Controlled decoherence of three-level rf-SQUID qubit with asymmetric potential*

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A scheme is proposed to controll the decoherence of three-level rf-SQUID qubit with asymmetric potential by designing an external electric circuit for superconductive flux qubit. The results show that it may not only raise the gate speed but also extend decoherence time for a three-level structure.

Keywords: gate speed, quantum NOT gates, decoherence time, controll

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1. Introduction

Over the past decade, a variety of qubits have been theoretically proposed and experimentally realised based on superconducting Josephson junctions.[1−7] One of valuable schemes is to use magnetic flux states in superconducting quantum interference devices (SQUID).[8−14] In a really physical system, the interaction between solid-state qubits and the environment is inevitable, which leads to decoherence. The process restricts the ability to maintain pure quantum states in quantum calculation.[15,16] Especially, the superconducting qubits always suffer the decoherence, such as relaxation and dephasing, resulting from a relatively short coherence time. Moreover, it is well-known that the decoherence strongly affects the gate speed and gate error rate.[17−19] Thus, environment-induced decoherence has been a main obstacle for the practical application of superconducting qubits in quantum computation. In the practical applications, in addition, it is important that a typical gate operation time must be much smaller than the decoherence time. Thus, there arises a fundamental issue of how to increase the decoherence time.

We know that the multilevel structure of the SQUID qubits is advantageous to the exaltation of the calculation speed of the qubits, but the intrinsic gate error and leakage to the noncomputational states cause significant errors even though the simple one-bit SQUID quantum gate operations.[20] As pointed out by Kis and Paspalakis,[21] for the sake of exalting the gate speed of the superconductive flux qubit, one should adopt a way of the three-level structure in quantum calculation. Based on the conventional configuration of SQUID qubits, Zhou *et al.*[22] proposed a three-level (Λ-type) rf-SQUID qubit. Here, the states of the qubit are the two lower flux states $|0\rangle$ and $|1\rangle$ of the Λ-type system. Thus the manipulation of the qubit is done with two microwave fields that couple the lower states ($|0\rangle$ and $|1\rangle$) to upper state $|e\rangle$. As the transition matrix elements, corresponding to the transitions $|0\rangle \leftrightarrow |e\rangle$ and $|1\rangle \leftrightarrow |e\rangle$, are larger than that of the transition, the three-level SQUID qubit has been shown to be more favourable than the conventional two-level SQUID qubit for implementing a NOT gate, where auxiliary level $|e\rangle$ is used to significantly increase the speed and reduce the errors of quantum gate operations. However, generally speaking, it is a contradictory issue for the exaltation of both the gate speed and the decoherence time at the same time.

It is known that numerous theoretical studies on the decoherence of superconducting qubits have typically relied on a widely used spin-boson model with pure two-level or multilevel dynamics. However, the environment reservoir is so complex that the simple phenomenological model can not cover all effects involved in it. In the present work, by combining network graph theory with the Caldeira–Leggett model in the multilevel quantum description method,[23−26] we analyse the expressions of the relaxation and the deco-

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herence times of the rf-SQUID qubit in the regime of the three-level structure. Then by designing an electric circuit of the superconducting flux qubit in the regime of three-level structure, we may control the spectral density of heat bath fluctuations to fit the real scenario of the decoherence process.

2. Model and the Hamiltonian

The Hamiltonian in the Caldeira–Leggett model is given by

$$H = H_{\rm S} + H_{\rm B} + H_{\rm SB}, \tag{1}$$

where $H_{\rm S}$ is the Hamiltonian of the rf-SQUID,

$$H_{\rm S} = -\frac{1}{2m}\frac{{\rm d}^2}{{\rm d}x^2} + \frac{1}{2}m\omega^2(x-x_{\rm e})^2 - \frac{m\omega^2}{4\pi^2}\beta\cos(2\pi x), \tag{2}$$

where $x = \Phi/\Phi_0$, $x_{\rm e} = \Phi_{\rm e}/\Phi_0$, $m = C\Phi_0^2$, $\omega^2 = 1/LC$ and $\beta = 2\pi LI_C/\Phi_0$. While Φ is total magnetic flux in the superconducting ring, $\Phi_{\rm e}$ is the magnetic flux externally applied to the SQUID, $\Phi_0 = h/2e$ is the flux quantum, $I_{\rm C}$ is the critical current of junction, L and C are the ring induction and the ring capacitance respectively, $H_{\rm B}$ is the Hamiltonian of the reservoir,

$$H_{\rm B} = \sum_\alpha \left(\frac{1}{2m_\alpha}\frac{{\rm d}^2}{{\rm d}q_\alpha^2} + \frac{1}{2}m_\alpha\omega_\alpha^2 q_\alpha^2\right), \tag{3}$$

which describes a bath with the position q_α, mass m_α and frequency ω_α of harmonic oscillators. The $H_{\rm SB}$ is the interaction between the quantum system and the reservoir,

$$H_{\rm SB} = -\Phi_0^{-1}\sum_\alpha \lambda_\alpha q_\alpha x, \tag{4}$$

where λ_α is the coupling coefficient.

