Entanglement dynamics of a double two-photon Jaynes–Cummings model with Kerr-like medium*

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In this paper, the entanglement dynamics of a double two-photon Jaynes–Cummings model with Kerr-like medium is investigated. It is shown that initial entanglement has an interesting subsequent time evolution, including the so-called entanglement sudden death effect. It is also shown analytically that the Kerr-like medium can repress entanglement sudden death and enhance the degree of atom–atom entanglement. A more interesting fact is that the Kerr effect is more obvious when each of the two cavities with have the Kerr-like medium than only one of them with the Kerr-like medium.

Keywords: entanglement sudden death, concurrence, two-photon Jaynes–Cummings model, Kerr-like medium

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1. Introduction

Entanglement[1−4] is not only a key concept for distinguishing between the quantum and the classical worlds, but has also been viewed as an indispensable resource for performing various intriguing global tasks in quantum computing and quantum information processing (QIP), such as quantum teleportation,[5−7] quantum dense coding,[8] and quantum cryptography.[9] However, it is very susceptible to decoherence in practical applications due to unavoidable interaction with the environment, and can even disappear completely in a finite time, an effect dubbed entanglement sudden death (ESD).

ESD has been extensively studied under various conditions[10−18] and has been recently demonstrated experimentally.[19] However, the abrupt disappearance of entanglement may be a disaster because most quantum information processes are based on entanglement, so it would be better to have some methods to repress the occurrence of ESD and enhance the degree of atom–atom entanglement. Work in this direction is under investigation. For example, Ref. [20] has shown that the direct measurement feedback method proposed by Wiseman can avoid ESD and further enhance the entanglement; Reference [21] has shown that the time of ESD can be controlled by the classical driving fields; Reference [22] has shown that quantum error correction may delay, but also cause ESD; Reference [23] has studied the effect of the dynamic Stark shift on the entanglement and examined the possibility of protecting the entanglement by the dynamic Stark shift in a double two-photon Jaynes–Cummings (JC) model.

In this paper, we consider a double two-photon JC model with a Kerr-like medium.[24−26] The difference between our model and that in Ref. [27] is that each cavity is filled with a nonlinear Kerr-like medium. So not only the interaction between B and b but also the interaction between A and a is described by using the two-photon JC model with an added Kerr-like medium. It is found that ESD is susceptible to initial state, one can not only control ESD but also enhance the degree of atom–atom entanglement by using the Kerr-like medium, and the Kerr effect is more apparent when both of the two cavities but only one of them contains Kerr-like medium.

2. The model

We consider a double two-photon JC model, which consists of two effective two-level atoms la-

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beled A and B, and two single-mode cavity fields labeled a and b (atom A interacting only with field a through two-photon processes and similarly for B and b). The excited and ground states of each atom will be designated by $|e\rangle$ and $|g\rangle$, respectively. Each cavity is filled with a nonlinear Kerr-like medium. The Kerr-like medium can be described by an anharmonic oscillator.[28] This situation can be realized physically as if the cavity contained two different species of Rydberg atom in which one behaves like a two-level atom undergoing two-photon transition, and the other behaves like an anharmonic oscillator in the single-mode field.

The Hamiltonian of the whole system in the rotating wave approximation (RWA) and in the interaction picture is (assuming $\hbar = 1$)

$$\begin{aligned} &H = H_1 + H_2, \\ &H_1 = g(a^{\dagger 2}\sigma_-^A + a^2\sigma_+^A) + \chi a^{\dagger 2}a^2, \\ &H_2 = g(b^{\dagger 2}\sigma_-^B + b^2\sigma_+^B) + \chi b^{\dagger 2}b^2, \end{aligned} \tag{1}$$

where $a^\dagger(a)$ is the photon creation (annihilation) operator of the cavity mode a, and $b^\dagger$ (b) is that of the cavity mode b; $\sigma_\pm(\sigma_+ = |e\rangle\langle g|, \sigma_- = |g\rangle\langle e|)$ are the raising and lowering operators of atom; g is the coupling constant between the cavity and atom; χ is the dispersive part of the third-order nonlinearity of Kerr-like medium.

