The propagation characteristic of the EPR entanglement for a composite non-degenerate parametric optical amplification system*

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This paper applies the minimum variance V_1 criterion to monitor the evolution of signal and idler modes of a composite non-degenerate optical parametric amplification (NOPA) system. The analytics and numerical calculation show the influence of the transition time, the vacuum fluctuations, and the thermal noise level on the EPR entanglement of the composite NOPA system. It finds that the entanglement and the squeezing degrade as the minimum variance V_1 increases.

Keywords: EPR entanglement, composite non-degenerate optical parametric amplification, the Fokker–Planck equation

PACC: 0367, 4250, 4265K

1. Introduction

In 1935, Einstein, Podolsky, and Rosen first brought forward the idea that quantum mechanics does not provide a complete description of physical reality, they argued in the measurement of correlations of continuous observables of two separated particles, that quantum mechanics results in contradiction with physics reality and called it the EPR paradox.[1] In 1951, Bohm presented a modified discrete version of the paradox, he considered a pair of spatially separated spin 1/2 particles which show high correlation between their various spin components,[2,3] and later stimulated Bell to derive his famous inequalities, now called the Bell inequalities.[4,5] Over the past twenty years, a variety of experiments have investigated the violations by quantum mechanics of the constraints imposed by local realism as codified by the Bell inequalities, these measurements have followed Bohm's suggestion, but they are not the original EPR paradox of continuous observables. On account of this, in 1988, Reid pointed out the experimental possibility of demonstrating of the original EPR paradox in the non-degenerate parametric optical amplification (NOPA) system.[6] Subsequently, a detailed analysis and derivation was given.[7] Later, The original EPR paradox was demonstrated experimentally by Ou *et al.*,[8,9] Silberhorn *et al.*[10] and Peng *et al.*[11,12] The importance of these researches lies in the quantum states entanglement and its application. Up to now, entanglement plays a key role in the field of quantum information processing. A sufficient condition for entanglement consists in the negativity of the partial transposition (NPT) of the quantum state of the two-party system.[13−15] The inseparability sufficient condition based on the second-order moments of position and momentum operators has been proposed by Simon[16] and Duan *et al.*[17] This condition has been proved to be necessary for the bipartite Gaussian state also. Recently, the necessary and sufficient conditions for the partial transposition of bipartite quantum states to be nonnegative have been provided by Shchukin and Vogel.[18] At present it needs to mention the minimum variance V_1 criterion given by Reid, in the demonstration of EPR paradox using NOPA system.[6,7] In the previous paper, we use the solution of the Fokker–Planck equation for NOPA system to deduce the condition of demonstration of the EPR paradox.[19] In this paper, we will apply the minimum variance V_1 criterion to monitor the evolution of sig-

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nal and idler modes of a composite NOPA system, our aim is to achieve a general understanding of the influence on the EPR entanglement due to propagation. It is well known that entanglement stems from squeezing, at the same time, we also calculate the squeezing. This paper is organized as follows. In Section 2, the Fokker–Planck solution of the NOPA system and the minimum variance V_1 criterion for violation of EPR paradox are briefly reviewed. In Section 3, the composite NOPA system is introduced. In Section 4, numerical calculation is derived. In Section 5, a summary of our results is given.

2. The Fokker–Planck solution of the NOPA system and the minimum variance V_1 criterion for violation of EPR paradox

We consider the NOPA system (the two modes with the same frequencies but different polarizations). Following the treatment of Ref. [20], the Hamiltonian describing this system is

$$H = H_0 + V + W,$$

where

$$\begin{aligned} H_0 &= \hbar\omega a_0^\dagger a_0 + \hbar\omega_1 a_1^\dagger a_1 + \hbar\omega_2 a_2^\dagger a_2 \\ &\quad + \mathrm{i}\,\hbar(E\omega^{-\mathrm{i}\,\omega_L t}a_0^\dagger - E^*\omega^{i\omega_L t}a_0) \\ &\quad + \sum_{j,i=1,2} \hbar\omega_j B_{ji}^\dagger B_{ji}, \\ V &= \hbar\sum k_j(B_{j1}a_1^\dagger + B_{j2}a_2^\dagger) \\ &\quad + \hbar\sum k_j^*(B_{j1}^\dagger a_1 + B_{j2}^\dagger a_2), \\ W &= \frac{\mathrm{i}\,\hbar}{2}(2\varepsilon a_1^\dagger a_2^\dagger - 2\varepsilon^* a_1 a_2), \end{aligned} \tag{1}$$

where the frequencies ω_0, ω_1 and ω_2 refer to the pump, signal and idler field respectively. Because frequency is degenerate, we get $\omega_1 + \omega_2 = \omega_0$ and $\hbar\omega_1 = \hbar\omega_2 = \hbar\omega_0/2$. The E is the amplitude of coherent driving field with frequency $\omega_L \approx \omega_0$, $\varepsilon = \chi\alpha_0$ is the gain factor, and α_0 is the amplitude of pump field.

