Study on eigenvalue space of hyperchaotic canonical four-dimensional Chua's circuit*

Li Guan-Lin(李冠林)† and Chen Xi-You(陈希有)

Department of Electrical and Electronics Engineering, Dalian University of Technology, Dalian 116023, *China*

(Received 14 July 2009; revised manuscript received 18 August 2009)

The eigenvalue space of the canonical four-dimensional Chua's circuit which can realize every eigenvalue for four-dimensional system is studied in this paper. First, the analytical relations between the circuit parameters and the eigenvalues of the system are established, and therefore all the circuit parameters can be determined explicitly by any given set of eigenvalues. Then, the eigenvalue space of the circuit is investigated in two cases by the nonlinear elements used. According to the types of the eigenvalues, some novel hyperchaotic attractors are presented. Further, the dynamic behaviours of the circuit are studied by the bifurcation diagrams and the Lyapunov spectra of the eigenvalues.

Keywords: four-dimensional Chua's circuit, eigenvalue space, hyperchaos

PACC: 0545

1. Introduction

Hyperchaos is defined as the dynamics of a chaotic system with more than one positive Lyapunov exponent, of which dynamical behaviours expand in more than one direction. Therefore, these systems have more complex attractors and can be used to improve the security of the messages masked by the chaotic signals.

Hyperchaos was first reported by Rössler in the ordinary differential equations for modeling chemical reactions.[1] Then, the experimental realization of hyperchaos in electrical circuit was first observed by Matsumoto *et al.*[2] Later, hyperchaos was found in some coupled chaotic systems,[3−5] and some higher-dimensional nonlinear systems,[6−16] even in some fractional order nonlinear systems.[17,18] Moreover, several control strategies used to control chaotic systems going to hyperchaos were also been proposed.[19,20]

Although some hyperchaotic systems have been presented, the dynamics of these systems and the routes going to hyperchaos should be further studied. The eigenvalue space of the canonical four-dimensional (4D) Chua's circuit is studied in this paper, which can realize every eigenvalue for 4D systems and is of considerable significance to the research on hyperchaos.

The rest of the present paper is organized as follows. The circuit structure and the ordinary differential equations of this system are introduced in Section 2. The relations between circuit parameters and eigenvalues are determined in Section 3. In Section 4, the eigenvalue parameters of the canonical 4D Chua's circuit are investigated in two cases in terms of the nonlinear elements used: one is the three-segment odd symmetrical piecewise-linear (PWL) element; the other is the two-segment one. From the types of the eigenvalues, some novel hyperchaotic attractors are proposed. The dynamic behaviours of the systems are studied by the bifurcation diagram of the eigenvalue parameters. The results of numerical simulation are shown in Section 4.

2. Canonical 4D Chua's circuit

Chua's circuit is a well-known piecewise-linear chaotic circuit.[21] Canonical Chua's circuit was first presented by Leon Chua and Lin,[22] which can realize every member of the Chua's circuit family. Then, higher-dimensional Chua's circuit was proposed by Gotz *et al.*[23]

The canonical n-dimensional Chua's circuit for even n is shown in Fig. 1, in which the nonlinear element N_R is either a two-segment PWL element or a three-segment odd symmetrical PWL element with the characteristics shown in Fig. 2.

*Project supported by the National Natural Science Foundation of China (Grant No. 50877007).

†Corresponding author. E-mail: lglhit@yahoo.com.cn

© 2010 Chinese Physical Society and IOP Publishing Ltd

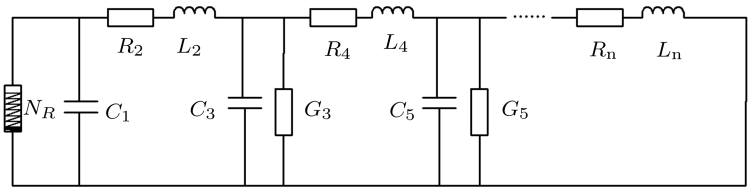


Fig. 1. An n-dimensional canonical Chua's circuit (n is even).

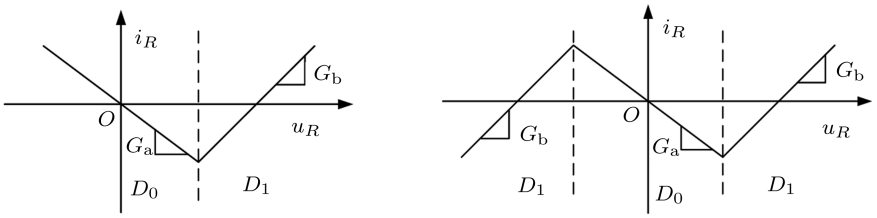


Fig. 2. Characteristics of the piecewise-linear element N_R. (a) N_R is two-segment; (b) N_R is three-segment odd symmetrical.