3. Atom–atom entanglement dynamics for the initial atomic state $|\Phi_{\rm atom}(0)\rangle = \cos\alpha\,|ee\rangle + \sin\alpha\,|gg\rangle$

For a bipartite state, there are some equivalent measures of the degree of entanglement. We choose the Wootters entanglement measure,[29,30] the concurrence C, defined as

$$C = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}), \tag{2}$$

where the quantities λ_1, λ_2, λ_3 and λ_4 are the eigenvalues in a decreasing order of the matrix $\tilde{\boldsymbol{\rho}} = \boldsymbol{\rho}(\sigma_y \otimes \sigma_y)\boldsymbol{\rho}^*(\sigma_y \otimes \sigma_y)$; $\boldsymbol{\rho}$ is the reduced density matrix representing the quantum state of the two atoms in this paper, and the matrix elements are taken with respect to the two atomic bases of $|ee\rangle$, $|eg\rangle$, $|ge\rangle$, $|gg\rangle$. The range of the concurrence is from 0 to 1. For two unentangled atoms $C = 0$, whereas $C = 1$ for two maximally entangled atoms.

First, we assume that the initial state of the two atoms is in a combination of the two Bell states $|\Phi^\pm\rangle$:[31,32]

$$|\Phi_{\rm atom}\rangle = \cos\alpha\,|ee\rangle + \sin\alpha\,|gg\rangle\,, \tag{3}$$

and the initial states of the two cavities are in vacuum state

$$|\Phi_{\rm c}\rangle = |00\rangle\,. \tag{4}$$

Therefore, the initial state for the total system is given by

$$|\Phi_{\rm I}(0)\rangle = (\cos\alpha\,|ee\rangle + \sin\alpha\,|gg\rangle) \otimes |00\rangle\,, \tag{5}$$

and the state of system at any time t can be written as

$$\begin{aligned} |\Phi_{\rm I}(t)\rangle \;=\; & y_1\,|ee00\rangle + y_2\,|gg00\rangle + y_3\,|eg02\rangle \\ & + y_4\,|ge20\rangle + y_5\,|gg22\rangle\,. \end{aligned} \tag{6}$$

Substituting the state $|\Phi_{\rm I}(t)\rangle$ into the Schrödinger equation in the interaction picture, one can obtain the equations of amplitudes (assuming $\hbar = 1$)

$$\begin{aligned} &\dot{y}_1 = -{\rm i}(\sqrt{2}gy_4 + \sqrt{2}gy_3), \\ &\dot{y}_2 = 0, \\ &\dot{y}_3 = -{\rm i}(\sqrt{2}gy_5 + \sqrt{2}gy_1 + 2\chi y_3), \\ &\dot{y}_4 = -{\rm i}(\sqrt{2}gy_1 + \sqrt{2}gy_5 + 2\chi y_4), \\ &\dot{y}_5 = -{\rm i}(\sqrt{2}gy_3 + \sqrt{2}gy_4 + 4\chi y_5). \end{aligned} \tag{7}$$

Then the probability amplitudes can be expressed by

$$\begin{aligned} y_1(t) &= \frac{\cos\alpha}{2\Delta^2}\left((r\Delta + 1 + r^2)\exp(-2{\rm i}\lambda^- gt) + (-r\Delta + 1 + r^2)\exp(-2{\rm i}\lambda^+ gt) + 2\exp(-2{\rm i}rgt)\right), \\ y_2(t) &= \sin\alpha, \\ y_3(t) &= \frac{\cos\alpha}{\sqrt{2}\Delta^2}\left(-\frac{1}{2}\lambda^+\exp(-2{\rm i}\lambda^- gt) - \frac{1}{2}\lambda^-\exp(-2{\rm i}\lambda^+ gt) + r\exp(-2{\rm i}rgt)\right), \\ y_4(t) &= \frac{\cos\alpha}{\sqrt{2}\Delta^2}\left(-\frac{1}{2}\lambda^+\exp(-2{\rm i}\lambda^- gt) - \frac{1}{2}\lambda^-\exp(-2{\rm i}\lambda^+ gt) + r\exp(-2{\rm i}rgt)\right), \end{aligned}$$