We set

$$b_1 = \frac{a_1 + a_2}{\sqrt{2}}, \quad b_2 = \frac{a_1 - a_2}{\sqrt{2}}. \tag{2}$$

In positive-P representation, b_1, b_2 can be expressed by

$$\beta_1 = \frac{\alpha_1 + \alpha_2}{\sqrt{2}}, \quad \tilde{\beta}_1 = \frac{\tilde{\alpha}_1 + \tilde{\alpha}_2}{\sqrt{2}}, \quad \beta_2 = \frac{\alpha_1 - \alpha_2}{\sqrt{2}}, \quad \tilde{\beta}_2 = \frac{\tilde{\alpha}_1 - \tilde{\alpha}_2}{\sqrt{2}}. \tag{3}$$

Refer to the pervious paper, the distribution function of system is[19]

$$\begin{aligned} p_1(\beta_1)\mathrm{d}\beta_1 &= \exp\left[\frac{-C_1(\beta_1 - \beta_{10}\exp[-(k-\varepsilon)t])^2}{1-\exp[-2(k-\varepsilon)t]}\right]\frac{\mathrm{d}\beta_1}{\int \exp\left[\frac{-C_1(\beta_1 - \beta_{10}\exp[-(k-\varepsilon)t])^2}{1-\exp[-2(k-\varepsilon)t]}\right]\mathrm{d}\beta_1}, \quad C_1 = \frac{k-\varepsilon}{\varepsilon + 2k\bar{n}}, \\ \tilde{p}_1(\tilde{\beta}_1)\mathrm{d}\tilde{\beta}_1 &= \exp\left[\frac{\tilde{C}_1(\tilde{\beta}_1 - \tilde{\beta}_{10}\exp[-(k+\varepsilon)t])^2}{1-\exp[-2(k+\varepsilon)t]}\right]\frac{\mathrm{d}\tilde{\beta}_1}{\int \exp\left[\frac{\tilde{C}_1(\tilde{\beta}_1 - \tilde{\beta}_{10}\exp[-(k+\varepsilon)t])^2}{1-\exp[-2(k+\varepsilon)t]}\right]\mathrm{d}\tilde{\beta}_1}, \quad \tilde{C}_1 = \frac{k+\varepsilon}{\varepsilon - 2k\bar{n}}, \\ p_2(\beta_2)\mathrm{d}\beta_2 &= \exp\left[\frac{-C_2(\beta_2 - \beta_{20}\exp[-(k+\varepsilon)t])^2}{1-\exp[-2(k+\varepsilon)t]}\right]\frac{\mathrm{d}\beta_2}{\int \exp\left[\frac{-C_2(\beta_2 - \beta_{20}\exp[-(k+\varepsilon)t])^2}{1-\exp[-2(k+\varepsilon)t]}\right]\mathrm{d}\beta_2}, \quad C_2 = \frac{k+\varepsilon}{\varepsilon - 2k\bar{n}}, \\ \tilde{p}_2(\tilde{\beta}_2)\mathrm{d}\tilde{\beta}_2 &= \exp\left[\frac{\tilde{C}_2(\tilde{\beta}_2 - \tilde{\beta}_{20}\exp[-(k-\varepsilon)t])^2}{1-\exp[-2(k-\varepsilon)t]}\right]\frac{\mathrm{d}\tilde{\beta}_2}{\int \exp\left[\frac{\tilde{C}_2(\tilde{\beta}_2 - \tilde{\beta}_{20}\exp[-(k-\varepsilon)t])^2}{1-\exp[-2(k-\varepsilon)t]}\right]\mathrm{d}\tilde{\beta}_2}, \quad \tilde{C}_2 = \frac{k-\varepsilon}{\varepsilon + 2k\bar{n}}, \end{aligned} \tag{4}$$

and the quadrature phase amplitudes are given by[7]

$$X_1 = \sqrt{2}(\beta_1 + \beta_2), \quad X_2 = \sqrt{2}(\tilde{\beta}_1 + \tilde{\beta}_2), \quad Y_1 = \sqrt{2}(\beta_1 - \beta_2), \quad Y_2 = \sqrt{2}(\tilde{\beta}_1 - \tilde{\beta}_2). \tag{5}$$

