ is convex.

(H_2) $\mathrm{L}V(t,X)\leq H(t,V(t,X))$ holds for $t\neq\tau_k$.

(H_3) There exists $\rho_0>0$, such that $X\in S_{\rho_0}$ implies $X+U(k,X)\in S_{\rho_0}$. What is more, $V(t,X+U(k,X))\leq\psi_k(V(t,X))$ holds for $t=\tau_k$.

Then the stability properties of the trivial solution in comparison system (3) imply the corresponding stability properties of the trivial solution in system (1) in the pth mean.

Proof

Step (1), to start with, we prove that the stability of trivial solution in system (3) implies the p-moment stability of trivial solution in system (1). Suppose that the trivial solution of system (1) is stable, then for any $\varepsilon>0$, there will exist $\delta=\delta(t_0,\varepsilon)>0$, such that when $0\leq\omega_0<\delta$, $\omega(t,t_0,\omega_0)<\beta(\varepsilon)$ holds for $t>t_0$, where $\omega(t,t_0,\omega_0)$ is an arbitrary solution of system (3). Choose $\omega_0=\alpha(\|X_0\|^p)$, then we will have the following claim.

Claim 1

For any trivial solution, $X(t)=X(t,t_0,X_0)$, of system (1), $\|X_0\|^p<\delta$ implies $E(\|X(t)\|^p)<\varepsilon$ for $t\geq t_0$.

In the following, we shall use a contradiction method to prove the above Claim. If Claim 1 is not true, then there exist one k, $t_1>t_0$ and a solution $X_1(t)=X_1(t,t_0,X_0)$ of Eq. (1), which satisfies $E(\|X_1(t_1)\|^p)\geq\varepsilon$, where $\|X_0\|^p<\delta$, and $t_1\in(\tau_{k-1},\tau_k]$. Since $\omega_0=\alpha(\|X_0\|^p)\geq V_0$, it follows from theory 1 that

$$E(V(t_1))\leq\omega_{\max}(t_1)\leq\omega_{\max}(t).$$

From the condition in theory 2, Jensen's inequality, and recalling that for any solution of Eq. (3), $\omega(t,t_0,\omega_0)<\beta(\varepsilon)$, what is more, $\omega_{\max}(t,t_0,\omega_0)<\beta(\varepsilon)$, we have

$$\beta(\varepsilon)\leq\beta(E(\|X_1(t_1)\|^p))\leq E(\beta(\|X_1(t_1)\|^p))$$
$$\leq E(V(t_1,X_1(t_1)))\leq\omega_{\max}(t,t_0,\omega_0)<\beta(\varepsilon),$$

which leads to a contradiction. Hence, Claim 1 is true and the trivial solution of system (1) is stable in the sense of p-moment.

Step (2), we prove that the asymptotical stability of the trivial solution in system (3) implies the asymptotical p-moment stability of the trivial solution in system (1) now. Suppose that the trivial solution of system (3) is asymptotical stable from which we know that the trivial solution of system (3) is attractive and stable. Following the proof of step (1), we know that the trivial solution of system (1) is p-moment stable.

Then for any $\varepsilon_1>0$, there is $\delta_1>0$, such that

$$\|X_0\|<\delta_1\text{ implies }E(\|X(t)\|^p)<\varepsilon,\quad t>t_0. \tag{4}$$

We then need to prove the attractive property. The trivial solution of system (3) is attractive, then

for any $\varepsilon > 0$, there exist $\delta_2 > 0$ and $T > 0$ such that $0 \leq \omega_0 \leq \delta_2$, which implies

$$\omega(t) = \omega(t, t_0, \omega_0) < \beta(\varepsilon) \text{ for all } t \geq t_0 + T. \quad (5)$$

Choose $\delta_0 = \min(\delta_1, \delta_2)$ and let $\|X_0\| < \delta_0$, $0 \leq \omega_0 = \alpha(\|X_0\|^p) \leq \delta_0$, $t > t_0 + T$, inequalities (4) and (5) hold at the same time. From theory 1 and $V(t_0, X_0) \leq \alpha(\|X_0\|^p)$, we have

$$E(V(t, X)) \leq \omega_{\max}(t, t_0, \alpha(\|X_0\|^p)) = \omega_{\max}(t). \quad (6)$$

