Distributed quantum information processing via quantum dot spins*

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We propose a scheme to engineer a non-local two-qubit phase gate between two remote quantum-dot spins. Along with one-qubit local operations, one can in principal perform various types of distributed quantum information processing. The scheme employs a photon with linearly polarisation interacting one after the other with two remote quantum-dot spins in cavities. Due to the optical spin selection rule, the photon obtains a Faraday rotation after the interaction process. By measuring the polarisation of the final output photon, a non-local two-qubit phase gate between the two remote quantum-dot spins is constituted. Our scheme may has very important applications in the distributed quantum information processing.

Keywords: quantum-dot spins, nonlocal quantum gates, polarised photon
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1. Introduction

In recent years, there has been intense interest in the study of electron spins trapped in semiconductor quantum dots(QD) due to the potential applications in quantum computation and quantum information processing.[1,2] The well-defined two states of spin up and down of an electron spin in a magnetic field form a qubit which has a long relaxation[3,4] and dephasing[5,6] time. The single-qubit rotation (i.e. coherent transition between the two Zeeman split levels of spin up and down of an electron) may be realised by the approaches of electron spin resonance(ESR)[7] or optical excitation.[8,9] Exchange coupling between QD spins provides the physical realization of two-qubit gates.[10,5] Readout of spin states may be done by the so-called spin-to-charge conversion scheme.[11,12] We may say that, up to now, most parts of Di Vincenzo's criteria[13] for constructing a quantum computer in spin-QD systems have been demonstrated at the proof-of-principle level.

Another important research domain in quantum information science is the quantum communication. In the well developed quantum communication tasks, including teleportation,[14,15] dense coding,[16−18] quantum cryptography,[19] remote quantum-state preparation[20,21] and quantum remote control,[22] etc, non-local entanglement is the essential ingredient. Thus, the establishment of nonlocal entanglement is very important work. In the process of creating entanglement between remote spins, photons are usually used as the media[23−27] for communicating quantum information duo to their fast speed and little decoherence in travels.

In a sense, the engineering of universal non-local logic gates between remote qubits is more important than the creation of non-local entanglement. Because, the universal non-local operations can be used to not only create various types of non-local entanglement including cluster states,[28] but also to directly fulfill various types of non-local quantum information processing, including the quantum communication tasks listed above and distributed quantum computation,[29] without using pre-prepared entanglement. The direct coupling between remote spins can even produce a better threshold for fault tolerant quantum computation.[30] Motivated by these ideas, we thus in this paper propose a theoretical method to produce a non-local phase gate between remote spins. For example, we highlight its applications in the creation

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of distributed cluster states and distributed quantum computation.

2. Interaction model

Our interaction model is mainly based on the idea[23] of giant Faraday rotation. A charged QD with only one excess electron is placed into the centre of a micro-pillar optical resonant cavity where the distributed Bragg reflectors and transverse index guiding provide the three-dimensional confinement of light (see Fig. 1(a) of Ref. [23]). Optical excitation can create a negatively charged exciton X^- that consists of two electrons bound to a hole. Due to the spin selection rule in the transition process, the interaction between light and X^- not only relies on the polarisation of light but also on the initial spin state of the excess electron. If the excess electron is initially spin up $|\uparrow\rangle$, only the left-handed circularly polarised light (L-light) can be resonantly absorbed to create X^- in state $|\uparrow\downarrow\Uparrow\rangle$ with the two electron spins antiparallel and the hole in spin state $|\Uparrow\rangle = |3/2\rangle$. While the right-handed circularly polarised light (R-light) does not interact with the X^- and only experiences an effect of vacuum cavity, as if the QD does not exist there. In this case, we thus say that the L-light feels the effect of a hot cavity and the R-light feels the effect of a cold cavity (see Fig. 1(b) of Ref. [23]). Contrarily, if the excess electron is initially spin down $|\downarrow\rangle$, then the R-light feels the effect of a hot cavity and creates the X^- in the state $|\uparrow\downarrow\Downarrow\rangle$ with the hole in the state $|\Downarrow\rangle = |-3/2\rangle$. While the L-light feels the effect of a cold cavity.