The conjugate variables X_1, X_2 and Y_1, Y_2 obey the commutation relation:

$$[X_1, X_2] = 2\,\mathrm{i}, \quad [Y_1, Y_2] = 2\,\mathrm{i}\,.$$

Considering the subsystems b_1, b_2 contributing to the quantum fluctuation of the total system and the vacuum fluctuations, we arrive at the quantum fluctuations which are

$$\begin{aligned}\left\langle(\Delta X_1)^2\right\rangle &= \left(\frac{\exp[-(k+\varepsilon)t]+\exp[-(k-\varepsilon)t]}{2}\right)^2+\left(\frac{\exp[-(k+\varepsilon)t]-\exp[-(k-\varepsilon)t]}{2}\right)^2\\ &+\frac{(2\bar{n}+1)k}{2(k-\varepsilon)}(1-\exp[-2(k-\varepsilon)t])+\frac{(2\bar{n}+1)k}{2(k+\varepsilon)}(1-\exp[-2(k+\varepsilon)t]),\end{aligned}\tag{6}$$

$$\begin{aligned}\left\langle(\Delta Y_2)^2\right\rangle &= \left(\frac{\exp[-(k-\varepsilon)t]+\exp[-(k+\varepsilon)t]}{2}\right)^2+\left(\frac{\exp[-(k-\varepsilon)t]-\exp[-(k+\varepsilon)t]}{2}\right)^2\\ &+\frac{(2\bar{n}+1)k}{2(k+\varepsilon)}(1-\exp[-2(k+\varepsilon)t])+\frac{(2\bar{n}+1)k}{2(k-\varepsilon)}(1-\exp[-2(k-\varepsilon)t]).\end{aligned}\tag{7}$$

The factor 1/2 has been inserted to manifest the probabilities of the system, whose half in the 'in phase state' and half in the 'out of phase state'.

In terms of Eqs. (6) and (7), we obtain the correlation between ΔX_1 and ΔY_2

$$\begin{aligned}\langle\Delta X_1\Delta Y_2\rangle &= \left(\frac{\exp[-(k+\varepsilon)t]+\exp[-(k-\varepsilon)t]}{2}\right)\times\left(\frac{\exp[-(k-\varepsilon)t]-\exp[-(k+\varepsilon)t]}{2}\right)\\ &-\left(\frac{\exp[-(k-\varepsilon)t]+\exp[-(k+\varepsilon)t]}{2}\right)\times\left(\frac{\exp[-(k+\varepsilon)t]-\exp[-(k-\varepsilon)t]}{2}\right)\\ &+(2\bar{n}+1)(1-\exp[-2kt]).\end{aligned}\tag{8}$$

Using Eqs. (6)–(8), we easily calculate the minimum variance V_1 which is[7]

$$V_1=\left\langle\Delta X_1^2\right\rangle-\langle\Delta X_1\Delta Y_2\rangle^2/\left\langle\Delta Y_2^2\right\rangle.\tag{9}$$

According to the statement of Ref. [7], a measurement of the idler amplitude $\hat{Y}_\phi(L)$ can be used to infer a value of the signal amplitude $\hat{X}_1^0(L)=g\hat{Y}_\phi(L)$ with certainty. The deviation of the scaled readout $\hat{X}_1^0(L)$ from the true signal amplitude $\hat{X}_1(L)$ is

$$\begin{aligned}V_1(g,\phi)&=\left\langle[\hat{X}_1(L)-\hat{X}_1^0(L)]^2\right\rangle\\ &=\left\langle[\hat{X}_1(L)-g\hat{Y}_\phi(L)]^2\right\rangle.\end{aligned}$$

The best estimate $\hat{X}_1^0(L)$ is such that the scaling factor g and the angle ϕ give a minimum $V_1(g,\phi)$, and deduce that $g=\langle X_1(L)Y_2(L)\rangle/\left\langle[Y_2(L)]^2\right\rangle$. If we divide V_1 by $2g$, the form of variance obtained $(\Delta\hat{X}_1(L)/\sqrt{2g}+g\Delta\hat{Y}_\phi(L)/\sqrt{2g})^2$ is similar to $(|a|\,\hat{x}_1+\hat{x}_2/a)^2$ discussed in Ref. [17]. Now we apply Eq. (7) to monitory the evolution of signal and idler modes of the NOPA system and think the violation of $V_1(g,\phi)\geq 1$ as the sufficient condition of the entanglement.

