Measurements of a periodically driving qubit using a quantum point contact*

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Using the method developed by Gurvitz [1996 *Phys. Rev.* B **53** 15932], we obtained the Bloch-type rate equations describing the entire system of a periodically driving qubit monitored by a quantum point contact detector. The results demonstrate that the isolated qubit can be kept in its initial state with a large driving frequency due to more difficult electron tunneling in qubit, and this initial state can always be measured at a small measurement-induced decoherence rate during a short time.

Keywords: periodically driving qubit, measurement, detector current, driving frequency
PACC: 0365, 0560

1. Introduction

A candidate for readout of the charge state in a quantum dot (QD) is the point contact (PC).[1−4] Recent studies showed that quantum PC can be used to study many quantum measurement problems of a two-state charge qubit or a multi-state system, such as measurement-induced relaxation and decoherence, energy exchange between the detector and the measured system, and so on.[5,6] When operating the PC readout for quantum dots, the PC current depends sensitively on the electrostatic environment, and a small fluctuation in the QD potential can be large enough to induce a measurable change in the PC current.[7] In recent experiments the terahertz radiation is introduced in quantum dots to raise the signal-to-noise ratio, which allows us to measure the charge state in an operation temperature up to 1.5 K by the PC detector.[8] It is shown that the PC as a charge state monitor has a very practical advantage.

In order to establish the optimal conditions for utilizing the PC as a more effective quantum detector, one needs to know how the PC detector influences the charge state. In Ref. [9], PC used to monitor an electrostatic qubit was investigated. It showed that occupation of the extra electron in different dots would have a distinct influence on transport current through the PC, which makes it possible to draw out the qubit state information from the ratio $I(t)/I_0$, where $I(t)$ is the average detector current and I_0 is the detector current in the absence of the qubit. The result supports that the qubit state can only be read out from the detector current with a large measurement-induced decoherence rate.

In this paper, we use the same model but let the qubit be driven by a periodic field. We discuss the influence of the periodic field on qubit motion and the improvement of the effectiveness of the detector for a small decoherence rate. Numerical results show that the electron oscillation in the qubit becomes more and more difficult with increasing periodically driving frequency, and the qubit can be pinned in its initial state with a large frequency. Importantly, we can measure this state during a short time at a small decoherence rate.

2. Point-contact detector near double dots

The model is shown schematically in Fig. 1, where the coupled double quantum dots (i.e. the qubit) with an extra electron stored, are driven by a periodic field $\Omega_0 \mathrm{e}^{\mathrm{i}\omega t}$. The PC detector is represented as a barrier, connected with two reservoirs at the chemical potentials μ_{L} and μ_{R}, respectively. The transmission probability of the barrier varies from T to T', depending on which quantum dot is occupied by the electron, see Figs. 1(a) and 1(b).

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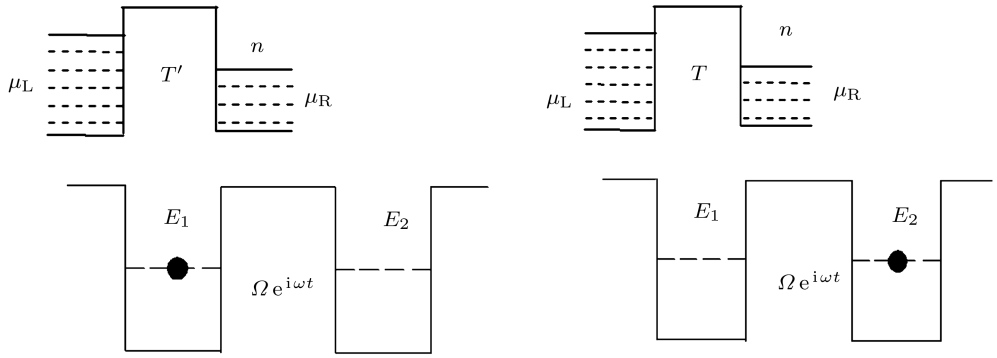


Fig. 1. A mesoscopic qubit (coupled quantum dots) driven by a periodic field, monitored continuously by the quantum PC. The parameter Ω_0 is the coupling amplitude and ω is the periodic driving frequency. The penetration coefficient of the PC varies from T to T' when an electron occupies the right dot (b) or left dot (a). The index n denotes the number of electrons accumulated in the collector at time t.