The circuit with $n = 4$ is the canonical 4-D Chua's circuit, shown in Fig. 3, and the characteristics of N'_R is curved in Fig. 4.

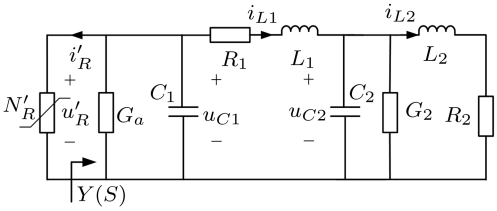


Fig. 3. Canonical 4-D piecewise-linear Chua's circuit.

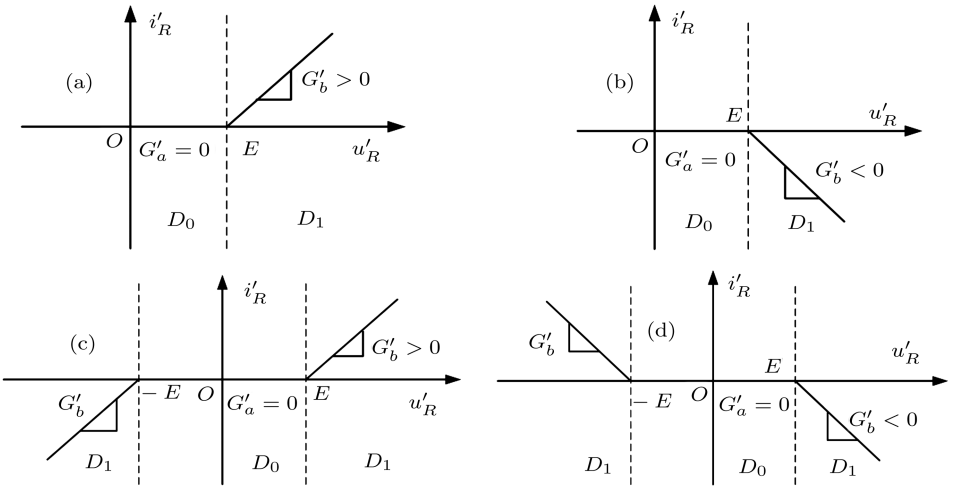


Fig. 4. Nonlinear characteristics of N'_R with zero conductance in region D_0. (a) $G'_b = G_b - G_a > 0$; (b) $G'_b = G_b - G_a < 0$; (c) $G'_b = G_b - G_a > 0$; (d) $G'_b = G_b - G_a < 0$.

Applying Kirchhoff's laws, the set of four coupled first-order differential equations describing the circuit can be obtained as

$$C_1\frac{\mathrm{d}u_{C1}}{\mathrm{d}t} = -G_a u_{C1} - i_{L1} - i'_R, \quad C_2\frac{\mathrm{d}u_{C2}}{\mathrm{d}t} = -G_2 u_{C2} + i_{L1} - i_{L2},$$
$$L_1\frac{\mathrm{d}i_{L1}}{\mathrm{d}t} = u_{C1} - u_{C2} - R_1 i_{L1}, \quad L_2\frac{\mathrm{d}i_{L2}}{\mathrm{d}t} = u_{C2} - i_{L2}R_2, \tag{1}$$

where u_{C1} and u_{C2} are the voltages across capacitors C_1 and C_2, respectively; i_{L1} and i_{L2} denote the currents flowing through inductances L_1 and L_2, respectively; i'_R represents the current flowing through N'_R.

The current i'_R in Fig. 4 can be expressed as follows:
for Figs. 4(a) and 4(b),

$$i'_R = f(u'_R) = \begin{cases} 0, & u'_R < E, \\ G'_b(u'_R - E), & u'_R \geq E, \end{cases} \quad (2)$$

for Figs. 4(c) and 4(d),

$$i'_R = f(u'_R) = \begin{cases} G'_b(u'_R - E), & u'_R \geq E, \\ 0, & -E < u'_R < E, \\ G'_b(u'_R + E), & u'_R \leq -E, \end{cases} \quad (3)$$

where $u'_R = u_{C1}$, and E is a constant.

The circuit shown in Fig. 3 is a canonical circuit because it can realize every eigenvalue in 4-D system. There are nine circuit parameters, i.e. G_a, G'_b, C_1, C_2, L_1, L_2, R_1, R_2, and G_2 in the circuit, so it has eight freedom degrees. Meanwhile, there are just four eigenvalues in each of two regions: D_0 and D_1.

3. Circuit parameters and eigenvalues

Because the dynamic characteristics of the circuit correspond to its eigenvalues, the relations between the circuit parameters and the eigenvalues of the canonical 4D Chua's circuit is established in this Section to explore the eigenvalue space of the circuit.

