Transient chaos in smooth memristor oscillator*

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(Received 15 September 2009; revised manuscript received 29 September 2009)

This paper presents a new smooth memristor oscillator, which is derived from Chua's oscillator by replacing Chua's diode with a flux-controlled memristor and a negative conductance. Novel parameters and initial conditions are dependent upon dynamical behaviours such as transient chaos and stable chaos with an intermittence period and are found in the smooth memristor oscillator. By using dynamical analysis approaches including time series, phase portraits and bifurcation diagrams, the dynamical behaviours of the proposed memristor oscillator are effectively investigated in this paper.

Keywords: dynamical behaviour, initial condition, memristor oscillator, transient chaos
PACC: 0545

1. Introduction

Memristor, characterised by a relation of the type $f(\varphi,\ q) = 0$, is a missing circuit element studied by Chua in 1971[1,2] and realised by Stan Williams's group of HP Labs in 2008.[3,4] The flux-controlled memristor is a passive two-terminal electronic device described by a nonlinear constitutive relation between the current flowing through the device and the voltage across the terminal of the device, i.e. $i = W(\varphi)v$, where the nonlinear function $W(\varphi)$, called the memductance, is defined as $W(\varphi) = \mathrm{d}q(\varphi)/\mathrm{d}\varphi$. It represents the slope of a scalar function $q = q(\varphi)$ and is called the memristor constitutive relation.

The memsistor based applications have attracted much attention recently. Itoh and Chua derived several oscillators from Chua's oscillators by replacing Chua's diodes with memristors characterised by a monotone-increasing and piecewise-linear nonlinearity.[5] In the present paper, we assume that the flux-controlled memristor is characterised by a smooth continuous cubic monotone-increasing nonlinearity as follows:

$$q(\varphi) = a\varphi + b\varphi^3, \tag{1}$$

where $a,\ b > 0$. In this case, the memductance $W(\varphi)$ is given by

$$W(\varphi) = \mathrm{d}q(\varphi)/\mathrm{d}\varphi = a + 3b\varphi^2. \tag{2}$$

Thus, by replacing the Chua's diode in Chua's oscillator with an active two-terminal memristive circuit[2] consisting of a negative conductance and a flux-controlled memristor characterised by Eq. (1), a smooth memristor oscillator is designed.

Different from general dynamical systems,[6−11] the memristor oscillator exhibits some novel nonlinear phenomena. The phenomenon that a 2-scroll transient chaos[12−16] evolves into a periodic orbit is investigated in detail, and a transition from transient chaotic to stable chaotic and intermittence periodic dynamics is revealed effectively. It is interesting to find that these novel dynamical behaviours are dependent on the parameters and initial conditions of the smooth memristor oscillator.

2. Smooth memristor Chua's oscillator

Figure 1 shows a memristor Chua's oscillator with an active memristive circuit, which is directly extended from Chua's oscillator with smooth equation[17] by replacing the Chua's diode with a smooth flux-controlled memristor and a negative conductance. The smooth flux-controlled memristor shown in Fig. 1 is a passive two-terminal electronic device described by Eq. (1).

*Project supported by the National Natural Science Foundation of China (Grant No. 60971090), and the Natural Science Foundations of Jiangsu Province, China (Grant No. BK2009105).
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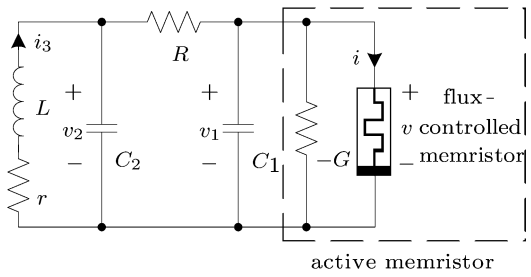


Fig. 1. Chua's oscillator with a flux-controlled memristor.