Now let us describe the entire setup shown in Fig. 1 by the tunneling Hamiltonian $H = H_{\rm q} + H_{\rm PC} + H_{\rm int}$.[10−13] Here we set both Planck's constant $\hbar$ and the electron charge e equal to unity. $H_{\rm q}$ and $H_{\rm PC}$ are the qubit and the PC Hamiltonians.

$$\begin{aligned} H_{\rm q} &= E_1 c_1^+ c_1 + E_2 c_2^+ c_2 \\ &\quad + \Omega_0 \left(c_1^+ c_2\, {\rm e}^{{\rm i}\omega t} + c_2^+ c_1\, {\rm e}^{-{\rm i}\omega t} \right), \\ H_{\rm PC} &= \sum_l E_l a_l^+ a_l + \sum_r E_r a_r^+ a_r \\ &\quad + \sum_{lr} \Omega_{lr} \left(a_l^+ a_r + a_r^+ a_l \right), \end{aligned} \tag{1}$$

where $c^+(c)$ is the creation (annihilation) operator for the electron in the qubit and $a^+(a)$ is the same operator for the detector; Ω_{lr} is the hopping amplitude between the states E_l and E_r in the left and right reservoirs; Ω_0 is the coupling amplitude and ω is the periodically driving frequency; $H_{\rm int}$ describes the interaction between the detector and the qubit. Since the presence of an electron in the left dot results in an effective increase of the PC barrier ($\Omega_{lr} \to \Omega_{lr} + \delta\Omega_{lr}$), the interaction term can be written as

$$H_{\rm int} = \sum_{lr} \delta\Omega_{lr} c_1^+ c_1 (a_l^+ a_r + a_r^+ a_l). \tag{2}$$

Initially all the levels in the reservoirs are filled up to the corresponding Fermi seas, with no electrons above the respective chemical potential, and the left dot is empty, while the right dot is occupied. Such a state is not stable, since the electron in the right dot would oscillate in the qubit, and the electrons would move from the left to the right reservoir. Thus the many-body wave function can be written in the occupation number representation as[9,10]

$$\begin{aligned} |\psi(t)\rangle \;=\; & \Bigg[b_1(t) c_1^+ + b_2(t) c_1^+ + \sum_{lr} b_{1lr}(t) c_1^+ a_r^+ a_l \\ & + \sum_{lr} b_{2lr}(t) c_2^+ a_r^+ a_l + \cdots \Bigg] |0\rangle\,, \end{aligned} \tag{3}$$

where $|0\rangle$ is called 'vacuum' state,[10] $b(t)$ are the time-dependent probability amplitudes for finding the system in the corresponding states, with the initial condition $b_2(0) = 1$ and all the other $b(0)$'s being zeros.

Now we start with the many-body Schrödinger equation ${\rm i}\,\dot{|\psi(t)\rangle} = H\,|\psi(t)\rangle$ for the entire system. It can be reduced to a Bloch-type rate equation describing the reduced density matrix $\sigma_{ij}^n(t)$, where $i, j = \{a, b\}$, and the index n denotes the number of electrons which have arrived at the right reservoirs by the time t. For example, $\sigma_{aa}^{(0)} = |b_1(t)|^2$ is the probability of finding no electron in the right reservoir and the left dot is occupied at time t, $\sigma_{bb}^{(1)} = \sum\limits_{lr} |b_{2lr}(t)|^2$ is the probability of finding one electron arriving in the right reservoir and the right dot is occupied. In general, the total probability for the electron to occupy the left dot is $\sigma_{aa}(t) = \sum\limits_n \sigma_{aa}^{(n)}(t)$, and the probability of the electron in the right dot is $\sigma_{bb}(t) = \sum\limits_n \sigma_{bb}^{(n)}(t)$. The corresponding off-diagonal density matrix elements $\sigma_{ab} = \sigma_{ba}^*$ describes the electron in the linear superposition of the states E_1 and E_2. As a result, we obtain the following Bloch-type equations describing the entire system