If we let a beam of probe light with frequency ω inject into the cavity, and assume a large cavity decay to make sure that the X^-, which is initially prepared in ground state, only be excited weakly so that $\langle\sigma_z\rangle \simeq -1$, we can then obtain the amplitude-reflection coefficient of the light,[23]

$$r(\omega) = 1 - \frac{\kappa\left[\mathrm{i}(\omega_{\mathrm{X}}-\omega)+\frac{\gamma}{2}\right]}{\left[\mathrm{i}(\omega_{\mathrm{X}}-\omega)+\frac{\gamma}{2}\right]\left[\mathrm{i}(\omega_{\mathrm{c}}-\omega)+\frac{\kappa}{2}+\frac{\kappa_{\mathrm{s}}}{2}\right]+g^2}, \tag{1}$$

where ω_{c} and ω_{X} are the frequencies of cavity mode and X^- transition respectively. g is the coupling strength between X^- and the cavity mode. $\gamma/2$ is the X^- dipole decay rate, $\kappa/2$ is the decay rate of cavity field into the input/output modes, and $\kappa_{\mathrm{s}}/2$ is the side leakage rate of the cavity. The cavity is assumed to be single-sided with back mirror completely reflective and front mirror partially reflective, the side leakage is very small compared to the main cavity decay, i.e., $\kappa_{\mathrm{s}}/2 \ll \kappa/2$. For cold cavity, the QD uncouples to the cavity mode, $g = 0$, thus the reflection coefficient becomes,

$$r_0(\omega) = \frac{\mathrm{i}(\omega_{\mathrm{c}}-\omega)-\frac{\kappa}{2}+\frac{\kappa_{\mathrm{s}}}{2}}{\mathrm{i}(\omega_{\mathrm{c}}-\omega)+\frac{\kappa}{2}+\frac{\kappa_{\mathrm{s}}}{2}}. \tag{2}$$

Similar input-output formulas for atom–cavity system have been derived.[31] The complex reflection coefficients indicate that the reflected light not only reduces in amplitude, but also obtains a phase shift with respect to the corresponding input light. When the excess electron initially lies in spin up, the L-light feels a hot cavity and gets a phase shift of φ_{h} after reflection, whereas the R-light feels a cold cavity and gets a phase shift of φ_0. The corresponding change can be described as,

$$\begin{aligned} |L\rangle|\uparrow\rangle &\to r_{\mathrm{h}}\,\mathrm{e}^{\mathrm{i}\varphi_{\mathrm{h}}}|L\rangle|\uparrow\rangle, \\ |R\rangle|\uparrow\rangle &\to r_0\,\mathrm{e}^{\mathrm{i}\varphi_0}|R\rangle|\uparrow\rangle, \end{aligned} \tag{3}$$

where $r_0 = |r_0(\omega)|$ and $\varphi_0 = \arg[r_0(\omega)]$ denote the reflection ratio and phase shift of the reflected light for cold cavity, and $r_{\mathrm{h}} = |r(\omega)|$ and $\varphi_{\mathrm{h}} = \arg[r(\omega)]$ for hot cavity respectively. Contrarily, when the excess electron initially lies in spin down, the L-light feels a cold cavity and gets a phase shift of φ_0 after reflection, whereas the R-light feels a hot cavity and gets a phase shift of φ_{h}. Thus the corresponding change can be described as,

$$\begin{aligned} |L\rangle|\downarrow\rangle &\to r_0\,\mathrm{e}^{\mathrm{i}\varphi_0}|L\rangle|\downarrow\rangle, \\ |R\rangle|\downarrow\rangle &\to r_{\mathrm{h}}\,\mathrm{e}^{\mathrm{i}\varphi_{\mathrm{h}}}|R\rangle|\downarrow\rangle. \end{aligned} \tag{4}$$

These spin-states dependent on the change of reflected light can also be described by a reflection operator,

$$\begin{aligned} \hat{r}(\omega) = {} & r_0\,\mathrm{e}^{\mathrm{i}\varphi_0}(|R\rangle\langle R|\otimes|\uparrow\rangle\langle\uparrow|+|L\rangle\langle L|\otimes|\downarrow\rangle\langle\downarrow|) \\ & + r_{\mathrm{h}}\,\mathrm{e}^{\mathrm{i}\varphi_{\mathrm{h}}}(|L\rangle\langle L|\otimes|\uparrow\rangle\langle\uparrow|+|R\rangle\langle R|\otimes|\downarrow\rangle\langle\downarrow|). \end{aligned} \tag{5}$$

In the ideal case where the cavity side leakage can be neglected ($\kappa_{\mathrm{s}} = 0$), thus the reflectance for the cold cavity is unity ($r_0 = 1$). For hot cavity, in the strong-coupling limit ($g \gg \kappa, \gamma$) and near the resonant regime ($\omega_{\mathrm{X}} = \omega_{\mathrm{c}}$, $|\omega - \omega_{\mathrm{c}}| \ll g$), we also have $r_{\mathrm{h}} \simeq 1$. Thus for a linearly polarised probe light which is the equally superposition of two circularly polarised components, $(|R\rangle + |L\rangle)/2$, after