From Fig. 1, we can obtain a set of four first-order differential equations, which define the relation among the four circuit variables (v_1, v_2, i_3, φ):[5]

$$\begin{cases} \dfrac{\mathrm{d}v_1}{\mathrm{d}t} = \dfrac{1}{RC_1}\left[v_2 - v_1 + GRv_1 - RW(\varphi)v_1\right], \\ \dfrac{\mathrm{d}v_2}{\mathrm{d}t} = \dfrac{1}{RC_2}\left[v_1 - v_2 + Ri_3\right], \\ \dfrac{\mathrm{d}i_3}{\mathrm{d}t} = -\dfrac{1}{L}v_2 - \dfrac{r}{L}i_3, \\ \dfrac{\mathrm{d}\varphi}{\mathrm{d}t} = v_1, \end{cases} \tag{3}$$

where the φ–q characteristic curve of the flux-controlled memristor is given by Eq. (1) and $W(\varphi) = \mathrm{d}q(\varphi)/\mathrm{d}\varphi$.

By letting $x = v_1$, $y = v_2$, $z = i_3$, $w = \varphi$, $\alpha = 1/C_1$, $\beta = 1/L$, $\gamma = r/L$, $\xi = G$, $C_2 = 1$, and $R = 1$, and defining the nonlinear functions $q(w)$ and $W(w)$ as

$$\begin{cases} q(w) = aw + bw^3, \\ W(w) = \mathrm{d}q(w)/\mathrm{d}w = a + 3bw^2, \end{cases} \tag{4}$$

state equations of (3) can be written in dimensionless form with a time scale factor k as follows:

$$\begin{cases} \dot{x} = k\alpha(y - x + \xi x - W(w)x), \\ \dot{y} = k(x - y + z), \\ \dot{z} = -k(\beta y + \gamma z), \\ \dot{w} = kx, \end{cases} \tag{5}$$

where $k > 0$.

Let $\alpha = 9.8$, $\beta = 100/7$, $\gamma = 0$, $\xi = 9/7$, $a = 1/7$, $b = 2/7$, and $k = 20$. For initial conditions (0, 10^{-10}, 0, 0), system (5) is chaotic and displays a symmetrical 2-scroll chaotic attractor as shown in Figs. 2(a) and 2(b), the corresponding Lyapunov exponents are $\mathrm{LE}_1 = 6.0135$, $\mathrm{LE}_2 = 0.0087$, $\mathrm{LE}_3 = -0.0118$, and $\mathrm{LE}_4 = -61.7401$, and the Lyapunov dimension is $d_\mathrm{L} = 2.0974$.

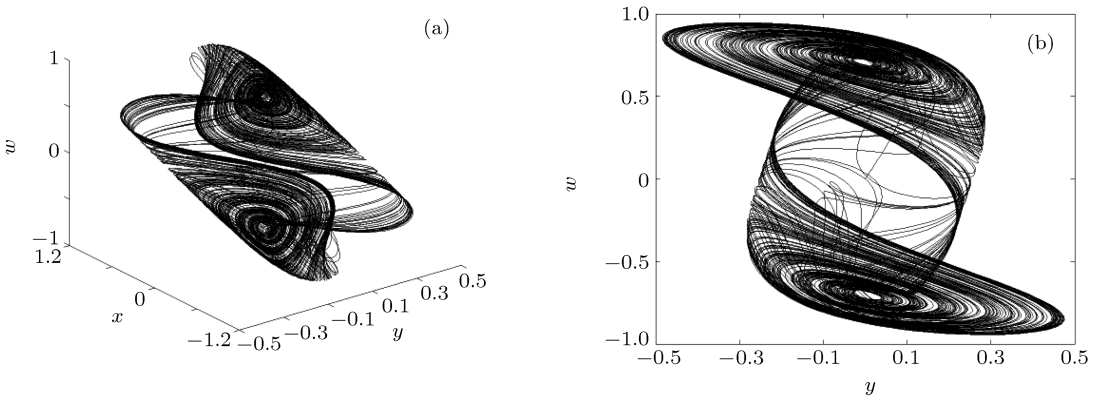


Fig. 2. Chaotic attractor of smooth memristor oscillator: (a) y–x–w; (b) y–w.