The eigenvalues in the regions D_0 and D_1 are denoted by μ_1–μ_4 and ν_1–ν_4, respectively. The characteristic polynomials of the system in the regions D_0 and D_1 are expressed as Eqs. (4) and (5),[24] respectively.

$$\text{In region } D_0: \quad F_0(s) = s^4 + \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0 = 0, \quad (4)$$

$$\text{In region } D_1: \quad F_1(s) = s^4 + \beta_3 s^3 + \beta_2 s^2 + \beta_1 s + \beta_0 = 0, \quad (5)$$

where

$$\alpha_3 = -\sum_{i=1}^{4} \mu_i; \quad \alpha_2 = \sum_{\substack{i=1,j=2 \\ (i<j)}}^{4} \mu_i\mu_j; \quad \alpha_1 = -\sum_{\substack{i=1,j=2,k=3 \\ (i<j<k)}}^{4} \mu_i\mu_j\mu_k; \quad \alpha_0 = \mu_1\mu_2\mu_3\mu_4;$$

$$\beta_3 = -\sum_{i=1}^{4} \nu_i; \quad \beta_2 = \sum_{\substack{i=1,j=2 \\ (i<j)}}^{4} \nu_i\nu_j; \quad \beta_1 = -\sum_{\substack{i=1,j=2,k=3 \\ (i<j<k)}}^{4} \nu_i\nu_j\nu_k; \quad \beta_0 = \nu_1\nu_2\nu_3\nu_4.$$

The linear one-port admittance function $Y(s)$ of the circuit shown in Fig. 3 can be found to be expressed as

$$\begin{aligned} Y(s) &= \frac{G'_b F_0(s)}{F_1(s) - F_0(s)} \\ &= \frac{G'_b(s^4 + \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0)}{(\beta_3 - \alpha_3)s^3 + (\beta_2 - \alpha_2)s^2 + (\beta_1 - \alpha_1)s + (\beta_0 - \alpha_0)} \\ &= \frac{\dfrac{G'_b}{\beta_3 - \alpha_3}(s^4 + \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0)}{s^3 + (\dfrac{\beta_2 - \alpha_2}{\beta_3 - \alpha_3})s^2 + \left(\dfrac{\beta_1 - \alpha_1}{\beta_3 - \alpha_3}\right)s + \left(\dfrac{\beta_0 - \alpha_0}{\beta_3 - \alpha_3}\right)}. \end{aligned} \quad (6)$$

Meanwhile, from the circuit analysis method, the admittance function $Y(s)$ for the circuit shown in Fig. 3 can also be expressed as below

$$Y(s) = G_a + C_1 s + \cfrac{1}{R_1 + Ls + \cfrac{1}{C_2 s + G_2 + \cfrac{1}{L_2 s + R_2}}}. \quad (7)$$

Therefore, the relations between the circuit parameters and the eigenvalues of the system can be established based on Eqs. (6) and (7). The following equations can be obtained:

$$\frac{G_b'}{\beta_3-\alpha_3}-C_1=0\ , \tag{8}$$

$$\frac{G_b'}{\beta_3-\alpha_3}\alpha_3-G_a-C_1\frac{\beta_2-\alpha_2}{\beta_3-\alpha_3}=0\ , \tag{9}$$

$$\frac{G_b'}{\beta_3-\alpha_3}\alpha_2-G_a\frac{\beta_2-\alpha_2}{\beta_3-\alpha_3} -C_1\frac{\beta_1-\alpha_1}{\beta_3-\alpha_3}=\frac{1}{L_1}, \tag{10}$$

$$\frac{G_b'}{\beta_3-\alpha_3}\alpha_1-G_a\frac{\beta_1-\alpha_1}{\beta_3-\alpha_3} -C_1\frac{\beta_0-\alpha_0}{\beta_3-\alpha_3}=\frac{1}{L_1}\left(\frac{R_2}{L_2}+\frac{G_2}{C_2}\right)\ , \tag{11}$$

$$\frac{G_b'}{\beta_3-\alpha_3}\alpha_0-G_a\frac{\beta_0-\alpha_0}{\beta_3-\alpha_3}=\frac{G_2R_2+1}{C_2L_2L_1}\ , \tag{12}$$

$$\frac{\beta_2-\alpha_2}{\beta_3-\alpha_3}=\frac{R_1}{L_1}+\frac{R_2}{L_2}+\frac{G_2}{C_2}\ , \tag{13}$$

$$\frac{\beta_1-\alpha_1}{\beta_3-\alpha_3}=\frac{R_1}{L_1}\left(\frac{R_2}{L_2}+\frac{G_2}{C_2}\right) +\frac{R_2G_2}{L_2C_2}+\frac{1}{L_2C_2}+\frac{1}{L_1C_2}\ , \tag{14}$$

$$\frac{\beta_0-\alpha_0}{\beta_3-\alpha_3}=\frac{R_1+R_2}{L_1L_2C_2}+\frac{R_1R_2G_2}{L_1L_2C_2}\ . \tag{15}$$