reflection by the cavity, the output light becomes $(\mathrm{e}^{\mathrm{i}\varphi_0}|R\rangle+\mathrm{e}^{\mathrm{i}\varphi_\mathrm{h}}|L\rangle)/2$ for the excess electron with spin up, or becomes $(\mathrm{e}^{\mathrm{i}\varphi_\mathrm{h}}|R\rangle+\mathrm{e}^{\mathrm{i}\varphi_0}|L\rangle)/2$ for the excess electron with spin down. A so-called Faraday rotation angle $\theta_\mathrm{F}^{\uparrow}=(\varphi_0-\varphi_\mathrm{h})/2$ or $\theta_\mathrm{F}^{\downarrow}=(\varphi_\mathrm{h}-\varphi_0)/2=-\theta_\mathrm{F}^{\uparrow}$ for the linearly polarised reflected light occurs. The reflection operator of Eq. (5) for this ideal case, up to a total phase, reduces to a phase-shift operator,

$$\hat{U}(\varphi)=\mathrm{e}^{\mathrm{i}\varphi(|L\rangle\langle L|\otimes|\uparrow\rangle\langle\uparrow|+|R\rangle\langle R|\otimes|\downarrow\rangle\langle\downarrow|)} \tag{6}$$

with $\varphi=\varphi_\mathrm{h}-\varphi_0$.

3. Nonlocal two-spin logic gates

The above principle of Faraday rotation can be used to engineer universal and nonlocal two-spin logic gates. Consider two spatially separated QD-cavity systems (see Fig. 3(a) of Ref. [23], each QD with a single excess electron. Let a single photon with linearly polarisation inject first into cavity one and then into cavity two where the photon interacts with QD one and QD two respectively. The final output photon is measured in two orthogonal polarisations. In the ideal case ($r_0=r_\mathrm{h}=1$), we can use phase-shift operators $\hat{U}(\varphi_1)$ and $\hat{U}(\varphi_2)$ to describe the evolution of the photon interacting with the two QD-cavity systems respectively. Assume that the excess electrons within the two QDs are initially in arbitrary spin states $|\psi_1\rangle$ and $|\psi_2\rangle$ respectively, then the state evolution for the photon and two QD electrons can be written as

$$\begin{aligned}|\Psi_\mathrm{out}\rangle &= \hat{U}(\varphi_2)\hat{U}(\varphi_1)\frac{1}{\sqrt{2}}(|R\rangle+|L\rangle)|\psi_1\rangle|\psi_2\rangle\\ &= \frac{1}{\sqrt{2}}[\mathrm{e}^{\mathrm{i}\varphi_2|\downarrow\rangle_{22}\langle\downarrow|}\mathrm{e}^{\mathrm{i}\varphi_1|\downarrow\rangle_{11}\langle\downarrow|}|R\rangle\\ &\quad+\mathrm{e}^{\mathrm{i}\varphi_2|\uparrow\rangle_{22}\langle\uparrow|}\mathrm{e}^{\mathrm{i}\varphi_1|\uparrow\rangle_{11}\langle\uparrow|}|L\rangle]|\psi_1\rangle|\psi_2\rangle,\end{aligned} \tag{7}$$

where $\{|\uparrow\rangle_1,|\downarrow\rangle_1\}$ and $\{|\uparrow\rangle_2,|\downarrow\rangle_2\}$ are the spin states of the two electrons in QD one and two respectively. By adjusting the frequency detuning to $\omega-\omega_\mathrm{c}\simeq\kappa/2$, we can make[23] $\varphi_1=\varphi_2=\pi/2$. Thus the above equation can be rewritten as

$$\begin{aligned}|\Psi_\mathrm{out}\rangle &= \left\{\frac{1}{2}\mathrm{e}^{\mathrm{i}\pi/4}(|R\rangle+\mathrm{i}|L\rangle)\otimes[|\uparrow\rangle_{11}\langle\uparrow|\otimes I^{(2)}\right.\\ &\quad+|\downarrow\rangle_{11}\langle\downarrow|\otimes\sigma_z^{(2)}]\\ &\quad+\frac{1}{2}\mathrm{e}^{-\mathrm{i}\pi/4}(|R\rangle-\mathrm{i}|L\rangle)\otimes\Big[|\uparrow\rangle_{11}\langle\uparrow|\otimes\sigma_z^{(2)}\\ &\quad\left.-|\downarrow\rangle_{11}\langle\downarrow|\otimes I^{(2)}\Big]\right\}|\psi_1\rangle|\psi_2\rangle,\end{aligned} \tag{8}$$

where $I^{(2)}$ and $\sigma_z^{(2)}=|\uparrow\rangle_{22}\langle\uparrow|-|\downarrow\rangle_{22}\langle\downarrow|$ are the identity and usual Pauli operators respectively for the electron spin in the second QD.