$$y_5(t) = \frac{\cos\alpha}{2\Delta^2}\left(\exp(-2\mathrm{i}\lambda^- gt) + \exp(-2\mathrm{i}\lambda^+ gt) - 2\exp(-2\mathrm{i}rgt)\right),$$
$$r = \frac{\chi}{g}, \quad \Delta = \sqrt{r^2+2}, \quad \lambda^{\pm} = r \pm \Delta. \tag{8}$$

From Eq. (6), we can obtain the atom–atom density operator $\boldsymbol{\rho}_{\mathrm{AB}}$. In the atomic basis $\{|ee\rangle, |eg\rangle, |ge\rangle, |gg\rangle\}$, $\boldsymbol{\rho}_{\mathrm{AB}}$ is written as

$$\boldsymbol{\rho}_{\mathrm{AB}} = \begin{pmatrix} y_1 y_1^* & 0 & 0 & y_1 y_2^* \\ 0 & y_3 y_3^* & 0 & 0 \\ 0 & 0 & y_4 y_4^* & 0 \\ y_2 y_1^* & 0 & 0 & y_2 y_2^* + y_5 y_5^* \end{pmatrix}. \tag{9}$$

It can be shown that the concurrence of this density matrix is given by

$$C^{\mathrm{AB}}(t) = 2\max(0, |y_1 y_2^*| - \sqrt{y_3 y_3^* y_4 y_4^*}). \tag{10}$$

The time evolution of the concurrences C^{AB} from Eq. (10) is plotted in Fig. 1 for different χ/g and α, where solid lines correspond to $\alpha = \pi/4$ (the maximally entangled initial state) and dashed lines correspond to $\alpha = \pi/6$ (the partially entangled initial state).

