Wavelet threshold method of resolving noise interference in periodic short-impulse signals chaotic detection*

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The chaotic oscillator has already been considered as a powerful method to detect weak signals, even weak signals accompanied with noises. However, many examples, analyses and simulations indicate that chaotic oscillator detection system cannot guarantee the immunity to noises (even white noise). In fact the randomness of noises has a serious or even a destructive effect on the detection results in many cases. To solve this problem, we present a new detecting method based on wavelet threshold processing that can detect the chaotic weak signal accompanied with noise. All theoretical analyses and simulation experiments indicate that the new method reduces the noise interferences to detection significantly, thereby making the corresponding chaotic oscillator that detects the weak signals accompanied with noises more stable and reliable.

Keywords: chaotic detection, periodic short-impulse signals, wavelet threshold

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1. Introduction

In resent years, the application of nonlinear chaotic system to detecting weak signals has become a popular topic in the fields of radar, guidance, satellite and communication.[1−10] The basic principle is that a chaotic system is extremely sensitive to the changes of initial conditions and parameters. The observable essential change in the dynamic behaviour of chaotic system results from the input of weak signals. The enhancement of sensitivity of the system could achieve the purpose of detecting weak signals.[11−14] However, there are some examples, analyses and simulation experiments, which indicate that some conclusions and results deviate from the actual circumstances.[15,16] Actually, almost all signals to be measured contain a lot of noises in practice. Furthermore, the effect of random noise on dynamic behaviour of chaotic system cannot be ignored; otherwise, it will make the changes of the phase diagram unstable and result in the failure of detection. There is no effective solution available that can be found in documents until now.

To solve the current problems above, we present a new hybrid detection system based on wavelet threshold processing, by which the chaotic oscillator can be detected effectively. The basic point of the system is lthe ater period of chaotic detection which is based on appropriate wavelet threshold processing of signals mixed with noises. All theoretical analyses and contrast experiments indicate that the niose interference to detection system reduces observably with using our method. Also this method improves the stability and the reliability of the corresponding chaotic oscillator for detecting weak signals accompanied with noise. Therefore, it is a new method that can detect weak signals accompanied with noises more effectively.

2. Chaotic detection method of periodic short-impulse signals

The chaotic system is used to detect weak signals and the physical mechanism of the system-control chaos by using periodic perturbation. The signals are detected when the system is converted from chaotic state to large-scale periodic state. There are two methods of inhibiting chaotic motion by using the periodic perturbation: periodic driving force perturbation and periodic parameter perturbation.

Periodic driving force perturbation requires a built-in signal of the chaotic detection system which has the same frequency and waveform as those of a to-

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be-measured signal. The combination of the built-in signal and the to-be-measured signal becomes the periodic driving force perturbation of the system. Then we can detect whether the weak signals are present by detecting the sensitivity to the change of periodic driving force perturbation. Though the mathematical expression is required in advance, this method could not be used to detect arbitrary signals.

Periodic parameters perturbation does not need to know the waveform of the to-be-measured signal. It suppresses the chaos by adding a periodic perturbation to the parameters of chaotic system. Lima and Pettini[17] used the method to suppress the chaotic motion of the Duffing oscillator successfully. They added a small periodic perturbation item, and we take it as a chaotic detection system model of periodic short-impulse signals as follows:

$$\ddot{x} + k\dot{x} - x + [1 + \alpha\delta_{\mathrm{T}}(t)]\, x^3 = \gamma\cos(\omega t)\,, \qquad (1)$$

where k is the damping ratio, $\alpha\delta_{\mathrm{T}}(t)$ is the periodic short-impulse to-be-measured signal, and $\gamma\cos(\omega t)$ is built-in signal of the system.

The Melnikov analysis and the calculations of the maximum Lyapunov exponent show that the map has chaotic motion in the sense of Smale horseshoe. When k is given, the threshold value is

$$\gamma_c = \gamma/k = \left|\sqrt{2}\left(3\pi^2 + 16\omega^2\right)^{3/2}/256\omega\pi\right|. \qquad (2)$$

When $\gamma > \gamma_{\mathrm{c}}$, the system enters into a chaotic state; until when $\gamma > \gamma_{\mathrm{d}}$, the system enters into a large periodic motion state.