Now, we measure the polarisation of the final output photon from cavity two in the two orthogonal basis $|+45^\circ\rangle=(|R\rangle+\mathrm{i}|L\rangle)/\sqrt{2}$ and $|-45^\circ\rangle=(|R\rangle-\mathrm{i}|L\rangle)/\sqrt{2}$. With probability of 1/2, equation (8) collapses to $[|\uparrow\rangle_{11}\langle\uparrow|\otimes I^{(2)}+|\downarrow\rangle_{11}\langle\downarrow|\otimes\sigma_z^{(2)}]|\psi_1\rangle|\psi_2\rangle$. Thus, a nonlocal two-qubit phase gate,

$$P=|\uparrow\rangle_{11}\langle\uparrow|\otimes I^{(2)}+|\downarrow\rangle_{11}\langle\downarrow|\otimes\sigma_z^{(2)} \tag{9}$$

for the two spatially separated electron spins is obtained. Alternatively, Eq. (8) also with probability of 1/2 collapses to $[|\uparrow\rangle_{11}\langle\uparrow|\otimes\sigma_z^{(2)}-|\downarrow\rangle_{11}\langle\downarrow|\otimes I^{(2)}]\}|\psi_1\rangle|\psi_2\rangle$ and another nonlocal two-spin phase gate,

$$P'=|\uparrow\rangle_{11}\langle\uparrow|\otimes\sigma_z^{(2)}-|\downarrow\rangle_{11}\langle\downarrow|\otimes I^{(2)} \tag{10}$$

is obtained. Noting relation $P'=\sigma_z^{(1)}\sigma_z^{(2)}P$, we can transfer gate P' into P [If we obtain P', we apply local gates $\sigma_z^{(1)}$ and $\sigma_z^{(2)}$ to spins one and two respectively, then P' transfers into P]. Therefore, no matter what polarisations ($|+45^\circ\rangle$ or $|-45^\circ\rangle$) be measured for the final output photon, we can always obtain the nonlocal two-spin phase gate (9).

Based on the nonlocal two-spin phase gate, we can further construct the well-known CN and SWAP gates between two remote spins. In fact, the CN gate is equivalent to the two-qubit phase gate. We only need to append two local Hadamard transformations on the second spin of the phase gate (9), then a nonlocal two-spin CN gate is gained, i.e.,

$$\mathrm{CN}_{12}=H_2PH_2, \tag{11}$$

where $H_2=[|\uparrow\rangle_{22}\langle\uparrow|+|\uparrow\rangle_{22}\langle\downarrow|+|\downarrow\rangle_{22}\langle\uparrow|-|\downarrow\rangle_{22}\langle\downarrow|]/\sqrt{2}$ is the Hadamard transformation performed on the second spin. The subscript $'12'$ in CN_{12} denotes that the first spin is control and the second one is target qubits. Finally, the nonlocal two-spin SWAP gate can be constructed principally in terms of three CN gates,

$$\mathrm{SWAP}=\mathrm{CN}_{12}\mathrm{CN}_{21}\mathrm{CN}_{12}. \tag{12}$$

In the above discussions, we have assumed the ideal condition, $r_0=r_\mathrm{h}=1$, which can be reached only in the strong-coupling regime ($g\gg\kappa,\gamma$) and without side leakage ($\kappa_\mathrm{s}=0$). In a real case, however, these conditions may not be satisfied completely. Thus r_0 and r_h will be little deviation from unity. This

leads to, instead of Eqs. (9) and (10), two real phase gates,

$$P_{\rm real} = |\uparrow\uparrow\rangle\langle\uparrow\uparrow| + \frac{r_0 r_{\rm h}(1+{\rm i})}{r_0^2+{\rm i}r_{\rm h}^2}[|\uparrow\downarrow\rangle\langle\uparrow\downarrow| + |\downarrow\uparrow\rangle\langle\downarrow\uparrow|] - \frac{{\rm i}r_0^2+r_{\rm h}^2}{r_0^2+{\rm i}r_{\rm h}^2}|\downarrow\downarrow\rangle\langle\downarrow\downarrow| \quad (13)$$

and

$$P'_{\rm real} = |\uparrow\uparrow\rangle\langle\uparrow\uparrow| + \frac{r_0 r_{\rm h}({\rm i}-1)}{r_0^2-{\rm i}r_{\rm h}^2}[|\uparrow\downarrow\rangle\langle\uparrow\downarrow| + |\downarrow\uparrow\rangle\langle\downarrow\uparrow|] + \frac{{\rm i}r_0^2-r_{\rm h}^2}{r_0^2-{\rm i}r_{\rm h}^2}|\downarrow\downarrow\rangle\langle\downarrow\downarrow|, \quad (14)$$

where the former and later spins denote the first and second spins mentioned above respectively. Obviously, in the ideal case with complete reflection ($r_0 = r_{\rm h} = 1$), these two equations reduce to P and P' respectively. In addition, we also find though the complete reflection is impossible, if the balanced reflection condition, $r_0 = r_{\rm h} \neq 1$, can be reached, the above equations also reduce to P and P' respectively. Such a balanced point exists when the side leakage is small (see Fig. 1). In real technical implementation, it is difficult to reach the complete reflection condition. Thus, this balanced point might provide a more practical application for the design of logic gates.