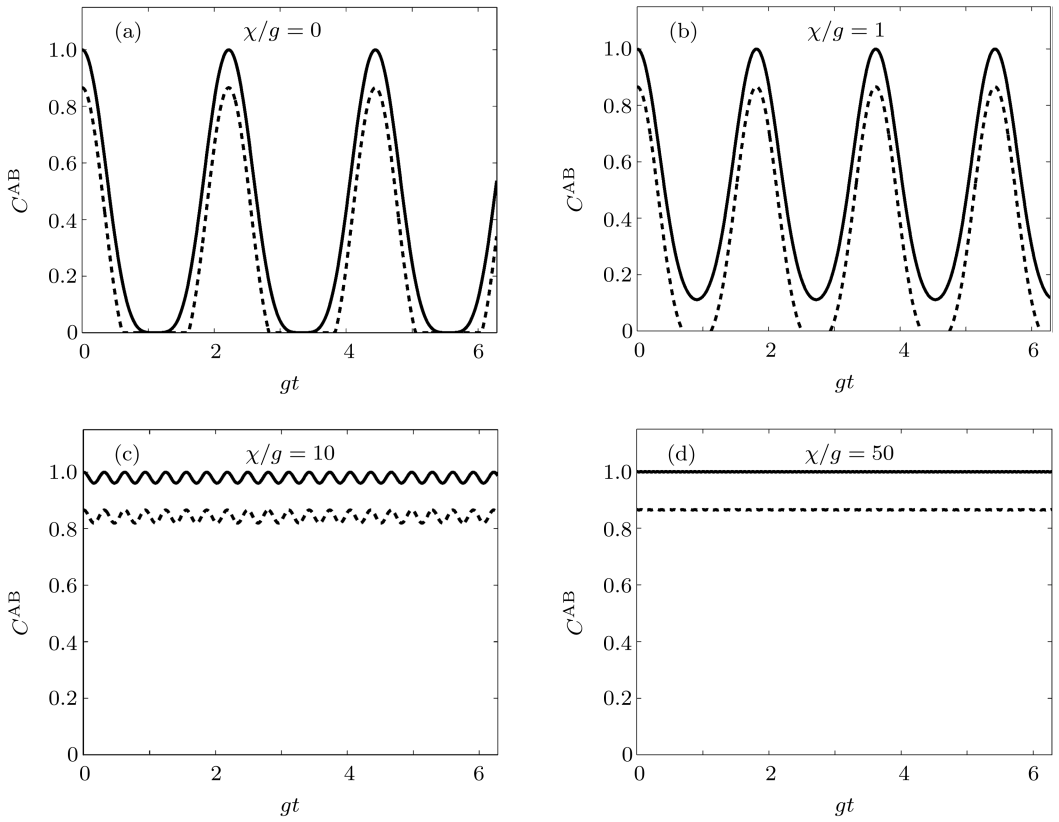


Fig. 1. The concurrence of $\boldsymbol{\rho}_{\mathrm{AB}}$ for atom–atom entanglement with the initial atomic state $|\Phi_{\mathrm{atom}}(0)\rangle = \cos\alpha\,|ee\rangle + \sin\alpha\,|gg\rangle$ for different χ/g, where solid lines correspond to $\alpha = \pi/4$ and dashed lines correspond to $\alpha = \pi/6$.

Figure 1(a) shows that in this initial state there is an ESD effect between two atoms when there is no Kerr-like medium ($\chi/g = 0$). It is apparent that the concurrence C^{AB} evolves periodically with a period $\pi/\sqrt{2}$, and the time interval for the zero entanglement when $\alpha = \pi/4$ is shorter than that when $\alpha = \pi/6$. In Fig. 1(b), corresponding to $\chi/g = 1$, it is observed that the evolution period of the concurrences C^{AB} decreases, and the time interval for the zero entanglement becomes shorter when $\alpha = \pi/6$. However, the

ESD disappears when $\alpha = \pi/4$. In Fig. 1(c), corresponding to $\chi/g = 10$, it is found that with the Kerr nonlinearity coefficient increasing gradually, the ESD completely disappears in either case, and the evolution period of the concurrences $C^{\rm AB}$ become much shorter than in Figs. 1(a) and 1(b).

This coincides with the formula of the evolution period

$$T = \frac{2\pi}{2\sqrt{\Delta}} = \frac{\pi}{\sqrt{2 + \left(\frac{\chi}{g}\right)^2}}. \tag{11}$$

From Fig. 1, it is also obvious that the larger the Kerr nonlinearity coefficient χ/g is, the bigger the minimum value of the atom–atom concurrence $C^{\rm AB}$ is. In addition, long-lived entanglement can be obtained when χ/g increases gradually. More interesting is the fact that, $C^{\rm AB}$ is always equal to 1 when $\chi/g = 50$ and $\alpha = \pi/4$ in Fig. 1(d). In this case, the two atoms can remain in a stationary entangled state and the degree of stationary entanglement equals their initial concurrence, which means that the Kerr-like medium can repress the ESD and increase the atom–atom entanglement efficiently. It might be very useful for entanglement preservation and QIP.

4. Atom–atom entanglement dynamics for the initial atomic state $|\Psi_{\rm atom}(0)\rangle = \cos\alpha\,|eg\rangle + \sin\alpha\,|ge\rangle$

Secondly, we assume that the initial state of the two atoms is in a combination of the other two Bell states $|\Psi^{\pm}\rangle$, but the initial states of the two cavities are still in vacuum state. Therefore, the initial condition of the whole system in this case can be expressed as

$$|\Psi_{\rm I}(0)\rangle = (\cos\alpha\,|eg\rangle + \sin\alpha\,|ge\rangle) \otimes |00\rangle\,, \tag{12}$$

the state of the total system at time t can be given by

$$\begin{aligned} |\Psi_{\rm I}(t)\rangle \;=\; & y_1\,|eg00\rangle + y_2\,|ge00\rangle \\ & + y_3\,|gg20\rangle + y_4\,|gg02\rangle\,. \end{aligned} \tag{13}$$