3. Controlled decoherence

In the Hilbert space spanned by the eigenstate $|m\rangle$ with eigenenergy E_m $(m = 1, 2, \ldots)$, the master equation from Hamiltonian (1) may be obtained in the Markovian approximation, i.e.

$$\dot\rho_{nm}(t) = -{\rm i}\,\omega_{nm}\rho_{nm}(t) - \sum_{kl} R_{nmkl}\rho_{kl}(t), \tag{5}$$

which is called Redfield equation, while $\rho_{mn} = \langle m|\,\hat\rho\,|n\rangle$ and $\omega_{nm} = \omega_n - \omega_m$. According to interacting Hamiltonian (4), the Redfield relaxation tensor is[27]

$$R_{nmkl} = \frac{1}{2\hbar^2}\Big[- x_{lmnk}J(\omega_{nk}) - x_{knml}J(\omega_{ml}) + \delta_{lm}\sum_i x_{niik}J(\omega_{ik}) + \delta_{nk}\sum_i x_{miil}J(\omega_{il})\Big], \tag{6}$$

where $x_{lmnk} = \langle l|\,x\,|m\rangle\,\langle n|\,x\,|k\rangle$. In the semiclassical approximation, the relaxation, decoherence and pure dephasing times of the system may be expressed in terms of the spectral density of the heat bath fluctuations $J(\omega)$.

Thus, the effect of a thermal bath on a quantum system may be characterised by a spectral density. It is known that in the two-level structure for the superconducting circuit containing flux qubit, by using the spin-boson mode, one proves that noise caused by fluctuations of spectral density can be computed from the frequency-dependent damping coefficient of the quantum Langevin equation. Thus, it is interesting to study the noise effects for three-level structure by using the Langevin equation.

Using the Kirchhoff law in the qubit circuit described by Fig. 1, the quantum Langevin equation may be expressed as

$$C\Phi_0\frac{{\rm d}^2x}{{\rm d}t^2} + \Phi_0\int_{t_0}^t {\rm d}\tau Y(t-\tau)\frac{{\rm d}x(\tau)}{{\rm d}\tau} + \frac{1}{\Phi_0}\frac{{\rm d}V(x)}{{\rm d}x} = I_{\rm B}(t), \tag{7}$$

where the damping coefficient $Y(t-\tau)$ is the equivalent admittance of the external circuits, and $I_{\rm B}(t)$ is the fluctuation force of undulation of ambient magnetic field. The spectral density of a bath is[28]

$$J(\omega) = \hbar\omega G(\omega)\left[1 + \coth\left(\frac{\hbar\omega}{2k_{\rm B}T}\right)\right], \tag{8}$$

where T is the ambient temperature. $G(\omega)$ is the real part of the frequency dependent admittance $Y(\omega)$ of the external circuit. The relaxation, pure dephasing and decoherence times of the system may be respectively expressed as

$$\frac{1}{T_1} = \frac{2\Phi_0^2}{\hbar}\,|x_{jk}|^2\,J(\omega_{jk}), \tag{9}$$

$$\frac{1}{T_\phi} = \frac{2\Phi_0^2}{\hbar}\,|x_{jj} - x_{kk}|^2\,J(\omega)|_{\omega\to 0}\,, \tag{10}$$

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\phi}, \tag{11}$$

where $x_{jk} = \langle j|\, x\, |k\rangle$ is the tunneling amplitude between the two wells.

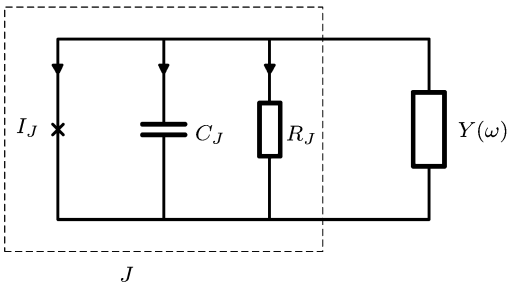


Fig. 1. Circuit of rf-SQUID flux qubit with external admittance $Y(\omega)$.