When detecting weak signals, first we should adjust the amplitude of the built-in signal from γ to critical value γ_{d}, and then add the to-be-measured periodic short-impulse signal to system. The to-be-measured signal perturbs nonlinear term x^3, and it will make the system change essentially from chaotic critical state to large-scale periodic state. The change is significant and easy to detect in that it is chosen as the reference to detect signals.

Next, we conduct a simulation experiment on periodic short-impulse signals, and the state equation of the system above is

$$\begin{cases} \dot{x} = y, \\ \dot{y} = x - [1 + \alpha\delta_{\mathrm{T}}(t) + z]\, x^3 - ky + \gamma\cos(\omega t)\,. \end{cases} \qquad (3)$$

Let the angular frequency of built-in signal $\omega = 1$ rad/s, $k = 0.5$, z is Gaussian white noise, $\delta_{\mathrm{T}}(t)$ is impulse to-be-measured signal whose period is 0.1 s and duty cycle is 0.1.

To start with, we search for the critical value γ_{d} of the system. Let the amplitude of the to-be-measured signal $\alpha = 0$, noise power be equal to 0. And we find that when the amplitude of built-in signal $\gamma = 0.82635$, the system is in a critical chaotic-state. The phase diagram of the system is shown in Fig. 1.

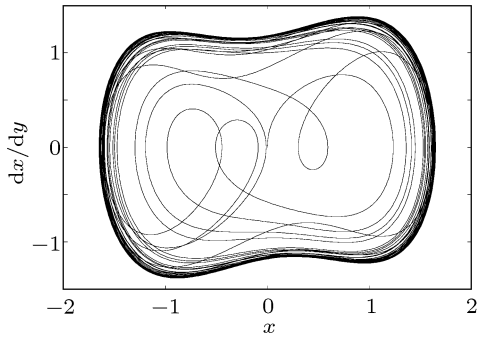


Fig. 1. Critical state phase diagram of system.

Then we just add periodic short-impulse signals to the system and adjust the signal amplitude. When the amplitude of the to-be-measured signal reaches 10^4, the system will change from critical state to large-scale periodic state (see Fig. 2). The change of system phase orbit signifies that the periodic short-impulse signal is detected.

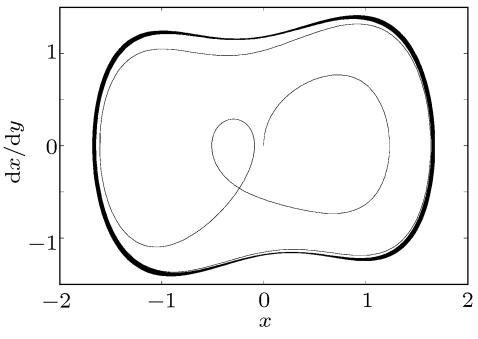


Fig. 2. Large-scale state phase diagram of system.

But in practice, weak signals are always immersed in noises. The disturbance of noise will result in extremely unstable dynamic behaviour of chaotic oscillator, and further more, leads to a great reduction of the detecting effect. Figure 3 shows the detection result of periodic short-impulse signals whose amplitude is 10^4, and the detection result is under the condition of Gaussian white noise whose power is 10^{-6} W

(SNR=−23 db, where SNR stands for signal-to-noise ratio).

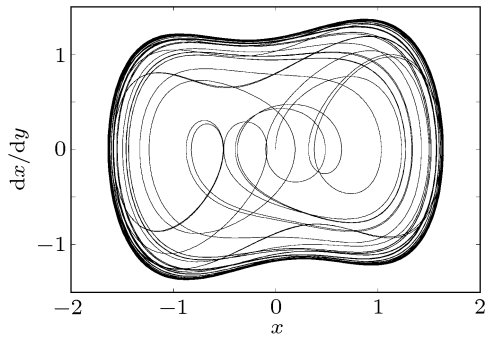


Fig. 3. Phase diagram of system.

We can see from the experiment above that because of the disturbance of noise, the large-scale state does not appear as we expected and the system still maintains chaotic state. As a result, it fails to detect the weak noise as detected before. We can see the failure percentage is 72% after 100-fold simulation experiments. The fact is that when the signal is weak, the randomness of noise has a serious or even a destructive effect on the effectiveness of detection results. It proves the faulty of the existing statements of 'chaotic oscillator detection system can fully guarantee the immunity to noise' and 'random noise influence on detection system can be ignored' which are based on the previously available knowledge.