$$\begin{aligned} \dot{\sigma_{aa}^n} =& - D'\sigma_{aa}^n + D'\sigma_{aa}^{n-1} \\ & + {\rm i}\Omega_0 \left(\sigma_{ab}^n\, {\rm e}^{-{\rm i}\omega t} - \sigma_{ba}^n\, {\rm e}^{{\rm i}\omega t} \right), \end{aligned} \tag{4a}$$

$$\dot{\sigma}^n_{bb} = -D\sigma^n_{bb} + D\sigma^{n-1}_{bb} + \mathrm{i}\Omega_0\left(\sigma^n_{ba}\,\mathrm{e}^{\mathrm{i}\omega t} - \sigma^n_{ab}\,\mathrm{e}^{-\mathrm{i}\omega t}\right), \quad (4b)$$

$$\dot{\sigma}^n_{ab} = \mathrm{i}\varepsilon\sigma^n_{ab} + \mathrm{i}\Omega_0\left(\sigma^n_{aa} - \sigma^n_{bb}\right)\mathrm{e}^{\mathrm{i}\omega t} - \frac{1}{2}(D+D')\sigma^n_{ab} + \sqrt{DD'}\sigma^{n-1}_{ab}, \quad (4c)$$

where $\varepsilon = E_2 - E_1$, and $D(D')$ is the transition rate of an electron hopping from the left to the right detector reservoirs, $D = T(\mu_\mathrm{L} - \mu_\mathrm{R})/2\pi$, $D' = T'(\mu_\mathrm{L} - \mu_\mathrm{R})/2\pi$. It follows from Eqs. (4a)–(4c) that for the detector current one obtains

$$I_\mathrm{d}(t) = \sum_n n[\sigma^n_{aa}(t) + \sigma^n_{bb}(t)] = D'\sigma_{aa}(t) + D\sigma_{bb}(t). \quad (5)$$

Performing summation in Eqs. (4) over the number of electrons arriving at the right reservoir (n), we derive the final equations for the electron density matrix

$$\dot{\sigma}_{aa} = \mathrm{i}\Omega_0\left(\sigma_{ab}\,\mathrm{e}^{-\mathrm{i}\omega t} - \sigma_{ba}\,\mathrm{e}^{\mathrm{i}\omega t}\right), \quad (6a)$$

$$\dot{\sigma}_{bb} = \mathrm{i}\Omega_0\left(\sigma_{ba}\,\mathrm{e}^{\mathrm{i}\omega t} - \sigma_{ab}\,\mathrm{e}^{-\mathrm{i}\omega t}\right), \quad (6b)$$

$$\dot{\sigma}_{ab} = \mathrm{i}\varepsilon\sigma_{ab} + \mathrm{i}\Omega_0\left(\sigma_{aa} - \sigma_{bb}\right)\mathrm{e}^{\mathrm{i}\omega t} - \frac{T_\mathrm{d}}{2}\sigma_{ab}. \quad (6c)$$

Here $T_\mathrm{d} = \left(\sqrt{D} - \sqrt{D'}\right)^2$ is the decoherence rate, generated by the detector. Apparently, the driving frequency dependence is induced in these equations while they are seemingly similar to the equations describing the electron oscillations in an electrostatic qubit.[9]

3. Discussion

As discussed by Gurvitz in Ref. [9], the electrostatic qubit state cannot be read out by PC at a small decoherence rate (T_d). We speculate that periodic driving is a probable way to improve the effectiveness of the detector. We, therefore, focus on the situation for $\omega \neq 0$, and examine in advance the dynamics of the qubit that decoupled from the PC detector, described by Eqs. (6) with $T_\mathrm{d} = 0$. Figure 2 shows that the scope of vibration reduces with increasing driving frequency, and the electron can be pinned to the initial state with large frequency. This phenomenon is not surprising since for $\varepsilon = 0$ the periodic field would result in an effective misalignment of the levels E_1 and E_2, thus destroy the resonant-tunneling condition. As a result, the probability of electron tunneling decreases.