From inequalities (5) and (6), and Jensen's inequality, we have

$$\beta(E(\|X(t)\|^p)) \leq E(\beta(\|X(t)\|^p)) \leq E(V(t, X)) \leq \omega_{\max}(t) \leq \beta(\varepsilon).$$

From the above inequality and the fact that $\beta(\cdot)$ is nondecreasing, we have

$$E(\|X(t)\|^p) < \varepsilon \text{ for } t > t_0 + T,$$

by which the trivial solution of system (1) is proven to be attractive. The trivial solution of system (1) is asymptotical p-moment stable then.

Theory 3

Let $g(t, \omega) = \dot{\lambda}(t)\omega$, $\lambda \in C^1(R_+, R_+)$, $\dot{\lambda}(t) \geq 0$, and $\psi_k(\omega) = d_k\omega$, $d_k \geq 0$ for all k, then the origin of system (3) will be asymptotical stable if

$$\lambda(\tau_{k+1}) + \ln(\gamma d_k) \leq \lambda(\tau_k) \text{ holds for all } k, \quad \gamma > 1.$$

Proof

The proof of this theory can be found in Ref. [2].

4. Impulsive synchronization of Chen system excited by parameter white-noise excitation

As an application of theory 2, we study the impulsive synchronization of two stochastic Chen systems with weak couplings in this section.

For the Chen system with parameter white-noise excitation

$$\begin{cases} \dot{x} = -ax + ay + \beta x \dfrac{\mathrm{d}w_t}{\mathrm{d}t}, \\ \dot{y} = (c - a)x + cy - xz + \beta y \dfrac{\mathrm{d}w_t}{\mathrm{d}t}, \\ \dot{z} = xy - bz + \beta z \dfrac{\mathrm{d}w_t}{\mathrm{d}t}, \end{cases} \quad (7)$$

where a, b, c and β are positive parameters, and w_t is normal Wiener process. When these parameters are chosen as $a = 3.0$, $b = 0.75$, $c = 2.8$, and $\beta = 0.01$, Chen system is chaotic. The phase portrait and corresponding time history are provided in Fig. 1.

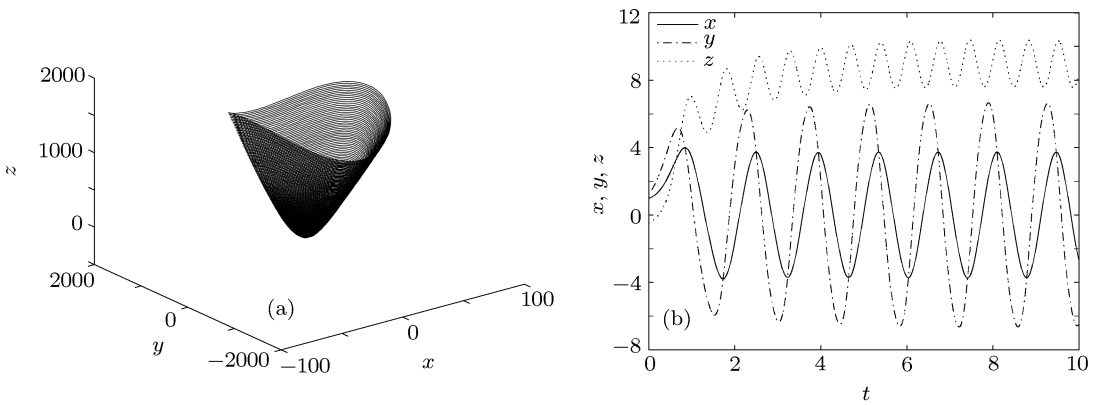


Fig. 1. (a) Phase portrait for system (7) and (b) its corresponding time history.