The squeezing $\left\langle(\Delta y)^2\right\rangle$ is calculated by[19]

$$\begin{aligned}\left\langle(\Delta y)^2\right\rangle&=\frac{1}{4}-\frac{\left\langle(\Delta\,\mathrm{i}\,\tilde{\beta})^2\right\rangle}{4}\\ &=\frac{1}{4}-\frac{1}{4}\left[\frac{\varepsilon-(2\bar{n}+1)k}{2(k+\varepsilon)}(1-\exp[-2(k+\varepsilon)t])\right.\\ &\left.+\frac{\varepsilon+(2\bar{n}+1)k}{2(k-\varepsilon)}(1-\exp[-2(k-\varepsilon)t])\right].\end{aligned}\tag{10}$$

3. The composite NOPA system

The composite NOPA system consists of two parametric amplifiers in series separated by the air gap, which is illustrated in Fig. 1. Our aim is to investigate the behaviour of the entanglement of the composite NOPA system. The analytics and numerical calculation show the influence of the transition time, the vacuum fluctuations, and the thermal noise level on the EPR entanglement of the composite system.

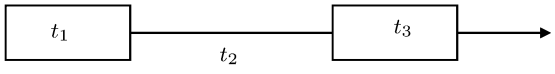


Fig. 1. The composite NOPA system.

We assume the ray outputs from the first NOPA system (transition time t_1, gain coefficient ε, and loss η), then passes through the air gap (transition time

t_2, loss λ), at last the ray is incident upon the second NOPA system (transition time t_3, gain coefficient ε, and loss η).

4. Numerical calculations

In the following numerical calculation, in the interval 0 to t_1, we express by the dashed curve, in the interval t_1 to t_2, we express by the solid curve, in the interval t_2 to t, we express by the dashed-dotted curve.

In terms of Eqs. (6)–(9), we calculate the minimum variance V_1 as a function of time t, the result is shown in Fig. 2(a), at the same time, we take the same parameters to evaluate the squeezing $\langle(\Delta y)^2\rangle$ versus time t, the result is shown in Fig. 2(b). Comparing Fig. 2(a) with Fig. 2(b), we find that when $t = 0$, the variance $V_1 = 1$, the squeezing $\langle(\Delta y)^2\rangle = 0.25$ is the initial vacuum fluctuations. In the interval 0 to t_1, the ray outputs from the first NOPA system, the variance V_1 decreases from 1 to 0.865, the squeezing $\langle(\Delta y)^2\rangle$ changes from 0.25 to 0.155. In the duration of $t > t_1$, the signal and idler modes enter into the air gap, the variance V_1 changes from 0.865 to 1.3 due to propagation, while the squeezing $\langle(\Delta y)^2\rangle$ experiences a little increase from 0.155 to 0.235. When $t > t_1 + t_2$, the two modes enter into the second NOPA system, the variance V_1 first experiences a little descent from 1.3 to 0.83, and then continuously rises up to 1, while the squeezing $\langle(\Delta y)^2\rangle$ changes monotonously from 0.235 to 0.065. The result has been obtained in a previous paper for one NOPA system.[22] The complex behaviour of the entanglement can be explained by the loss $\eta \neq 0$. In order to discuss the influence of the transition time, we reduce the time t_2 passing through the air gap, other parameters are the same as Fig. 2(a), we take $t_2 - t_1 \leq 0.5$, the background noise decreases, and the two modes enter into the second NOPA system ahead of time, the variance V_1 decreases to the minimum value 0.52, this can be seen from Fig. 3. From the above analysis, we should adopt the small air gap transition time t_2 in order to obtain a small variance, namely high entanglement.