3. Chaotic detection based on wavelet threshold method

The cases above are ignored in the previously documented work. Obviously, noise disturbance may not be ignored any more. Consequently, we consider the application of wavelet decomposition to the signal mixed with noise by the wavelet threshold method. When we set an appropriate threshold-function value to be the wavelet coefficient, the reconstructed signal will sufficiently approach the to-be-measured weak signal considering mean square deviation. Consequently this method reduces the influence of noise, further more, improves the stability and reliability of the corresponding chaotic oscillator to detect weak signals accompanied with noise. In the following paragraphs, we will show a new detecting method based on wavelet threshold processing.

3.1. Threshold estimation

Firstly, we need to estimate the signals mixed with noises, that is, filter the noise influence from the disturbed signals as much as possible. The structure diagram is shown in Fig. 4.

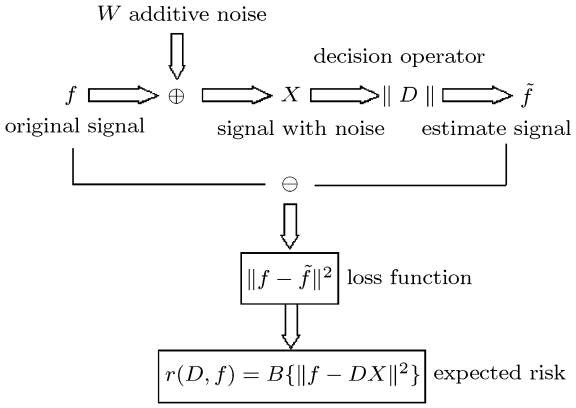


Fig. 4. Structure diagram of estimating signals accompanied with noise.

It can be described as

$$\inf_{D\in O_N} E\left\{\|f-DX\|^2\right\}$$
$$\text{s.t.} X[n]=f[n]+W[n], n=0,1,\ldots,N-1, \quad (4)$$

where O_N is a set of operators from C^N to C^N, $f[n]$ is the original signal, $W[n]$ is the Gaussian white noise whose mean square deviation is δ^2, and $X[n]$ is the mixed signal. Our aim is to find out a decision operator D, which makes the expected risk between estimation result DX and original signal f minimal or as least as possible.

However, operator set is quite large. If the preknowledge of corresponding signals and noise is lacking, the optimization of decision operator D is hard to realize exactly. But aiming at specific problem in a certain range, we can narrow the range of decision operator D to reduce the complexity. We consider a special nonlinear operator $O_{\rm d}$, which is diagonal under an orthonormal basis B of C^N. If set B is expressed as a sparse signal, the diagonal is almost optimal in all the nonlinear operators.[18]

The signal with noise $X=f+W$ can be decomposed under basis B as follows:

$$X=\sum_{m=0}^{N-1}\langle X,g_m\rangle g_m. \quad (5)$$

Set $X_{\rm B}[m]=\langle X,g_m\rangle$, $f_{\rm B}[m]=\langle f,g_m\rangle$, and $W_{\rm B}[m]=\langle W,g_m\rangle$, so $X_{\rm B}[n]=f_{\rm B}[n]+W_{\rm B}[n]$.

$D \in O_{\mathrm{d}}$, so under basis B, the mixed signals is estimated to be

$$DX = \sum_{m=0}^{N-1} d_m \left(X_{\mathrm{B}} [m] \right) g_m. \tag{6}$$

For $d_m(0) = 0$, we set $d_m \left(X_{\mathrm{B}} [m] \right) = a[m] X_{\mathrm{B}} [m]$. In the function, $a[m]$ depends on $X_{\mathrm{B}} [m]$, otherwise, diagonal operator D is a linear operator.

We need to find out an $a[m]$ which makes the estimation risk $r(D, f)$ minimal. We have

$$\begin{aligned} r(D, f) &= E\left\{ \|f - DX\|^2 \right\} \\ &= \sum_{m=0}^{N-1} E\left\{ |f_{\mathrm{B}} [m] - a[m] X_{\mathrm{B}} [m]|^2 \right\}. \end{aligned} \tag{7}$$

For $X_{\mathrm{B}} = f_{\mathrm{B}} + W_{\mathrm{B}}$ and $E\left\{ |W_{\mathrm{B}} [m]|^2 \right\} = \delta^2$, we have

$$\begin{aligned} &E\left\{ |f_{\mathrm{B}} [m] - a[m] X_{\mathrm{B}} [m]|^2 \right\} \\ &= |f_{\mathrm{B}} [m]|^2 (1 - a[m])^2 + \delta^2 a[m]^2. \end{aligned} \tag{8}$$