In order to find nine circuit parameters from eight equations, one of the circuit parameters should be known beforehand. For example, let C_1 be known, then, the other circuit parameters can be deduced as follows:

$$G_b'=C_1(\beta_3-\alpha_3)\ , \tag{16}$$

$$G_a=C_1\left(\alpha_3-\frac{\beta_2-\alpha_2}{\beta_3-\alpha_3}\right), \tag{17}$$

$$L_1=\frac{1}{C_1\alpha_2-G_a\dfrac{\beta_2-\alpha_2}{\beta_3-\alpha_3}-C_1\dfrac{\beta_1-\alpha_1}{\beta_3-\alpha_3}}, \tag{18}$$

$$R_1=L_1\left(\frac{\beta_2-\alpha_2}{\beta_3-\alpha_3}-k\right), \tag{19}$$

$$C_2=\frac{1}{L_1\left(\dfrac{\beta_1-\alpha_1}{\beta_3-\alpha_3}-\dfrac{R_1}{L_1}k-p\right)}, \tag{20}$$

$$G_2=C_2(k-m), \tag{21}$$

$$L_2=\frac{1}{C_2\left[p-(k-m)m\right]}, \tag{22}$$

$$R_2=mL_2, \tag{23}$$

where,

$$k=L_1\left(C_1\alpha_1-G_a\frac{\beta_1-\alpha_1}{\beta_3-\alpha_3}-C_1\frac{\beta_0-\alpha_0}{\beta_3-\alpha_3}\right),$$

$$p=L_1\left(C_1\alpha_0-G_a\frac{\beta_0-\alpha_0}{\beta_3-\alpha_3}\right),$$

$$m=\left(\frac{\beta_0-\alpha_0}{\beta_3-\alpha_3}-\frac{R_1}{L_1}p\right)L_1C_2.$$

Thus, the relations between the circuit parameters and the eigenvalues of the system can be established. From Eqs. (8)–(15), all the circuit parameters can be determined by any given set of eigenvalues.

4. Eigenvalue space

According to the relations established in Section 3, the eigenvalue space and the types of the circuit are studied in two cases by the nonlinear elements used in this section.

4.1. Applying three-segment odd symmetrical PWL element

The nonlinear characteristic of the three-segment odd symmetrical PWL element (component I for short) is shown in Figs. 4(c) and 4(d), and the eigenvalues in regions D_0 and D_1 are proposed in Table 1. In order to ensure that the circuit parameters are real numbers, the eigenvalue parameters must be set to be real or a pair of conjugate complex numbers.

Table 1. Circuit parameters and Lyapunov exponents of the circuit with different types of eigenvalues when applying nonlinear component I.

different types of eigenvalues			circuit parameters ($C_1=1$)	Lyapunov exponents (λ_1–λ_4)
group	D_0 region (μ_1–μ_4)	D_1 region (v_1–v_4)		
(1)	0.5 ± 0.08j	-50	$G_a=2.6629,\quad G_b'=49.07$	0.055
	-0.05 ± 10j	-0.01	$L_1=0.0111,\quad R_1=-0.021$	0.007
		0.92 ± 4j	$C_2=5.7493,\quad G_2=-9.6549$	0.000
			$L_2=0.5508,\quad R_2=0.0072$	-2.403

Table 1 (*Continued*)

	different types of eigenvalues		circuit parameters ($C_1 = 1$)	Lyapunov exponents (λ_1–λ_4)
group	D_0 region (μ_1–μ_4)	D_1 region (v_1–v_4)		
(2)	0.85 ± 0.5j ± 5j	-58 0.2 0.28 ± 3.86j	$G_a = -0.7675$, $G'_b = 58.94$ $L_1 = 0.1027$, $R_1 = 0.1914$ $C_2 = 0.5264$, $G_2 = -1.2579$ $L_2 = 1.5072$, $R_2 = -0.6127$	0.081 0.032 0.000 −8.929
(3)	0.5 ± 0.2j -0.05 ± 10j	1.6 ± 7j -10 ± 3.15j	$G_a = -0.7469$, $G'_b = 17.7$ $L_1 = 0.0178$, $R_1 = 0.1201$ $C_2 = 0.6549$, $G_2 = -6.6997$ $L_2 = 0.0391$, $R_2 = 0.1310$	0.170 0.000 −0.296 −0.952
(4)	0.8 0.8 0.55 ± 10.1j	-50 0.01 0.92 ± 4j	$G_a = 0.8467$, $G'_b = 50.85$ $L_1 = 0.0114$, $R_1 = -0.0171$ $C_2 = 5.4999$, $G_2 = -11.1372$ $L_2 = 0.2528$, $R_2 = -0.005$	0.069 0.017 0.000, −3.045
(5)	−50 0.1 0.92 ± 4j	1.0 0.6 0.55 ± 10j	$G_a = 51.6580$, $G'_b = -50.76$ $L_1 = 0.0116$, $R_1 = -0.0180$ $C_2 = 5.3690$, $G_2 = -10.4272$ $L_2 = 0.3570$, $R_2 = -0.0382$	0.086 0.019 0.000 −3.685
(6)	−0.8 0.8 0.15 ± 10.1j	-50 -0.2 0.5 ± 4j	$G_a = 2.2322$, $G'_b = 49.5$ $L_1 = 0.011$, $R_1 = -0.023$ $C_2 = 5.6130$, $G_2 = -3.5143$ $L_2 = -0.2469$, $R_2 = -0.0438$	0.041 0.000 −0.045 −4.596
(7)	−30 0.05 0.18 ± 4j	2.1 ± 1j ± 5j	$G_a = 30.3790$, $G'_b = -33.79$ $L_1 = 0.0963$, $R_1 = 0.3057$ $C_2 = 0.5632$, $G_2 = -0.9756$ $L_2 = 0.2333$, $R_2 = -0.5206$	0.118 0.098 0.000 −6.556
(8)	1.0 1.2 −25 −25	−10 0.1 0.2 ± 4j	$G_a = 34.6115$, $G'_b = -38.3$ $L_1 = 0.0102$, $R_1 = 0.1346$ $C_2 = -2.5376$, $G_2 = -0.7302$ $L_2 = -0.5806$, $R_2 = 0.1835$	0.014 0.000 −0.888 −1.849