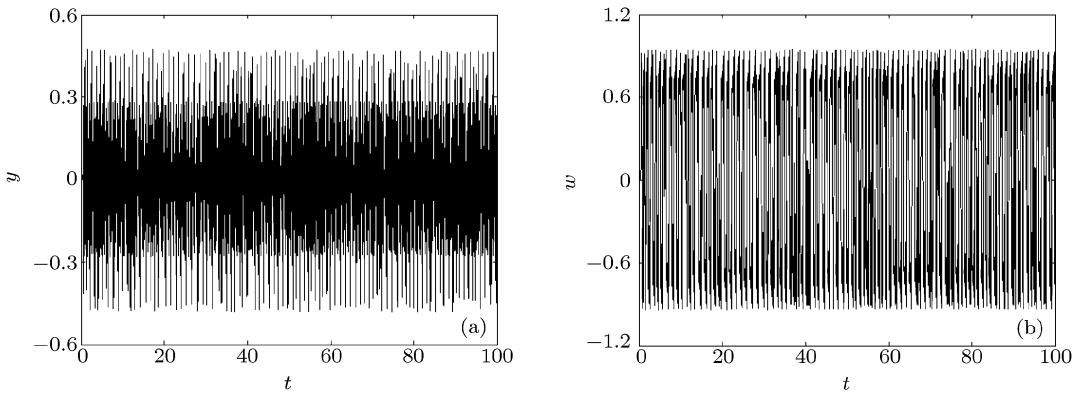


Fig. 3. Plots of state variables y (a) and w (b) as a function of time.

Figure 3 shows the plots of state variables y and w as a function of time, in which their trajectories are aperiodic with apparent random behaviour and consistent for transient and stable states.

The equilibrium state of system (5) is given by set $A = \{(x, y, z, w)|x = y = z = 0,\ w = c\}$, which corresponds to the w-axis. Here, c is a real constant. The Jacobian matrix $\boldsymbol{J}_A$ at this equilibrium set is given by

$$\boldsymbol{J}_{\rm A} = \left[\begin{array}{cccc} k\alpha(-1+\xi-W(w)) & k\alpha & 0 & -6k\alpha bxw \\ k & -k & k & 0 \\ 0 & -k\beta & -k\gamma & 0 \\ k & 0 & 0 & 0 \end{array}\right]\Bigg|_A . \tag{6}$$

When $\beta = 100/7$, $\gamma = 0$, $\xi = 9/7$, $a = 1/7$, $b = 2/7$, and $k = 20$ and keep parameters α and c as variables, the four eigenvalues λ_i ($i = 1, 2, 3, 4$) of the equilibrium set A are characterised by an unstable or stable saddle-focus except for the zero eigenvalue, which are listed in Table 1 for typical values of constant c, where $\alpha = 9.8$. Thus the dynamical behaviours of the smooth memristor oscillator are dependent on the initial conditions of state variables. Such characteristics well reflect the unique properties of the memristor circuit.

Table 1. Four values of eigenvalue λ_i ($i = 1, 2, 3, 4$) of the equilibrium set A ($\alpha = 9.8$).

$\|c\|$	λ_1	$\lambda_{2,3}$	λ_4
0	49.1406	$-20.5703 \pm$ j53.2243	0
0.4082	0	$-10 \pm$ j41.1617	0
0.45	–19.2752	$-3.3724 \pm$ j42.1106	0
0.481	–30.8655	$\pm$j44.8508	0
0.8	–107.3994	$3.9397 \pm$ j64.9262	0
1.0722	–185.1349	$\pm$j71.3931	0
1.2	–230.5755	–1.6723 $\pm$j72.7923	0

3. Transient chaos in memristor oscillator

When $\beta = 100/7$, $\gamma = 0$, $\xi = 9/7$, $a = 1/7$, $b = 2/7$, and $k = 20$, and initial conditions (0.1, 0, 0, 0) are kept constant while parameter α is varying, the bifurcation diagrams of the state variable w are shown in Fig. 4. Figures 4(a) and 4(b) show the transient bifurcation behaviours and stable bifurcation behaviours respectively, from which it can be observed that the transient chaotic behaviours mainly happen in the regions $9.56 \leq \alpha \leq 9.71$ and $10 \leq \alpha \leq 10.64$, and the stable chaotic and intermittence periodic behaviours mostly occur in the region $9.26 \leq \alpha \leq 9.5$.