In a kind of impulsive synchronization configuration, the driving system is given by Eq. (7). Let $\bar{X} = (\bar{x}, \bar{y}, \bar{z})$ be state variables of driven system, then the driven system will be given by

$$\begin{cases} \dot{\bar{x}} = -a\bar{x} + a\bar{y} + \beta\bar{x} \dfrac{\mathrm{d}w_t}{\mathrm{d}t}, \\ \dot{\bar{y}} = (c - a)\bar{x} + c\bar{y} - \bar{x}\bar{z} + \beta\bar{y} \dfrac{\mathrm{d}w_t}{\mathrm{d}t}, \\ \dot{\bar{z}} = \bar{x}\bar{y} - b\bar{z} + \beta\bar{z} \dfrac{\mathrm{d}w_t}{\mathrm{d}t}. \end{cases} \quad (8)$$

At discrete instants, τ_i, $i = 1, 2, \ldots$, state variables of the driving system are transmitted to the driven system and the state variables of the driven system are subjected to a sudden changes at these instants. In this sense, the driven system is described by impulsive differential equation as

$$\begin{cases} \dot{\bar{x}} = -a\bar{x} + a\bar{y} + \beta\bar{x}\frac{\mathrm{d}w_t}{\mathrm{d}t}, \\ \dot{\bar{y}} = (c-a)\bar{x} + c\bar{y} - \bar{x}\bar{z} + \beta\bar{y}\dfrac{\mathrm{d}w_t}{\mathrm{d}t}, \quad t \neq \tau_i, \\ \dot{\bar{z}} = \bar{x}\bar{y} - b\bar{z} + \beta\bar{z}\dfrac{\mathrm{d}w_t}{\mathrm{d}t}, \\ \Delta\bar{x}|_{t=\tau_i} = -\boldsymbol{B}e, \quad i = 1, 2, \ldots, \end{cases} \tag{9}$$

where $\boldsymbol{B} = \begin{pmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{pmatrix}$. If the synchronization error is described as

$$\begin{aligned} \boldsymbol{e}(t)^{\mathrm{T}} &= (e_x(t), e_y(t), e_z(t)) \\ &= (x(t) - \bar{x}(t), y(t) - \bar{y}(t), z(t) - \bar{z}(t)), \end{aligned}$$

then the error system of the impulsive synchronization is given by

$$\begin{cases} \dot{e}_x = -ae_x + ae_y + \beta e_x\dfrac{\mathrm{d}w_t}{\mathrm{d}t}, \\ \dot{e}_y = (c-a)e_x + ce_y - e_xe_z + \beta e_y\dfrac{\mathrm{d}w_t}{\mathrm{d}t}, \quad t \neq \tau_i, \\ \dot{e}_z = e_xe_y - be_z + \beta e_z\dfrac{\mathrm{d}w_t}{\mathrm{d}t}, \\ \Delta e|_{t=\tau_i} = e(\tau_i^+) - e(\tau_i) = Be, \quad i = 1, 2, \ldots. \end{cases} \tag{10}$$

In the following, we find out whether system (8) satisfies the conditions of theory 2.

Let $V(t, e) = e_x^2 + e_y^2 + e_z^2$, it is easy to find that condition (H_1) of theory 2 holds.

$$\begin{aligned} \mathrm{L}(V(t,e)) &= -2ae_x^2 + 2ce_xe_y + 2ce_y^2 - 2be_z^2 \\ &\quad + \beta^2(e_x^2 + e_y^2 + e_z^2) \\ &\leq (\beta^2 + c - 2a)e_x^2 + (\beta^2 + 3c)e_y^2 \\ &\quad + (\beta^2 - 2b)e_z^2. \end{aligned}$$

Let $\dot{\lambda}_1(t) = \max\{|\beta^2 + c - 2a|, |\beta^2 + 3c|, |\beta^2 - 2b|\}$, and $M = \dot{\lambda}_1(t)\delta$, then we will have $\mathrm{L}(V(t,e)) \leq \dot{\lambda}_1(t)V(t,e)$, for all $t \in R_{t_0}\backslash\Gamma$. And for $i = 0, 1, 2, \ldots$, we have

$$\begin{aligned} V(\tau_i, e^2(\tau_i^+)) &= (1+k_1)^2e_x^2(\tau_i) + (1+k_2)^2e_y^2(\tau_i) \\ &\quad + (1+k_3)^2e_z^2(\tau_i) \\ &\leq (1+k)^2V(\tau_i, e(\tau_i)). \end{aligned}$$