Eigenvalues of groups (1), (2), (4), (5) and (7) are corresponding to hyperchaotic systems, for they have two positive Lyapunov exponents. Phase portraits of the hyperchaotic systems are drawn in Fig. 5.

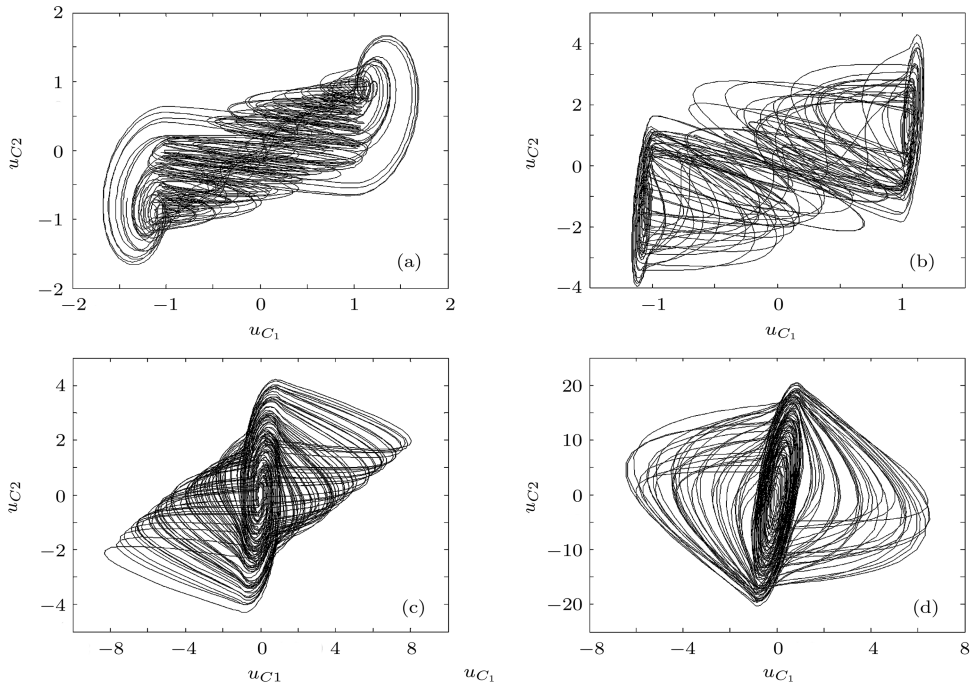


Fig. 5. Hyperchaotic attractors of the system with different types of eigenvalues when applying the nonlinear component I. (a) Group 1, (b) Group 2, (c) Group 5, (d) Group 7.

The dynamic behaviours of the circuit are studied by the bifurcation diagrams of the eigenvalue parameters.

The bifurcation parameters may be the real eigenvalues, the real part or the imaginary part of the complex eigenvalues.

Case 1 The eigenvalues in the region D_0 are $\mu_{1,2} = u_{1a} \pm 0.5\text{j}$, $\mu_{3,4} = \pm 5\text{j}$, where u_{1a} is the real part of the complex eigenvalue varying from 0.85 to 3.3; the eigenvalues in the region D_1 are $v_1 = -58$, $v_2 = 0.2$ and $v_{3,4} = 0.28 \pm 3.86\text{j}$. Figure 6(a) shows a one-parameter bifurcation diagram drawn in (u_{1a}, i_{L1}) plane. The Lyapunov spectrum in the (u_{1a}, $\lambda_{1,2,3}$) plane is shown in Fig. 6(b).