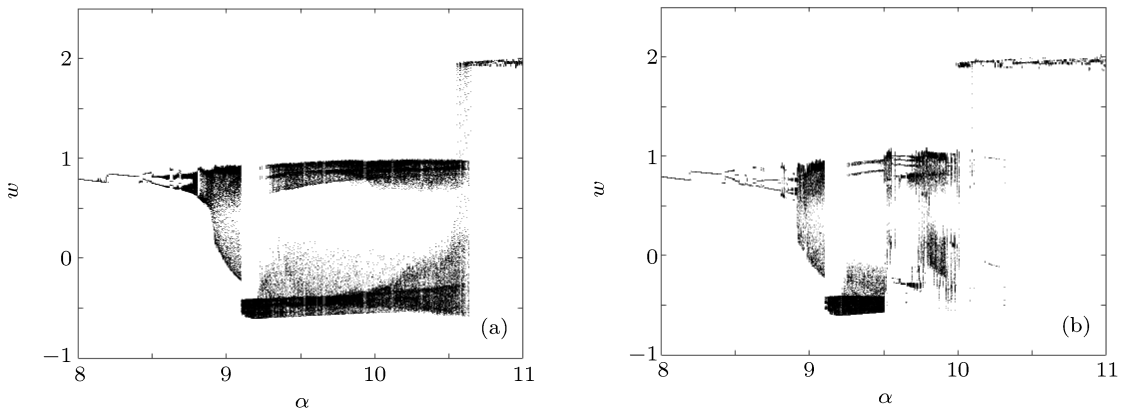


Fig. 4. Bifurcation diagrams of w with value α of the memristor oscillator increasing: (a) transient behaviours; (b) stable behaviours.

The appearance of chaos on finite time scales is known as transient chaos. The phenomenon of transient chaos accompanied with boundary crisis is frequently encountered in dynamical systems. In particular, such a

phenomenon can be observed in many dynamical systems such as the Logistic maps,[12] the Rössler system,[13] the Lorenz system,[14] and the four-dimensional hyperchaotic system.[15]

In a boundary crisis, with the increase of the control parameter p, the distance between a strange attractor and the boundary of its basin of attraction in the phase space decreases until they touch each other at a critical value ($p = p_c$). At this point the attractor also touches an unstable periodic orbit and the chaotic attractor exhibits a crisis. For $p = p_c$, the chaotic attractor no longer exists and will be replaced by a chaotic transient. In the initial stage of this regime, the system behaviours are virtually undistinguishable from chaotic, but then the system rapidly passes to another stable state (attractor) that can be stationary, periodic, or chaotic as well.[18]

When $\alpha = 10.5$, the phenomenon of transient chaos occurs in system (5). Figure 5 shows the time series of state variables y and w in system (5). Obviously, this time series is different from one generated by a chaotic system. The first part in the interval $t \in [0\text{ s}, 82.5\text{ s}]$ is so disordered and chaotic, and the second part after the time $t = 82.5$ s is ordered and periodic. The projections of the phase portrait of the transient chaotic attractor and stable periodic limit cycle on y–w plane are shown in Figs. 6(a) and 6(b) respectively. Figure 6(a) illustrates a 2-scroll chaotic attractor reflecting the basic dynamic behaviour on a finite time scale, while Fig. 6(b) represents the formed periodic orbit ultimately after a chaotic transient.

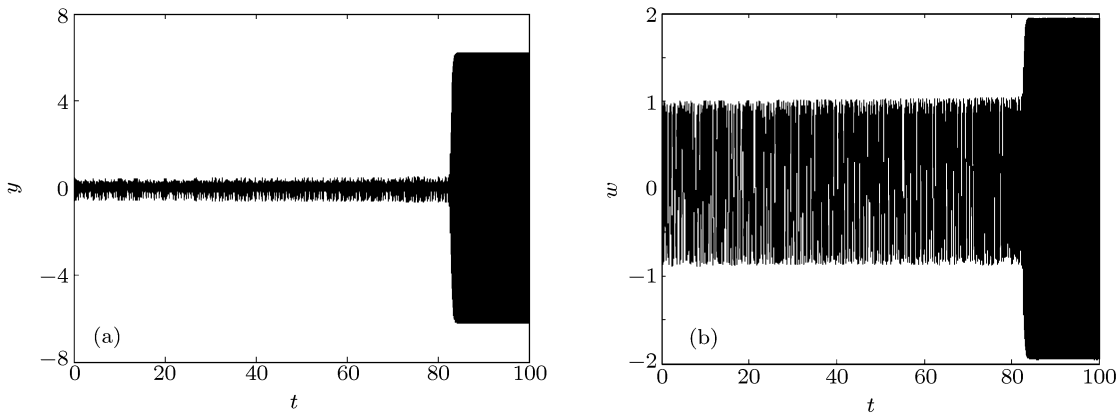


Fig. 5. Plots of state variables as a function of time: (a) y–t; (b) w–t.