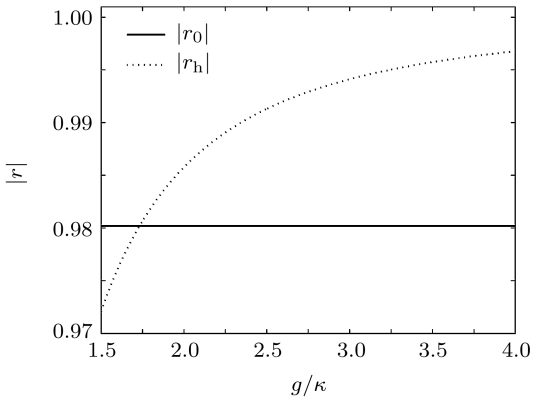


Fig. 1. The reflection rates r_0 for cold cavity and $r_{\rm h}$ for hot cavity vs the coupling strength for $\kappa_{\rm s} = 0.02\kappa$, $\omega - \omega_{\rm c} = \kappa/2$, $\omega_{\rm X} = \omega_{\rm c}$ and $\gamma/\kappa = 0.1$. The curves are cut off for $g < 1.5\kappa$, as $\varphi = \varphi_{\rm h} - \varphi_0 = \pi/2$ is not achievable in this regime.[23] It is shown that when $g/\kappa = 1.73$, the balanced refection condition $r_0 = r_{\rm h}$ is reached.

We may introduce the quantity $D(\hat{U}_{\rm real}, \hat{U}_{\rm ideal}) = \sum_{m,n} |(\hat{U}_{\rm real})_{mn} - (\hat{U}_{\rm ideal})_{mn}|$ to describe the error of a real operator $\hat{U}_{\rm real}$ from its ideal counterpart $\hat{U}_{\rm ideal}$. After a simply deduction, we find,

$$D(P_{\rm real}, P) = \frac{|r_0 - r_{\rm h}|}{\sqrt{r_0^4 + r_{\rm h}^4}}[2\sqrt{r_0^2 + r_{\rm h}^2} + \sqrt{2}(r_0 + r_{\rm h})], \quad (15)$$

which is a small quantity of the first order of $|r_0 - r_{\rm h}|$. For complete reflection or balanced reflection, the error vanishes and $P_{\rm real} = P$. Similar result is also valid for $D(P'_{\rm real}, P')$.

In addition, we may also use the average fidelity,[32] $\bar{F}(\hat{U}_{\rm real}, \hat{U}_{\rm ideal}) = \overline{|\langle\Psi|\hat{U}^+_{\rm real}\hat{U}_{\rm ideal}|\Psi\rangle|}$, to measure the error of a real operator $\hat{U}_{\rm real}$ from its ideal counterpart $\hat{U}_{\rm ideal}$. For a two-qubit system, the average is taken over all of the two-qubit pure states $|\Psi\rangle$. But for simplicity, we here use two-qubit product states, $|\Psi\rangle = \bigotimes_{j=1}^{2}[\cos(\theta_j/2)|\uparrow\rangle_j + {\rm e}^{{\rm i}\phi_j}\sin(\theta_j/2)|\downarrow\rangle_j]$, averaged over the individual Bloch spheres, to estimate the average fidelity between P and $P_{\rm real}$ of Eqs. (9) and (13). The numerical result is shown in Fig. 2, where we set $\omega - \omega_{\rm c} = \kappa/2$ and $\omega_{\rm X} = \omega_{\rm c}$ by considering the conditions for realising the phase gate, and $\gamma/\kappa = 0.1$ is assumed. It is shown that for small cavity-side leakage, say $\kappa_{\rm s} < 0.05\kappa$, the fidelity is close to unity.

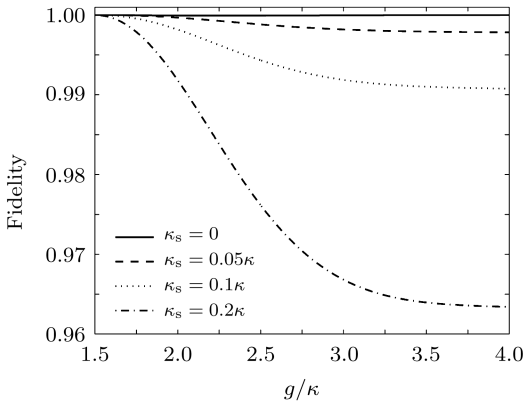


Fig. 2. The average fidelity between the real and ideal phase gates of Eqs. (13) and (9) vs the coupling strength of different cavity-side leakage ($\kappa_{\rm s} = 0$ solid, $\kappa_{\rm s} = 0.05\kappa$ dash, $\kappa_{\rm s} = 0.1\kappa$ dot, and $\kappa_{\rm s} = 0.2\kappa$ dash dot). We set parameters as $\omega - \omega_{\rm c} = \kappa/2$, $\omega_{\rm X} = \omega_{\rm c}$ and $\gamma/\kappa = 0.1$. It is shown that for small cavity-side leakage, say $\kappa_{\rm s} < 0.05\kappa$, the fidelity is close to unity.