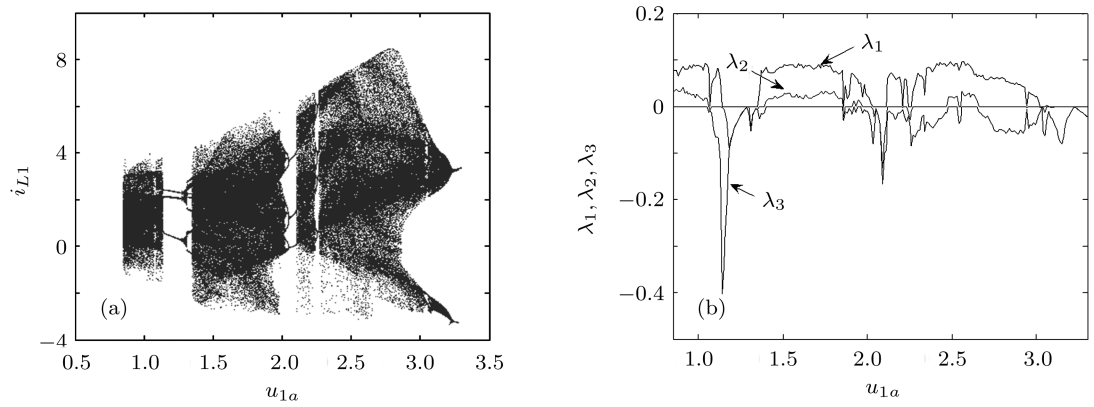


Fig. 6. Dynamics of the system when the eigenvalues are $\mu_{1,2} = u_{1a} \pm 0.5\text{j}$, $\mu_{3,4} = \pm 5\text{j}$ in region D_0 and $v_1 = -58$, $v_2 = 0.2$, $v_{3,4} = 0.28 \pm 3.86\text{j}$ in region D_1, where u_{1a} varies from 0.85 to 3.3. (a) one-parameter bifurcation diagram drawn in (u_{1a}, i_{L1}) plane, (b) Lyapunov spectrum in (u_{1a}, $\lambda_{1,2,3}$) plane.

Case 2: The eigenvalues in the region D_0 are $\mu_{1,2} = 0.85 \pm u_{1b}\cdot\text{j}$, $u_{3,4} = \pm 5\text{j}$, where u_{1b} is the imaginary part of the complex eigenvalue varying from 0.5 to 3; the eigenvalues in the region D_1 are $v_1 = -58$, $v_2 = 0.2$ and $v_{3,4} = 0.28 \pm 3.86\text{j}$. Figure 7(a) shows a one-parameter bifurcation diagram drawn in (u_{1b}, i_{L1}) plane. The Lyapunov spectrum in the (u_{1b}, $\lambda_{1,2,3}$) plane is shown in Fig. 7(b).

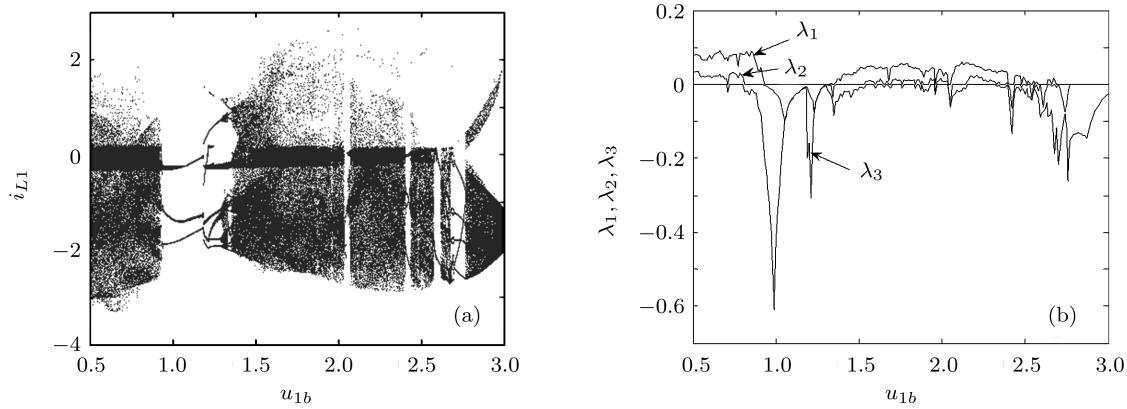


Fig. 7. Dynamics of the system when the eigenvalues are $\mu_{1,2} = 0.85 \pm u_{1b}\cdot\text{j}$, $\mu_{3,4} = \pm 5\text{j}$ in region D_0 and $v_1 = -58$, $v_2 = 0.2$, $v_{3,4} = 0.28 \pm 3.86\text{j}$ in region D_1, where u_{1b} varies from 0.5 to 3. (a) one parameter bifurcation diagram drawn in (u_{1b}, i_{L1}) plane, (b) Lyapunov spectrum in (u_{1b}, $\lambda_{1,2,3}$) plane.