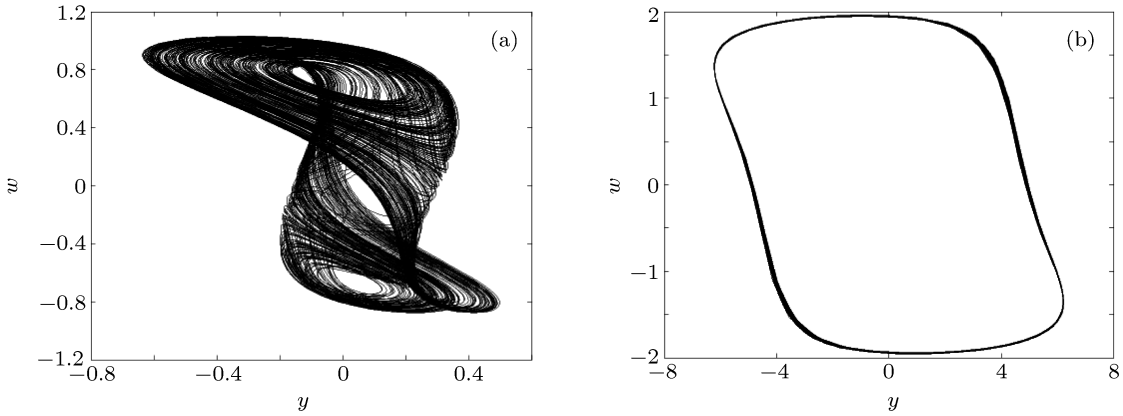


Fig. 6. Phase portraits of system (5) on y–w plane: (a) transient chaotic attractor; (b) stable periodic limit cycle.

It can be seen that the orbit of system (5) originating from initial values (0.1, 0, 0, 0) will return to a periodic limit cycle with period-1 eventually through the evolution in 82.5 s.

Similar phenomenon of transient chaos can be observed at another typical parameter value $\alpha = 9.6$. Figures 7(a) and 7(b) show the projections of the phase portrait on y–w plane of the transient chaotic attractor and a stable period-4 limit cycle respectively. Different from the phenomenon described in Fig. 6, the evolution from chaotic to periodic behaviours in Fig. 7 is smooth with the increase of time.

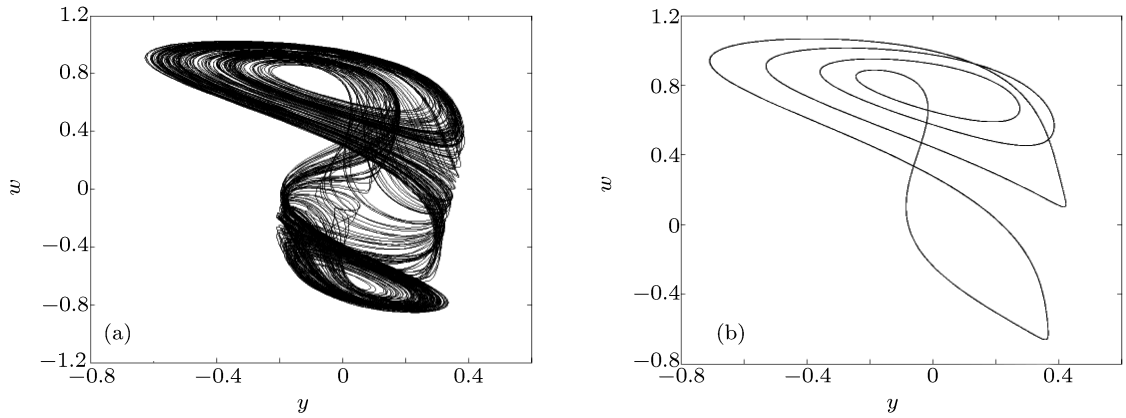


Fig. 7. Projections of the phase portrait on y–w plane of system (5): (a) transient chaotic attractor; (b) stable periodic limit cycle.