Let $\lambda_2(t) = (1+k)^2 = \max\{(1+k_1)^2, (1+k_2)^2, (1+k_3)^2\}$, $-2 < k < 0$, then condition (H_3) of theory 2 will hold. So we have the comparison system

$$\begin{cases} \dot{\omega} = \dot{\lambda}_1(t)\omega, \quad t \neq \tau_i, \\ \omega(\tau_i^+) = (1+k)^2\omega(\tau_i), \quad i = 1, 2, \ldots, \\ \omega(t_0^+) = \omega_0 \geq 0, \end{cases} \tag{11}$$

where $\omega = (\omega_1, \omega_2, \omega_3)^{\mathrm{T}}$, so the stability properties of trivial solution of comparison system (11) imply the corresponding stability properties of the trivial solution of system (10) in the pth mean. Let the impulses be equidistant and separated by interval δ. From theory 3, if $\dot{\lambda}_1(t)\delta + \ln(\gamma(1+k)^2) \leq 0$ holds, i.e.

$$0 \leq \delta \leq -\frac{\ln\gamma + 2\ln(1+k)}{\dot{\lambda}_1(t)}$$

is satisfied, then system (11) is asymptotically stable, from which it follows that system (10) is asymptotically p-moment stable.

Let $a = 3.0$, $b = 0.75$, $c = 2.8$, $\beta = 0.01$, $\gamma = 1$, then $\dot{\lambda}_1(t) = 84.0001$. Figure 2 shows the stable region of Chen system. The whole region under the curve is a stable region.

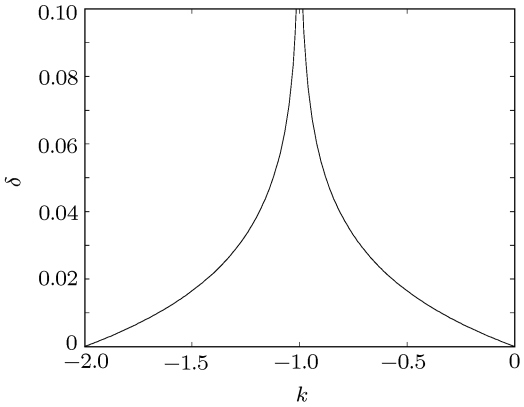


Fig. 2. Estimated boundaries of stable region in Chen system.

Numerical experimental results are shown in Fig.3 with $k_1 = -1.5$, $k_2 = k_3 = -1.2$ and $\delta = 0.02$, which are under the curve in Fig.2. The solid line shows $e_x(t)$, the dot line denotes $e_y(t)$, the dash-dot line represents $e_z(t)$. One can see that the system approaches the origin as time goes by.

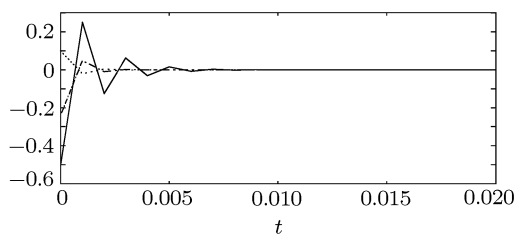


Fig. 3. Time evolution of system (10).

5. Conclusion

In this paper, the asymptotical p-moment stability of the stochastic impulsive differential system is studied, a comparison theory about asymptotical p-moment stability is given under the traditional assumption in the stochastic impulsive dynamical system, which is a nice tool in the study of stability problems. From the comparison theory, we can find out whether the stochastic impulsive differential system is stable just by studying the stability of a deterministic comparison system. As an application of this theory, we study the chaos synchronization problem of the stochastic Chen system by using the impulsive method for the first time. Finally, numerical simulation confirms the feasibility of the impulsive synchronization method in the stochastic differential system.

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