The cases above show the dynamics of the canonical 4D Chua's circuit corresponding to different eigenvalues. Figures 6(b) and 7(b) show the first three maximal Lyapunov exponents. When $\lambda_1 = 0$, and $\lambda_2, \lambda_3 < 0$, the circuit corresponds to periodic orbit; when $\lambda_1 = \lambda_2 = 0$, and $\lambda_3 < 0$, it corresponds to quasi-periodic orbit; when $\lambda_1 > 0$, $\lambda_2 = 0$, and $\lambda_3 < 0$, it corresponds to chaotic attractor; when $\lambda_1, \lambda_2 > 0$, $\lambda_3 = 0$, it corresponds to hyper-chaotic attractor. The other bifurcation diagrams and the Lyapunov spectrums can be drawn by the other bifurcation parameters and the eigenvalues.

4.2. Applying two-segment PWL element

The other case is for applying the two-segment PWL element (component II for short) as shown in Figs. 4(a) and 4(b). Two types of eigenvalues in regions D_0 and D_1 are proposed in Table 2. Eigenvalue of group (1) is corresponding to hyperchaotic systems. Phase portraits of these systems are shown in Fig. 8. The dynamics of the system can also be studied by the bifurcation diagram of the eigenvalue parameters, shown in Fig. 9.

Table 2. Circuit parameters and Lyapunov exponents of the circuit with different types of eigenvalues with applying the nonlinear component II.

different types of eigenvalues			circuit parameters ($C_1 = 1$)	Lyapunov exponents (LE)
group	D_0 region (μ_1–μ_4)	D_1 region (v_1–v_4)		
(1)	0.1 ± 5j 0.2 ± 0.8j	-50 0 0.2 ± 1.9j	$G_a = 0.2390$, $G_b' = 50.2$ $L_1 = 0.0452$, $R_1 = -0.0160$ $C_2 = 7.649$, $G_2 = -3.5327$ $L_2 = 0.1716$, $R_2 = -0.0038$	0.0244 0.0189 0.000 -3.5961
(2)	1 -10.8 1 ± 10.8j	-55 -10 0.5 ± 3.5j	$G_a = 0.5$, $G_b' = 56.2$ $L_1 = 0.01$, $R_1 = -0.031$ $C_2 = 3.3944$, $G_2 = 23.3637$ $L_2 = -0.0078$, $R_2 = -0.0275$	0.048 0.00 -0.90 -4.62

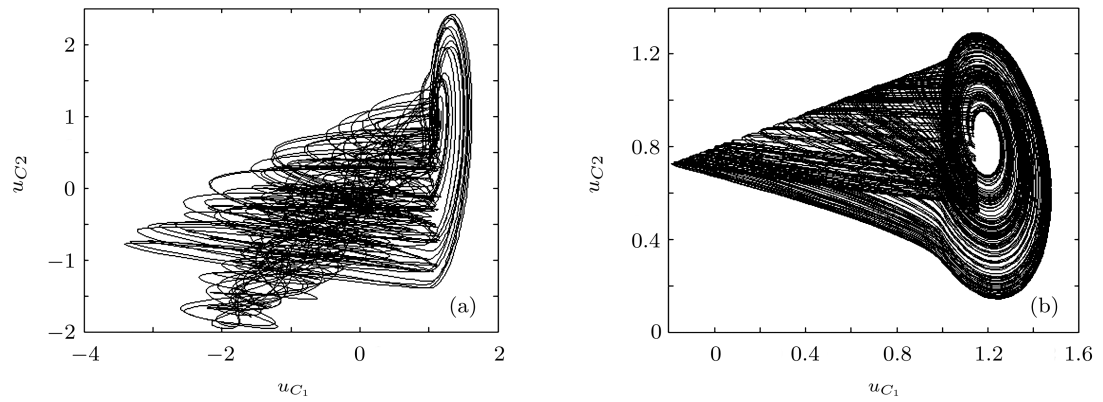


Fig. 8. Phase portraits of the circuit with different types of eigenvalues with applying the nonlinear component II. (a) Group 1, (b) Group 2.