Substituting state $|\Psi_{\rm I}(t)\rangle$ into the Schrödinger equation in the interaction picture, we can obtain the equations of amplitudes(assuming $\hbar = 1$)

$$\begin{aligned} \dot{y}_1 &= -{\rm i}\sqrt{2}gy_3, \\ \dot{y}_2 &= -{\rm i}\sqrt{2}gy_4, \\ \dot{y}_3 &= -{\rm i}(\sqrt{2}gy_1 + 2\chi y_3), \\ \dot{y}_4 &= -{\rm i}(\sqrt{2}gy_2 + 2\chi y_4). \end{aligned} \tag{14}$$

Solving the differential equations (14), the probability amplitudes can be expressed as

$$\begin{aligned} y_1(t) &= \frac{1}{2}\cos\alpha\left(\varepsilon^{+}\exp(-{\rm i}\lambda^{-}gt) + \varepsilon^{-}\exp(-{\rm i}\lambda^{+}gt)\right), \\ y_2(t) &= \frac{1}{2}\sin\alpha\left(\varepsilon^{+}\exp(-{\rm i}\lambda^{-}gt) + \varepsilon^{-}\exp(-{\rm i}\lambda^{+}gt)\right), \\ y_3(t) &= \frac{1}{\sqrt{2}\Delta}\cos\alpha\left(\exp(-{\rm i}\lambda^{+}gt) - \exp(-{\rm i}\lambda^{-}gt)\right), \\ y_4(t) &= \frac{1}{\sqrt{2}\Delta}\cos\alpha\left(\exp(-{\rm i}\lambda^{+}gt) - \exp(-{\rm i}\lambda^{-}gt)\right). \end{aligned} \tag{15}$$

with

$$\begin{aligned} r &= \frac{\chi}{g}, \quad \Delta = \sqrt{r^2+2}, \\ \varepsilon^{\pm} &= 1 \pm \frac{r}{\Delta}, \quad \lambda^{\pm} = r \pm \Delta. \end{aligned}$$

Then the atom–atom density operator $\rho_{\rm AB}$ can be obtained from Eq. (13). In the atomic basis $\{|ee\rangle\,, |eg\rangle\,, |ge\rangle\,, |gg\rangle\}$, $\boldsymbol{\rho}_{\rm AB}$ is written as

$$\boldsymbol{\rho}_{\rm AB} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & y_1y_1^* & y_1y_2^* & 0 \\ 0 & y_2y_1^* & y_2y_2^* & 0 \\ 0 & 0 & 0 & y_3y_3^* + y_4y_4^* \end{pmatrix}. \tag{16}$$

It can be shown that the concurrence of this density matrix is given by

$$C^{\rm AB}(t) = \max(0, 2\,|y_1y_2^*|). \tag{17}$$

The time evolution of the concurrence $C^{\rm AB}$ from Eq. (17) is plotted in Fig. 2 for different χ/g and α, where solid lines correspond to $\alpha = \pi/4$ (the maximally entangled initial state) and dashed lines correspond to $\alpha = \pi/6$ (the partially entangled initial state), too. It is obvious that there is no ESD between the two atoms A and B in this case. Similar conclusions have been obtained in Refs. [27], [31], and [32].

Figure 2 also shows another property of entanglement dynamics of two atoms. From Fig. 2(a), we can see that, when each cavity without Kerr-like medium (corresponding to $\chi/g = 0$), the evolution of the concurrence $C^{\rm AB}$ fluctuates in the range of 0-1 regularly. In Fig. 2(b), with the increase of Kerr nonlinearity coefficient (corresponding to $\chi/g = 1$), it is found that

the fluctuation of the concurrence declines and the mean value of concurrence increases. This trend becomes more obvious in Figs. 2(c) and 2(d), where the coefficient χ/g becomes large. In a word, the Kerr-like medium can keep ESD in control; with strong Kerr-like medium in each of the two cavities, the entanglement between the two atoms can be maintained approximately constant.