It is worthwhile to point out that the success of the above nonlocal logic gates is based on the observation of the final output photon. In practical implementation, there are many factors to induce photon loss (including cavity mirror absorption and scattering, photon loss in the propagation, the inefficiency of detector, etc.). Thus the probability for creating above

nonlocal logic gates will be less than unity. The failure rate duo to X^- decay is about $\gamma/g \simeq 4\%$. Single-photon counting efficiency may be close to unity.[33] Assume the failure rate due to the cavity mirror absorption and scattering, as well as photon loss in the propagation, to be 6%, then the final successful probability for the non-local phase gate (9) is about $(1-4\%)^2 \times (1-6\%) = 86.63\%$, where the square is due to two QDs involved. The successful probability for the swap gate (12) is thus $(86.63\%)^3 = 65\%$.

4. Creation of distributed cluster states

The nonlocal two-spin logic gates gained in the previous section may have important applications in quantum information fields. Along with local one-spin gates, one can in principle perform arbitrary distributed quantum information processing. To highlight it, for example, we intend to illuminate the applications in the creation of distributed cluster states and distributed quantum computation in this and next sections respectively.

Cluster states are pure quantum states of two-level systems (qubits) located on a cluster $\mathcal{C}$ which have novel entanglement properties. Their property, creation and application have been investigated extensively in recent years.[28] It is convenient to use the notion of stabiliser for the definition of cluster states.[34] We consider a cluster $\mathcal{C}$ of spin qubits which forms a d-dimensional space lattice ($d = 1, 2,$ or 3). For any site a of the lattice, one defines a operator

$$K^{(a)} = \sigma_x^{(a)} \bigotimes_{b\in \mathrm{nghb}(a)} \sigma_z^{(b)}, \tag{16}$$

where nghb(a) is the set of all neighbouring lattice sites of a. The operators $\{K^{(a)}, a \in C\}$ form a complete family of commuting operators on the lattice. A cluster state is any of their common eigenstates

$$K^{(a)}|\phi_{\{\kappa\}}\rangle_C = (-1)^{\kappa_a}|\phi_{\{\kappa\}}\rangle_C, \tag{17}$$

where $\kappa_a \in \{0,1\}$ and $\{\kappa\} = \{\kappa_a | a \in C\}$ is a set of binary parameters that specify the cluster state. A cluster state $|\phi_{\{\kappa\}}\rangle_C$ is completely specified by the eigenvalue Eqs. (17) and all states are equally suitable for quantum computation. The cluster state associated with all eigenvalues being 1 can be written as (without being normalised)[35]

$$\bigotimes_{a\in C}(|\uparrow\rangle_a + |\downarrow\rangle_a \bigotimes_{\gamma\in\Gamma} \sigma_z^{(a+\gamma)}) \tag{18}$$

with the choice $\Gamma = \{1\}$ for $d = 1$, $\Gamma = \{(1,0),(0,1)\}$ for $d = 2$ and $\Gamma = \{(1,0,0),(0,1,0),(0,0,1)\}$ for $d = 3$.

The phase gate of Eq. (9) can conveniently be used to create cluster states for distributed spin lattice. For simplicity, we only discuss the creation of the cluster with the form of Eq. (18). The process can be divided into two steps. First, a spin-product state $|+\rangle_{\mathcal{C}} = \bigotimes_{a\in\mathcal{C}} |+\rangle_a$ is prepared, where $|+\rangle_a = (1/\sqrt{2})(|\uparrow\rangle_a + |\downarrow\rangle_a)$ is the eigenstate of σ_x for spin at site a. Secondly, the unitary transformation $S^{(\mathcal{C})} = \prod_{<a,b>} P^{ab}$ is applied to the state $|+\rangle_{\mathcal{C}}$. Where $P^{ab} = |\uparrow\rangle_{aa}\langle\uparrow| \otimes I^{(b)} + |\downarrow\rangle_{aa}\langle\downarrow| \otimes \sigma_z^{(b)}$ is the nonlocal phase gate of Eq. (9). And $< a, b >= \{a, b \in \mathcal{C} | b - a \in \Gamma\}$. After these two simple steps, a distributed spin cluster state with the form of Eq. (18) is created immediately.