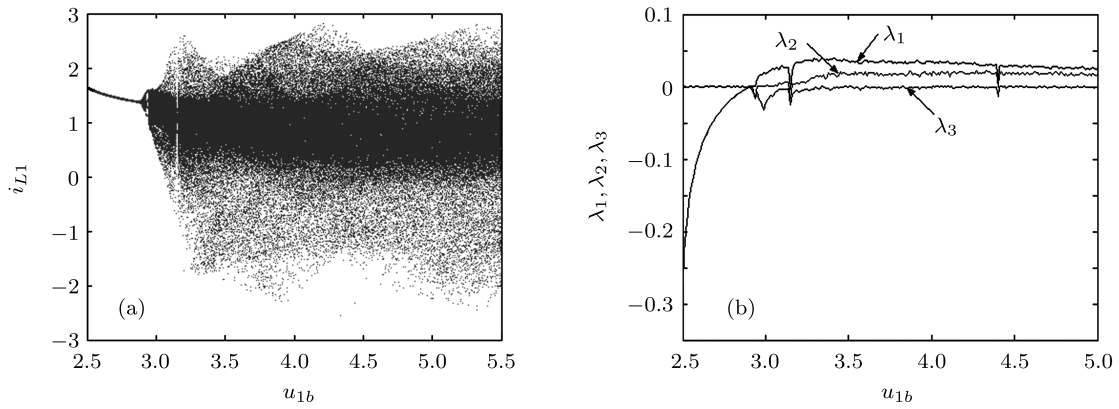


Fig. 9. Dynamics of the system when eigenvalues are $\mu_{1,2} = 0.1 \pm u_{1b}$j, and $u_{3,4} = 0.2 \pm 0.8$j in region D_0 and $v_1 = -50$, $v_2 = 0$ and $v_{3,4} = 0.2 \pm 1.9$j in region D_1, where u_{1b} varies from 2.5 to 5. (a) one parameter bifurcation diagram drawn in (u_{1b}, i_{L1}) plane, (b) Lyapunov spectrum in the (u_{1b}, $\lambda_{1,2,3}$) plane.

5. Conclusions

The eigenvalue space of the canonical 4D Chua's circuit is studied in this paper. The relations between the circuit parameters and the eigenvalues of the system are established. All the circuit parameters can be determined explicitly from any given set of eigenvalues. Therefore, the dynamics of the circuit can be studied by the bifurcation diagram of the eigenvalues. The bifurcation parameter may be the real eigenvalue, the real part or the imaginary part of the complex eigenvalue. Some sets of eigenvalues corresponding to hyperchaotic attractors are proposed and analysed in this paper. According to the bifurcation diagrams of the eigenvalues, the routes to hyperchaos may be further investigated.

References

[1] Rössler O E 1979 *Phys. Lett.* A **71** 155
[2] Matsumoto T, Chua L O and Kobayashi K 1986 *IEEE Trans. Circ. and Syst.* **33** 1143
[3] Kapitaniak T, Chua L O and Zhong G Q 1994 *IEEE Trans. Circ. and Syst.* I **41** 499
[4] Barbara C and Silvano C 2002 *Int. J. Circ. Theor. Appl.* **30** 625
[5] Cenys A, Tamasevicius A, Baziliauskas A, Krivickas R and Lindberg E 2003 *Chaos, Solitons and Fractals* **17** 349
[6] Tamasevicius A, Namajunas A and Cenys A 1996 *Electron. Lett.* **32** 957
[7] Tsubone T and Saito T 1998 *IEEE Trans. Circ. and Syst.* I **45** 889
[8] Takahashi Y, Nakano H and Saito T 2004 *IEEE Trans. Circ. and Syst.* II **51** 468
[9] Thamilmaran K, Lakshmanan M and Venkatesan A 2004 *Int. J. Bifurca. and Chaos* **14** 221
[10] Chen Z, Yang Y, Qi G and Yuan Z 2007 *Phys. Lett.* A **360** 696
[11] Yu S, Lü J and Chen G 2007 *Phys. Lett.* A **364** 244
[12] Wang F Q and Liu C X 2006 *Chin. Phys.* **15** 963
[13] Liu C X 2007 *Chin. Phys.* **16** 3279
[14] Wang G Y, Liu J B and Zheng X 2007 *Chin. Phys.* **16** 2278
[15] Liu C X and Liu L 2009 *Chin. Phys.* B **18** 2188
[16] Yu S and Yu Z 2008 *Acta Phys. Sin.* **57** 6859 (in Chinese)
[17] Ahmad W M 2005 *Chaos, Solitons and Fractals* **26** 1459
[18] Zhou P, Wei L and Cheng X 2009 *Chin. Phys.* B **18** 2674
[19] Li Y, Chen G and Tang W 2005 *IEEE Trans. Circ. and Syst.* II **52** 204
[20] Wu X, Wang J, Lu J and Iu H 2007 *Chaos, Solitons and Fractals* **32** 1485
[21] Chua L O and Komuro M 1986 *IEEE Trans. Circ. and Syst.* **33** 1073
[22] Chua L O and Lin G N 1990 *IEEE Trans. Circ. and Syst.* **37** 885
[23] Gotz M, Feldmann U and Schwarz W 1993 *IEEE Trans. Circ. and Syst.* I **40** 854
[24] Scanlan S O 2001 *IEEE Trans. Circ. and Syst.* I **48** 1057