When

$$a[m] = \frac{|f_{\mathrm{B}} [m]|^2}{|f_{\mathrm{B}} [m]|^2 + \delta^2},$$

the risk is minimum. In this case,

$$\begin{aligned} r_{\inf}(D, f) &= E\left\{ \|f - DX\|^2 \right\} \\ &= \sum_{m=0}^{N-1} \frac{|f_{\mathrm{B}} [m]|^2}{|f_{\mathrm{B}} [m]|^2 + \delta^2}. \end{aligned} \tag{9}$$

In practice, because $a[m]$ depends on $|f_{\mathrm{B}} [m]|$ and it is unknown, $a[m]$ cannot be calculated. As a result, risk $r_{\inf}$ is a lower bound that is very hard to obtain by calculating. However, if we use an appropriate method, the difficulty above is avoidable in a practically feasible range.

When d_m is a threshold function, we have

$$d_m(x) = \begin{cases} x - T, & \text{if } x \geq T, \\ x + T, & \text{if } x \leq -T, \\ 0, & \text{if } |x| \leq T, \end{cases} \tag{10}$$

The theorem[19] below proves that choosing an appropriate T, the risk of threshold estimation will be close to $r_{\inf}$.

Theorem Set $T = \delta\sqrt{2 \log_{\mathrm{e}} N}$, for all the$N \geq 4$, the risk of threshold estimation r_t satisfies

$$r_t \leq (2 \log_{\mathrm{e}} N + 1) \left(\delta^2 + r_{\inf} \right). \tag{11}$$

In all the diagonal operator estimation of B, factor $2 \log_{\mathrm{e}} N$ is optimized as

$$\lim_{N \to \infty} \inf_{D \in O_{\mathrm{d}}} \sup_{f \in C^N} \frac{E\left\{ \|f - DX\|^2 \right\}}{\delta^2 + r_{\inf}} \frac{1}{2 \log_{\mathrm{e}} N} = 1. \tag{12}$$

3.2. Wavelet threshold estimation

Next, we combine the wavelet theory and the threshold estimation method together, to form a new mixed detection system for the chaotic oscillator detection of weak signals accompanied with noises. Normalize the support set of continuous signal $f(t)$ in the interval $[0, 1]$, and then its discrete sampling signal $f = (f[n])_{n=0,\ldots,N-1}$ is

$$f[n] = f(n/N) = 0, \ldots, N - 1. \tag{13}$$

Considering discrete wavelet function ψ in the interval $[0, 1]$ and the corresponding scaling function is ϕ. Because discrete signal $f = (f[n])_{n=0,\ldots,N-1}$ is the sample of continuous signal $f(t)$ by internal $N^{-1} = 2^L$, we can relate it with the approximation of continuous signal $f(t)$ on the scale-space V_L. Then the scale parameter of wavelet function 2^j changes from $2^{L+1} = 2N^{-1}$ to $2^J < 1$. In addition, we can prove

$$B = \left\{ \{\phi_{J,m}\}_{0 \leq m < 2^{-J}} \{\psi_{j,m}\}_{L < j \leq J, 0 \leq m < 2^{-j}} \right\} \tag{14}$$

to be a set of discrete wavelet basis on C^N. So the threshold estimation decision operator under the wavelet basis is

$$\begin{aligned} DX = &\sum_{j=L+1}^{J} \sum_{m=0}^{2^{-j}} \rho_{\mathrm{T}} \left(\langle X, \psi_{j,m} \rangle \right) \psi_{j,m} \\ &+ \sum_{m=0}^{2^{-J}} \rho_{\mathrm{T}} \left(\langle X, \phi_{J,m} \rangle \right) \phi_{J,m}, \end{aligned} \tag{15}$$

where ρ_{T} is threshold function defined above.

Then we combine impulse signals whose period is 0.1 s and duty cycle is 0.1 with Gaussian white noise whose chaos power is 10^{-6} W together. Then we apply the wavelet threshold processing. The experimental result is shown in Fig. 5.