Furthermore, novel phenomena of stable chaotic and intermittence periodic behaviours appear in memristor Chua's oscillator. As shown in Figs. 8 and 9 where $\alpha = 9.3$, the chaotic attractor has a transition from 2-scroll chaotic to eventual chaotic and intermittence periodic behaviours with the increase of time, and there is a coexisting intermittent periodic orbit with period-3 within chaos in the stable state of the memristor oscillator. The results of a large number of numerical simulations demonstrate that the appearing of the coexisting phenomenon of stable chaotic and intermittence periodic motion strongly depends not only on the parameters but also on the initial positions.

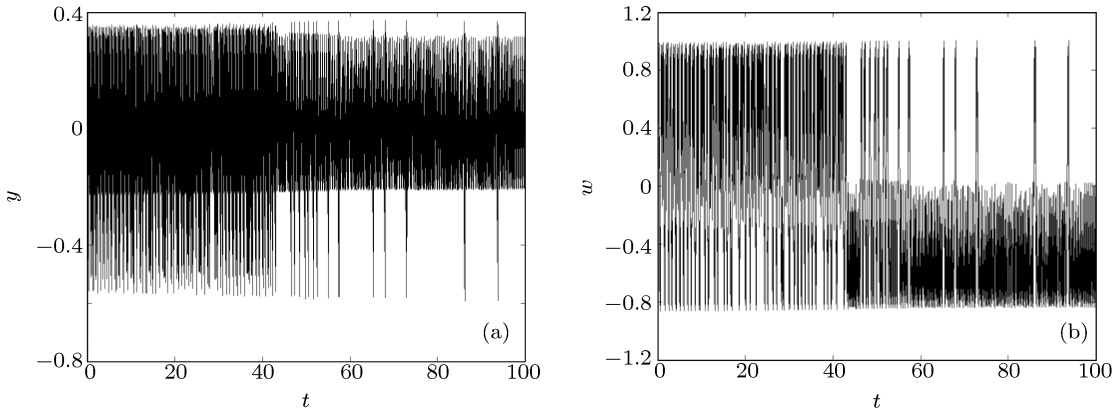


Fig. 8. Plots of state variables y (a) and w (b) as a function of time t.

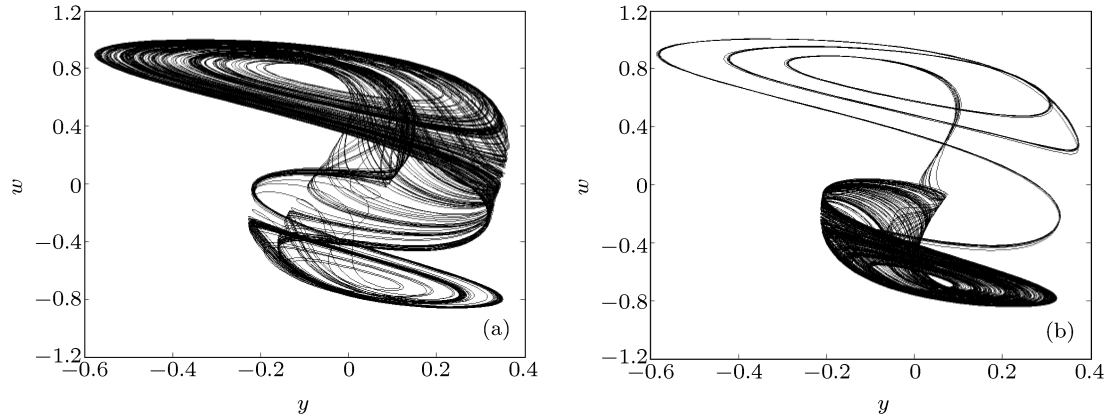


Fig. 9. Transition from 2-scroll chaotic to chaotic and intermittence periodic behaviours: (a) transient chaotic attractor; (b) stable chaotic attractor with intermittence period 3.

4. Conclusions

In this paper a smooth memristor Chua's oscillator, which is directly extended from a Chua's oscillator by replacing the Chua's diode with a smooth flux-controlled memristor and a negative conductance, is presented and studied. The novel oscillator can generate a 2-scroll chaotic attractor and has complex nonlinear dynamics such as the transient chaos and stable chaos with an intermittence period under different parameters and initial conditions. The research results demonstrate that the introduction of the smooth flux-controlled memristor makes the dynamical behaviours more complicated and completely different from the existing chaotic system. From this point of view, it is believed that the new smooth memristor oscillator deserves further investigation in the near future.

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