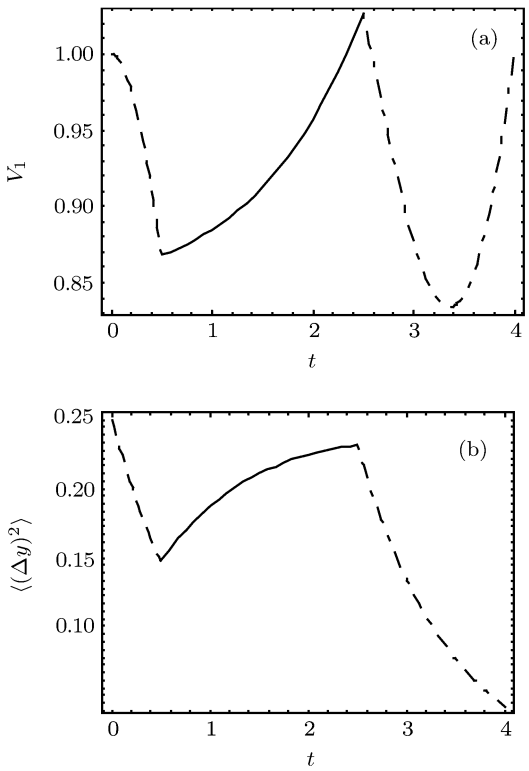


Fig. 2. (a) The minimum variance V_1 of the composite NOPA system as a function of time t, (b) The squeezing $\langle(\Delta y)^2\rangle$ of the composite NOPA system verse time t with $t_1 = 0.5$, $t_2 = 2.5$, $t_3 = 4$, $\eta = 0.1$, $\lambda = 0.01$, $\bar{n} = 0$.

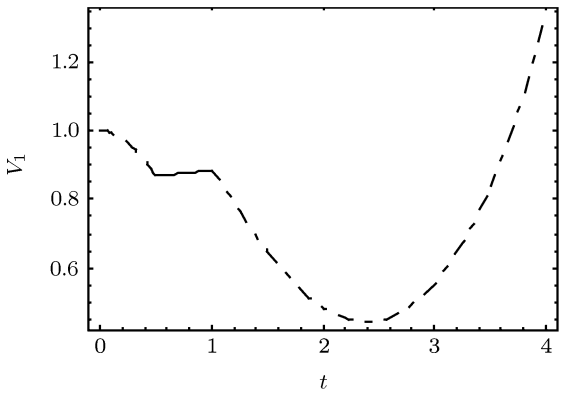


Fig. 3. The minimum variance V_1 of the composite NOPA system versus time t with $t_1 = 0.5$, $t_2 = 1$, $t_3 = 4$, $\eta = 0.1$, $\lambda = 0.01$, $\bar{n} = 0$.

In the following, we discuss the effect of the thermal noise $\bar{n}$, other parameters are the same as Fig. 2(a), we take $\bar{n} = 1$, this can be seen from Fig. 4.

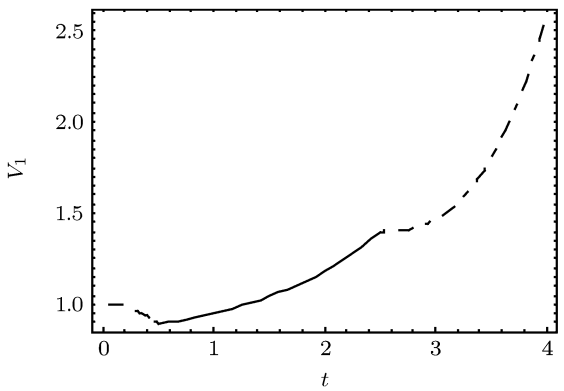


Fig. 4. The minimum variance V_1 of the composite NOPA system verse time t with $t_1 = 0.5$, $t_2 = 2.5$, $t_3 = 4$, $\eta = 0.1$, $\lambda = 0.01$, $\bar{n} = 1$.

By comparing it with Fig. 2(a) ($\bar{n} = 0$, without the thermal noise), we find that the variance V_1 increases

to a very high level. From the above analysis, we should adopt the small thermal noise $\bar{n}$ in order to obtain a high entanglement.

5. Conclusions

In this paper, we apply the criterion of minimum variance V_1 to monitor the evolution of the signal and idler modes of a composite NOPA system. Through numerical calculation and analysis, we find that: (i) in the duration of propagation, the variance V_1 increases, entanglement and squeezing become weak, the second NOPA system works at high vacuum fluctuations and high thermal noise level results in the signal, idler modes arrive at the second NOPA system's time delay, it is unfavourable for achieving a high degree of entanglement; (ii) in order to achieve a high entanglement, the length of the second NOPA system must be controlled, and the loss η as small as possible; (iii) the thermal noise $\bar{n}$ must be controlled by the low level. These propagation characteristics of EPR entanglement for a composite NOPA system provide a method of parameter control for the high entanglement.

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