From Eqs. (9)–(11), we see that the relaxation and the dephasing times of qubit are determined by the spectral densities at the transition frequency $\omega = |\omega_{jk}|$ around low frequency $\omega = 0$. In addition, the relaxation time is proportional to the modulus square of the transition matrix element, i.e., $1/|x_{jk}|^2$, while the dephasing time is proportional to $1/(x_{jj} - x_{kk})^2$. Moreover, we see that when $T_\phi \to \infty$, the decoherence time T_2 decreases with the increase of the coupled matrix element x_{jk}. Therefore, in order to make sure of the correctness of quantum computation at a given decoherence time, we must enhance the operating velocity. On the other hand, in order to exalt the decoherence time, the overlapping integral x_{jk} must be a very small number. This corresponds to the case where the two wells are far from each other. Therefore, there appears a contradictory issue of the simultaneous exaltation of gate speed and coherence time.

In order to solve the problem, it may be useful to compare the gate speed of three-level with two-level quantum NOT gates for an asymmetric potential of rf-SQUID.[29] The evolutions of the gate speed for the three-level and the two-level systems with field amplitude are shown in Figs. 2(a) and 2(b) respectively. The curves indicate the microwave pulse duration for the complete transition of the system with the same parameters from the initial state$|0\rangle$to the state $|1\rangle$, which are functions of the normalised field amplitude. Fig. (a) shows the speed of NOT gate versus microwave amplitude for the three-level qubit, and Fig. (b) displays the speed of NOT gate versus microwave amplitude for the two-level qubit. It is obvious that, under the same parameter as those in Fig. 1, the scheme for three-level qubit scheme is much faster than that for the two-level qubit.

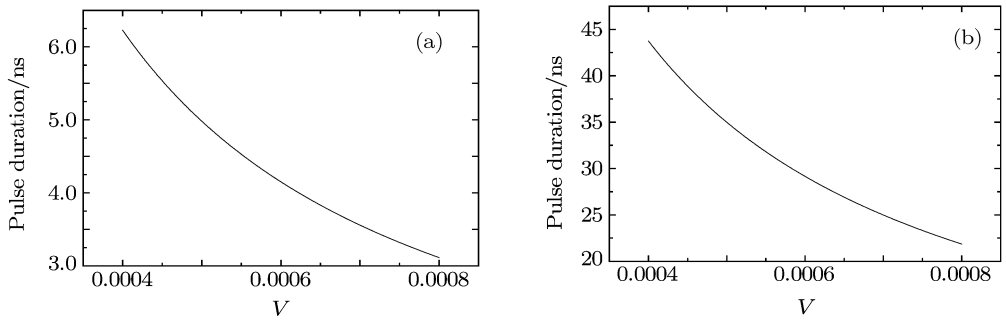


Fig. 2. Comparison of the NOT gate speed between the three-level system (a) and two-level system (b).

The result may be easy to understand because the coupling matrix elements $\langle 0|\, x\, |e\rangle$ and $\langle 1|\, x\, |e\rangle$ are about two orders of magnitude greater than the coupling matrix element $\langle 0|\, x\, |1\rangle$, where $|e\rangle$ is the auxiliary level. This property is universal for flux qubits since the state $|e\rangle$ is delocalised and this significantly overlaps the wavefunctions of states between $|0\rangle$ and state $|1\rangle$. In contrast, the wavefunctions of states between $|0\rangle$ and state $|1\rangle$ have essentially no overlap because they are the computational basis of the flux qubit. Hence, $\langle 0|\, x\, |1\rangle$ is always exponentially smaller than $\langle 0|\, x\, |e\rangle$ and $\langle 1|\, x\, |e\rangle$, which explains why the second order process described here is faster than the conventional first order process.

Different from in the two-level structure,[30] in the three-level scheme, one may have two manipulation processes. First, one microwave pulse with frequency $\omega_{\rm a}$ lasts time $t_{0\,\rm e}$ and disappears at the moment when the transition $|0\rangle \to |e\rangle$ is completed. At the same moment a second microwave pulse with frequency $\omega_{\rm b}$

is applied and lasts time t_{e1} to complete the transition $|e\rangle \rightarrow |1\rangle$. The two microwave pulses are of the same amplitude. Second, after the action of microwave pulse with amplitude ε and frequency ω_a lasts time t_{0e}, let the action of another microwave with frequency ω_b last time t_{e1}. Thus, the total decoherence time for the three-level scheme is given by

$$T_{2\Lambda} = \frac{\hbar}{\Phi_0^2 |x_{0e}|^2 \omega_{0e} G(\omega_{0e})} + \frac{\hbar}{\Phi_0^2 |x_{e1}|^2 \omega_{e1} G(\omega_{e1})}. \quad (12)$$

An actual SQUID always has limited decay impedance, therefore it has a limited decoherence time. At low temperatures, the dephasing time is inverse with respect to temperature. For an extremely high temperature, furthermore, setting $T_\phi \rightarrow \infty$, there is $T_2 = 2T_1$. Different from the gate speed, the decoherence time will reduce with the raise of coupling matrix elements. Fortunately, one may extend the decoherence time to the regime of three-level structure by controlling the external circuit. Around the low temperature, from Eqs. (9)–(12), we have

$$T_2 = \frac{\hbar}{\Phi_0^2 |x_{jk}|^2 \omega_{jk} G(\omega_{jk})}, \quad (13)$$

which determines the coupling matrix elements. Thus, the corresponding gate speed is obtained. From Eqs. (12) and (13) it follows that if the function, $f(\omega) = \omega G(\omega)$, is a decreasing function of frequency ω, one may improve the decoherence time.