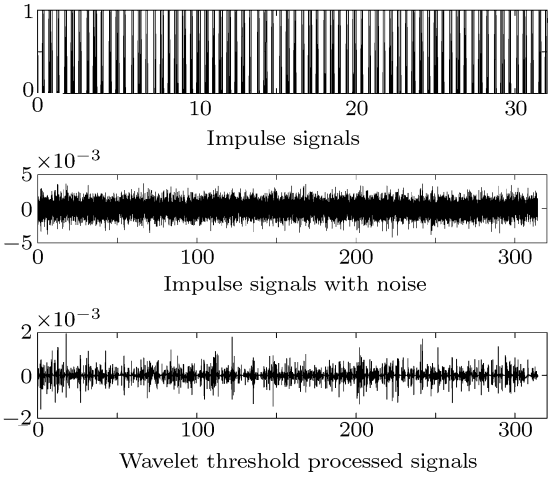


Fig. 5. Effect diagram of wavelet threshold estimation.

Finally, we combine the wavelet processing, the threshold estimating and the chaotic oscillator together, to form a new mixed detection system. Figure 6 shows the basic principle of the chaotic detection of weak signal accompanied with noises which is on the basis of wavelet threshold processing.

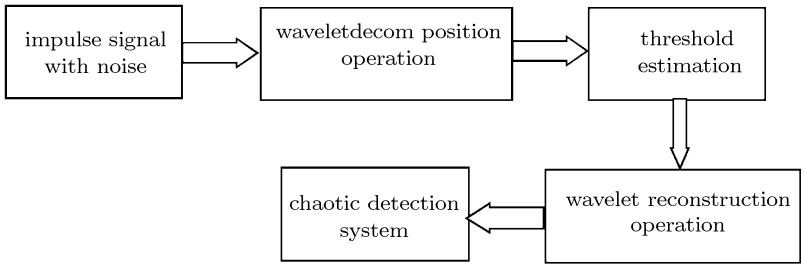


Fig. 6. Basic principle of the chaotic detection of weak signal accompanied with noise.

In the above discussion, we have proved in theory that after wavelet threshold processing of signals mixed with noises, it mostly approximates the weak signals in the sense of mean square deviation, and the influence of noise reduces to a least extent. Next, we input mixed signals which have been wavelet threshold processed into a chaotic oscillator. Comparing the new mixed detection system with the traditional detection system by simulating experiments, we validate the detecting effect of the new mixed detection system on periodic short-impulse signals accompanied with noise.

4. Simulation and analysis

Figure 7 shows the comparison of experiment results between the traditional detection system and the new mixed detection system on condition that the two systems have the same periodic short-impulse signal amplitude (α= 10^4) and the same Gaussian white noise power (10^{-6} W).

The left column shows the time-domain oscillogram and phase diagram of traditional detection system, we can see that there should be a large-scale periodic state in the case of no noises, but it does not appear. The system is still in chaotic state. As a result, it fails to detect the weak periodic short-impulse signal whose amplitude is α= 10^4, the SNR is -23 db.

The right column reflects the detecting result of new detection system. We can see from the time-domain oscillogram that the system has entered into a large-scale periodic state. But the phase diagram is different from the conventional phase diagram of large-scale periodic state (because of the limited space of the present paper, the analyses of the causes and the effects caused by different wavelet thresholds on the wavelet filtering state between chaotic and large-scale periods, will be discussed in the next paper). Technically, we refer to the state as 'sub-level large-scale periodic state derived from wavelet', or for short 'sub-cycled state'. The state is observably different from the corresponding 'sub-layer chaotic state that originates from wavelet' phase diagram. We can take the changes as a reference of signal detection. In fact, we can see from the time-domain oscillogram and the phase diagram in the right column that the new mixed detection system can successfully detecte periodic short-impulse signals which could not be detected in the traditional way. We find that the success rate of detecting weak signals accompanied with noise by using the new mixed detection system reaches 87% by 100-fold simulation experiments, which proves that the new mixed detection system obviously exceeds the traditional chaotic oscillator detection system in sensitivity.

In fact, if we fully consider the periodicity of signal and the randomness of noise, we can preprocess the to-be-measured signal with several methods, such as cross-correlation, synchronization-accumulation or sample-integral. The purpose is to restrain noise and increase SNR. We conbine the time-domain signal processing which is an effective conventional method and the chaos detection together, then we can constitute a detection system with a better performance.