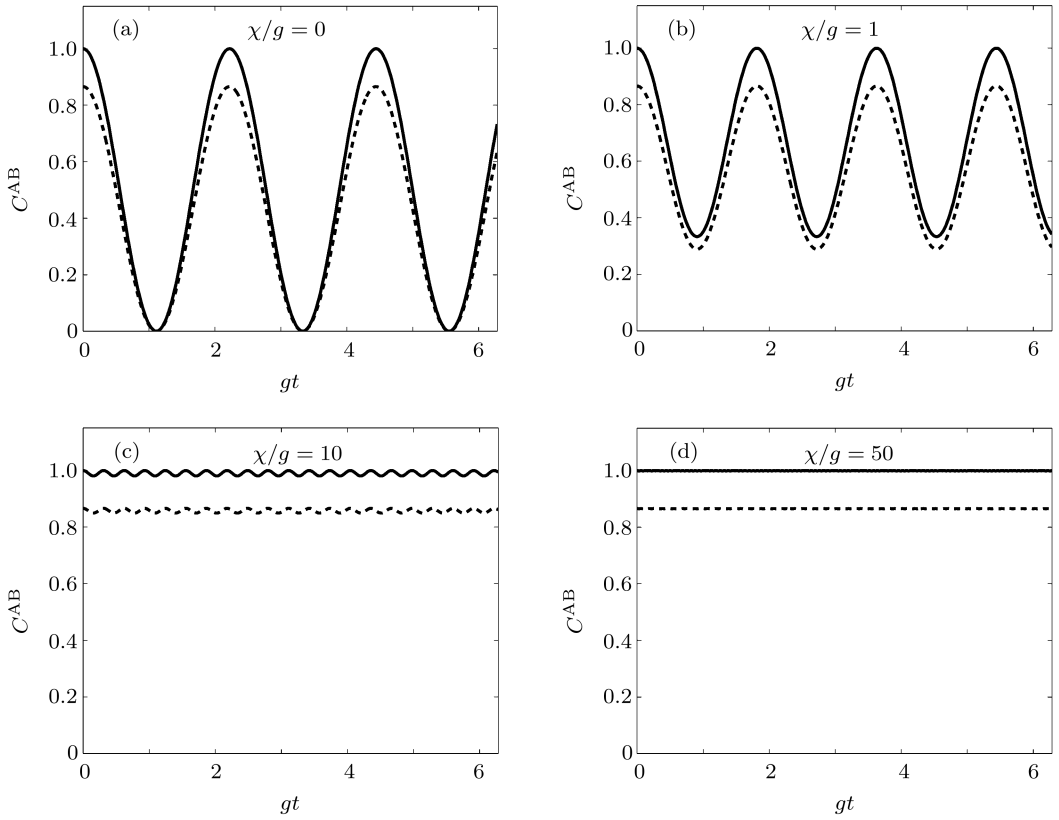


Fig. 2. The concurrence for atom–atom entanglement with the initial atomic state $|\Psi_{\mathrm{atom}}(0)\rangle = \cos\alpha\,|eg\rangle + \sin\alpha\,|ge\rangle$ for different χ/g, where solid lines correspond to $\alpha = \pi/4$ and dashed lines correspond to $\alpha = \pi/6$.

5. Conclusion

In this paper, the entanglement dynamics of a double two-photon JC model with Kerr-like medium is investigated by using the entanglement degree concurrence. For two types of atomic initial states, it is found that there occurs the ESD effect for the atomic initial state $|\Phi^{\pm}\rangle$, while it does not for atomic initial states $|\Psi^{\pm}\rangle$. By adjusting the Kerr nonlinear coefficient in the cavity, one can stabilize the entanglement between the two atoms and can effectively repress the appearance of ESD. It is also found that the Kerr effect is more obvious when each of the two cavities but not only one of them is filled with Kerr-like medium as in Ref. [27]. These results may be useful for QIP based on the entanglement.

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