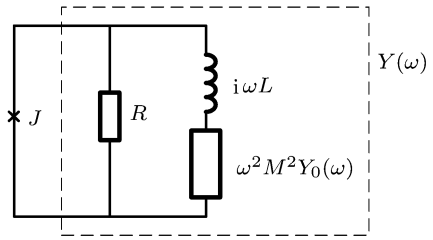


Fig. 3. A simple circuit mode containing flux qubit. The left is circuit of qubit, the right is the circuit that controls qubit by coupling mutual-inductor.

According to the ideas, in order to obtain not only a fast speed of the gate but also a longer decoherence time, we construct an electric circuit like Figs. 3 and 4, where the mutual-inductor coefficient between primary and secondary circuits is M, $Y_0(\omega) = G_0(\omega) + \mathrm{i}\, B_0(\omega)$ is admittance of secondary circuit. According to Fig. 3, we have

$$Y(\omega) = \frac{1}{R} + \frac{\omega^2 M^2 G_0 - \mathrm{i}(\omega L + \omega^2 M^2 B_0)}{\omega^4 M^4 G_0^2 + (\omega L + \omega^2 M^2 B_0)^2}. \quad (14)$$

Using Eq. (14) under the condition $R \rightarrow \infty$, Eq. (13) becomes

$$T_2 = \frac{\hbar[\omega_{jk}^4 M^4 G_0^2(\omega_{jk}) + (\omega_{jk} L + \omega_{jk}^2 M^2 B_0(\omega_{jk}))^2]}{\Phi_0^2 |x_{jk}|^2 \omega_{jk}^3 M^2 G_0(\omega_{jk})}. \quad (15)$$

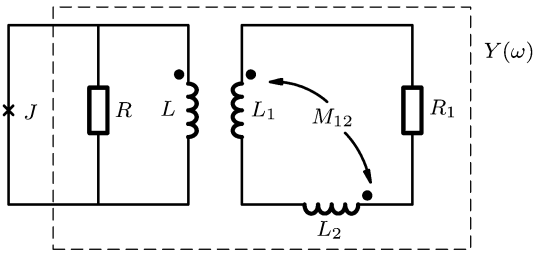


Fig. 4. A particular circuit mode containing flux qubit. In primary circuit, resistor R and inductor L are in parallel, mutual-inductor coefficient between primary circuit and secondary circuit is M; In secondary circuit, resistor R_1, inductor L_1 and L_2 are in series, mutual-inductor coefficient between inductors L_1 and L_2 is M_{12}.

Adjusting mutual-inductor coefficient between inductors L_1 and L_2 and letting them satisfy the relation $L_1 + L_2 - 2M_{12} = 0$, we find

$$Y(\omega) = \left(\frac{1}{R} + \frac{M^2 R_1}{\omega^2 M^4 + R_1^2 L^2}\right) - \mathrm{i}\,\frac{R_1^2 L}{\omega^3 M^4 + \omega R_1^2 L^2}. \quad (16)$$

Obviously, in order to assure a longer decoherence time, $f(\omega) = \omega R_e(Y(\omega))$ should decrease with the increase of frequency ω. Under the condition $R \rightarrow \infty$, Eq. (16) should satisfy the following relation:

$$\omega > \frac{R_1 L}{M^2}. \quad (17)$$

Figure 4 shows how a particular circuit mode containing the flux qubit controls the frequency dependent admittance $Y(\omega)$. By adjusting the parameters R_1 and M, where M is controlled by L_1 in the secondary circuit and the coupling coefficient between the primary and secondary circuits, one may find a solution satisfied Eq. (17).

Setting $L = 10^2$ pH, $M = 10^3$ pH and $R_1 = 10^2\ \Omega$, we have $T_2 = 156$ ns and $T_{2\Lambda} = 5528$ ns.

Obviously, $T_{2\Lambda}$ is much larger than T_2. We find that for a longer time of decoherence, R_1 should be approximately proportional to M^2, i.e., $R_1 \approx 10^{-20} M^2$.

4. Conclusion

From our results it follows that although there is a contradiction between the raising of gate speed and the exalting of decoherence time, it may be conquered by rationally designing the electric circuit. In the rf-SQUID flux qubit with an external circuit, by adopting the three-level structure, one may not only raise the gate speed but also extend the decoherence time under a reasonable choice of parameters in the external electric circuit.

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