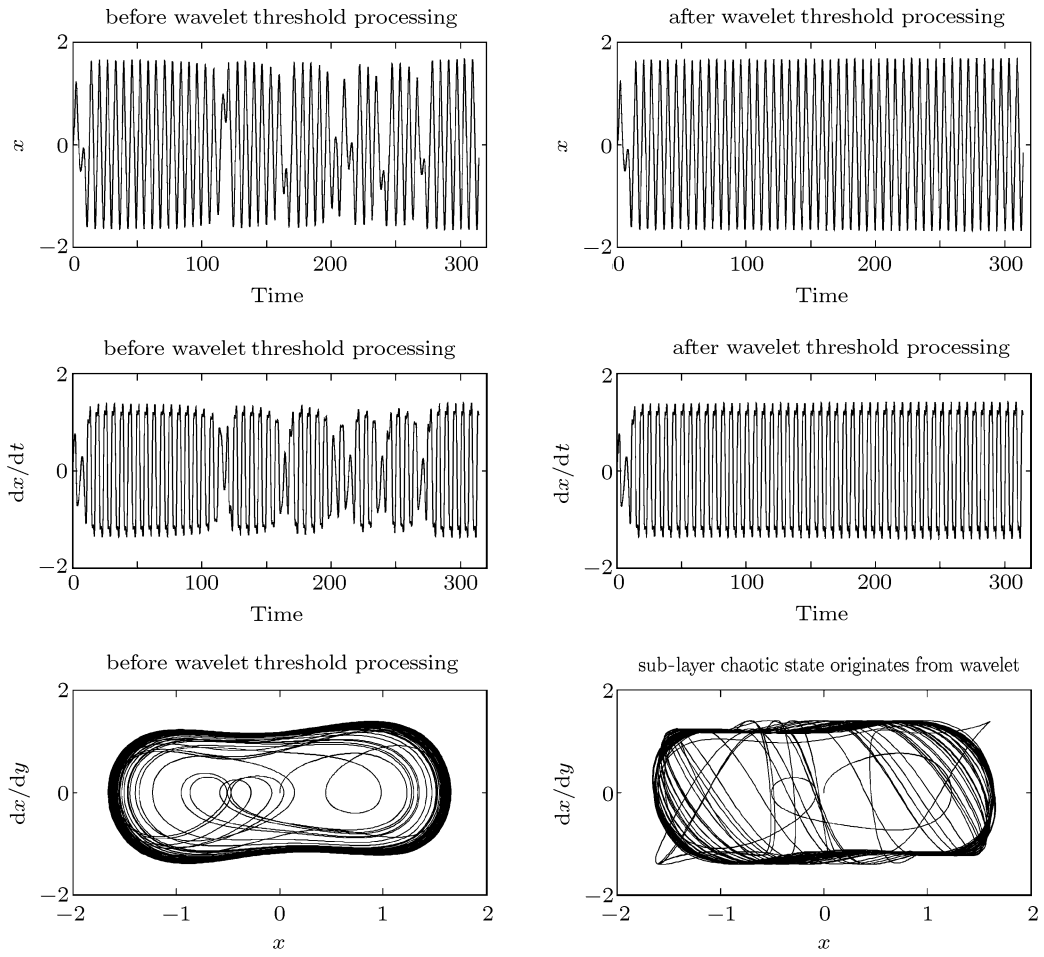


Fig. 7. Comparison of experimental results between two detection systems.

5. Conclusions

The way of chaotic detection of weak signals accompanied with noise has been widely accepted to detect signals in theory. However, some successful attributes in theory might be unsatisfactory in practice even only in computer simulation. That is because in theoretical analysis, all hypotheses are ideal, but the chaotic system is very sensitive to any changes of non-ideal actual conditions. Furthermore, in actual conditions, the changes of phase are not a threshold or an interval whose width can be ignored in general understanding or imagination. It will change with specific conditions, and we cannot ignore these changes. As a result of these changes, it is difficult to judge the intermediate state, then hard to judge the detected results. To solve the problems, we indicate a solution based on the basic of wavelet, and put forward a new method to form a mixed detection system for detecting weak signals accompanied with noises. With the new method, the serious even destructive interference by random noise is observably reduced. Furthermore, it improves the stability and the reliability of chaotic oscillator of detecting weak signals accompanied with noise. All theoretic analyses and computer simulations prove the feasibility of the new method of detecting weak signals accompanied with noises.

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