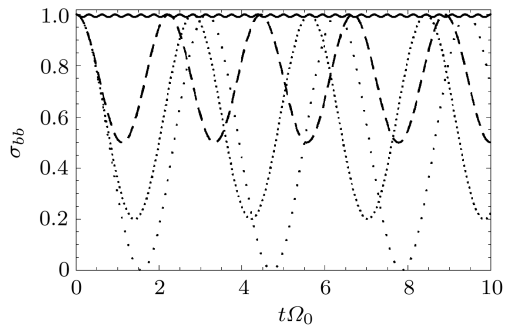


Fig. 2. Time-dependent electron-occupation probability $\sigma_{bb}(t)$ with $T_\mathrm{d} = 0$ for different driving frequencies: $\omega = 0$ (dotted line), Ω_0 (densely dotted line), $2\Omega_0$ (dashed line), and $20\Omega_0$ (solid line).

Now we consider how the detector measures the qubit at a small decoherence rate, for the transition rate $D = 10\Omega_0$ and $D' = 5\Omega_0$. The above results lead us to expect that when the qubit is monitored by the PC, the large driving frequency would also imprison the electron in the right dot, i.e. in the initial state as in Fig. 1(b). Then we can rewrite Eq. (5) as $I_\mathrm{d}(t) \approx D\sigma_{bb}(t)$. In this case, the detector current and the qubit oscillations will have the same variation trend. The result of our calculations for $\omega = 20\Omega_0$, confirms these qualitative arguments as shown in Fig. 3. Therefore, for small T_d, the electron can be imprisoned by controlling the periodic field, and this state can also be measured by PC detector.

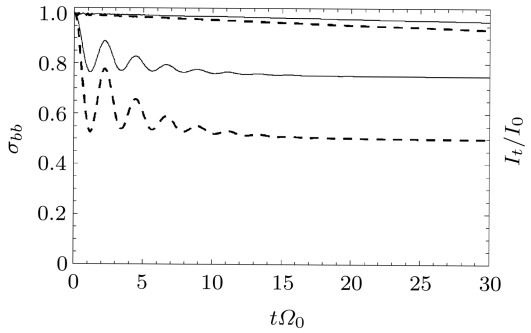


Fig. 3. Time-dependent of electron-occupation probability $\sigma_{bb}(t)$ (dashed line) and the ratio $I(t)/I_0$ (solid line) for $D = 10\Omega_0$ and $D' = 5\Omega_0$. From bottom to top, the curves correspond to driving frequency $\omega = 2\Omega_0$ and $20\Omega_0$.

However, in Fig. 3, we can see that an almost invisible split between the current curve and probability curve for $\omega = 20\Omega_0$. Actually, the split would be more obvious as the time continues, and then the qubit state cannot be known by the current. This phenomenon can be interpreted by the decoherence induced by quantum measurement.[14] The continuous monitoring of one of the dots destroys the off-diagonal

coherences of the electron density matrix. As a result, it becomes the statistical mixture: $\sigma_{jj'}(t) \to (1/2)\,\delta_{jj'}$ for $t \to \infty$. This discussion can be substantiated in Fig. 3 for $\omega = 2\Omega_0$.

Can the qubit state be directly extracted from the PC current for other initial state? Consider for arbitrary initial conditions $\sigma_{aa}(0) = 1-p$ and $\sigma_{bb}(0) = p$, $(0 \leq p \leq 1)$. Large driving frequency would almost keep the electron in the initial state at the beginning of a period of time, thus the ratio $I(t)/I_0$ can be rewritten as $(1-p)/2+p$, from which the probability p can be known. Therefore, regardless of the initial state, we can also read out the qubit initial state from the detector current during the short time.

4. Summary

In this paper, the PC detector for the measurements of a periodically driven qubit has been proposed. We find that the qubit motion can be controlled by the periodic field, and keeps its initial state with a large driving frequency. Besides, the qubit initial state can be monitored by the PC detector during a short time at a small coherence rate. However, the qubit state cannot be known from the detector current with time continuing, since the continuous monitoring of one of the dots destroys the off-diagonal elements (coherence) of the electron mixture matrix, which brings the electron away from the initial state.

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