Our method has many prominent virtues. First, it is independent of the concrete structure of the given spin lattice. It applies to the generation of various geometric structures of 1D, 2D and 3D spin-cluster states, which only needs to perform one time of the phase gate of Eq. (9) between all neighbouring spins. Secondly, the creation of the cluster state is a two-step process in time. All the phase operations P^{ab} in $S^{(\mathcal{C})}$ commute mutually and they can therefore be carried out simultaneously. The initial individual preparation of the spins in $|+\rangle_C$ can also be done in parallel. Of course, due to the photon loss in practical implementation, the nonlocal phase gate (9) is actually probabilistic. Thus the efficiency for creating cluster states will decrease rapidly as the size of cluster states increases.

5. Distributed quantum computation

Distributed quantum computation is the development of the notion of classical network computation. This model of network computation can sufficiently make use of the resource of hardware at different places. The non-local logic gates developed in section III can also be used to this kind of distributed quantum computation. Let us consider a quantum computation which can be divided into several subroutines so that each subroutine can be run on a separate quantum processer. These processers may be

placed at different nodes of a computational network and may exchange classical data or even quantum information with a selected central processer. When the processers finished their individual computations, they send the computational results (classical or quantum) to the central processer. Obviously, the non-local swap gate of (12) can realise the exchange or transfer of quantum information between individual and central processers. Distributed quantum computation is particularly fitted to the tasks of repetitive quantum computation, such as parameter estimation with a prescribed precision.[29,36,37]

In the distributed quantum computation, the inputs for the individual processers may be independent of each other, correlated or even entangled. It has already been shown that, for certain tasks, the use of entangled input states may be more advantageous than that of disentangled states.[29] The universal non-local logic gate of Eqs. (9) or (11) can be used expediently to create the required non-local entangled states for the inputs of a given distributed quantum computation. Alternatively, we can also use the non-local swap operation of Eq. (12) to create these non-local entangled states. The process is very simple and perspicuous. We can first let the central processer create a local entangled state. Then we use the swap operation of (12) to transfer the quantum information to individual processers. In this way, a non-local entangled states for the input of a given distributed quantum computation is created.

We can even engineer a so-called distributed lattice register (see Fig. 3) for quantum computers. The lattice register may have arbitrary size, i.e., consist of arbitrary number of qubits. Each qubit is formed by a QD spin inside an optical cavity. All the QD spins are separated for a moderate distance so that they are independent of each other. However, we can perform a universal logic operation between any a pair of spins by using non-local operations (9), (11) or (12). The advantage of this lattice register is that the number of qubits in a register may assemble to a very large level in principal, and one can perform any required logic operation directly between any two non-neighbouring qubits.

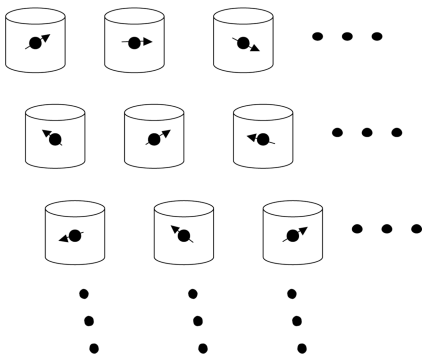


Fig. 3. Lattice register model for quantum computers. Each qubit is formed by a QD spin inside an optical cavity. The logic operation between any pair of spins can be realised through the transfer of a photon according to the scheme proposed in the text.

6. Conclusion and discussion

Based on the idea of giant Faraday rotation proposed by Hu *et al.*,[23] we have proposed a scheme for engineering a universal and non-local phase gate between two remote QD spins. We have also estimated the error of the constructed gate due to the side leakage of the cavity. Especially, there is a balanced reflection point at where the error of the gate from cavity-side leakage vanishes.

The obtained nonlocal logic gates have very important rules in quantum information science. Along with single spin operations, they enable us to perform various types of distributed quantum information processing. We have only demonstrated the applications in the creation of distributed cluster states and distributed quantum computation as an example. In fact, it can be used to finish any distributed quantum information processing finished by nonlocal entanglement, such as the transfer of quantum states, teleportation, remote quantum-state preparation and quantum remote control etc. The advantage of using nonlocal logic gates instead of nonlocal entanglement for distributed quantum information processing is that the decoherence from nonlocal-entangled quantum channels can be avoided.

References

[1] Loss D and Di Vincenzo D P 1998 *Phys. Rev.* A **57** 120

[2] Hanson R, Kouwenhoven L P, Petta J R, Tarucha S and Vandersypen L M K 2007 *Rev. Mod. Phys.* **79** 1217

[3] Elzerman J M, Hanson R, Willems van Beveren L H, Witkamp B, Vandersypen L M K and Kouwenhoven L P 2004 *Nature* **430** 431

[4] Fujisawa T, Austing D G, Tokura Y, Hirayama Y and

Tarucha S 2002 *Nature* **419** 278
[5] Petta J R, Johnson A C, Taylor J M, Laird E A, Yacoby A, Lukin D M, Marcus C M, Hanson M P and Gossard A C 2005 *Science* **309** 2180
[6] Laird E A, Petta J R, Johnson A C, Marcus C M, Yacoby A, Hanson M P and Gossard A C 2006 *Phys. Rev. Lett.* **97** 056801
[7] Koppens F H L, Buizert C, Tielrooij K J, Vink I T, Nowack K C, Meunier T, Kouwenhoven L P and Vandersypen L M K 2006 *Nature* **442** 766
[8] Imamoglu A, Awschalom D D, Burkard G, Di Vincenzo D P, Loss D, Sherwin M and Small A 1999 *Phys. Rev. Lett.* **83** 4204
[9] Gupta J A, Knobel R, Samarth N and Awschalom D D 2001 *Science* **292** 2458
[10] Burkard G, Loss D and Di Vincenzo D P 1999 *Phys. Rev.* B **59** 2070
[11] Engel H A, Golovach V N, Loss D, Vandersypen L M K, Elzerman J M, Hanson R and Kouwenhoven L P 2004 *Phys. Rev. Lett.* **93** 106804
[12] Hanson R, Willems van Beveren L H, Vink I T, Elzerman J M, Naber W J M, Koppens F H L, Kouwenhoven L P and Vandersypen L M K 2005 *Phys. Rev. Lett.* **94** 196802
[13] Di Vincenzo D P 2000 *Fortschr. Phys.* **48** 771
[14] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
[15] Li H C, Lin X and Yang R C 2007 *Chin. Phys.* **16** 624
[16] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881
[17] Cheng W W, Huang Y X, Li H and Liu T K 2007 *Chin. Phys.* **16** 38
[18] Cao S, Fang M F, Liao X P and Zheng X J 2008 *Chin. Phys.* B **17** 431
[19] Ekert A K 1991 *Phys. Rev. Lett.* **67** 661
[20] Bennett C H, Di Vincenzo D P, Shor P W, Smolin J A, Terhal B M and Wootters W K 2001 *Phys. Rev. Lett.* **87** 077902
Bennett C H, Di Vincenzo D P, Shor P W, Smolin J A, Terhal B M and Wootters W K 2002 *Phys. Rev. Lett.* **88** 099902(E)
[21] Berry D W and Sanders B C 2003 *Phys. Rev. Lett.* **90** 057901
[22] Huelga S F, Vaccaro J A, Chefles A and Plenio M B 2001 *Phys. Rev.* A **63** 042303
[23] Hu C Y, Young A, O'Brien J L, Munro W J and Rarity J G 2008 *Phys. Rev.* B **78** 085307
[24] Simon C, Niquet Y M, Caillet X, Eymery J, Poizat J P and Gérard J M 2007 *Phys. Rev.* B **75** 081302(R)
[25] Simon C and Irvine W T M 2003 *Phys. Rev. Lett.* **91** 110405
[26] Leuenberger M N, Flatté M E and Awschalom D D 2005 *Phys. Rev. Lett.* **94** 107401
[27] Childress L, Taylor J M, Sorensen A S and Lukin M D 2006 *Phys. Rev. Lett.* **96** 070504
Childress L, Taylor J M, Sorensen A S and Lukin M D 2005 *Phys. Rev.* A **72** 052330
[28] Hein M, Dür W, Eisert J, Raussendorf R, van Den Nest M and Briegel H J 2006 quant-ph/0602096
[29] Cirac J I, Ekert A K, Huelga S F and Macchiavello C 1999 *Phys. Rev.* A **59** 4249
[30] Svore K M, Terhal B M and Di Vincenzo D P 2005 *Phys. Rev.* A **72** 022317
[31] An J H, Feng M and Oh C H 2009 *Phys. Rev.* A **79** 032303
[32] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge, England: Cambridge University Press)
[33] Becker W, Bergmann A, Konig K and Tirlapur U 2001 *Proc. SPIE Int. Soc. Opt. Eng.* **4262** 414
[34] Raussendorf R, Browne D E and Briegel H J 2003 *Phys. Rev.* A **68** 022312
[35] Briegel H J and Raussendorf R 2001 *Phys. Rev. Lett.* **86** 910
[36] Bollinger J J, Itano W M, Wineland D J and Heinzen D J 1996 *Phys. Rev.* A **54** R4649
[37] Huelga S F, Macchiavello C, Pellizzari T, Ekert A K, Plenio M B and Cirac J I 1997 *Phys. Rev. Lett.* **79** 3865