Turbulence-induced changes in degree of polarization, degree of coherence and spectrum of partially coherent electromagnetic beams*

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Based on a recently formulated unified theory of coherence and polarization, a method is described to study turbulence-induced changes in the polarization, the coherence and the spectrum of partially coherent electromagnetic beams on propagation. The electromagnetic Gaussian Schell-model beam is taken as a typical example of partially coherent electromagnetic beams, and the closed-form expressions for the degree of polarization, the degree of coherence and the spectrum of electromagnetic Gaussian Schell-model beams propagating through atmospheric turbulence are derived in the quadratic approximation of Rytov's phase structure function. Some interesting results are obtained, which are illustrated by numerical examples and are explained in physics.

Keywords: unified theory of coherence and polarization, turbulence, partially coherent electromagnetic beams, degree of polarization, degree of coherence, spectrum

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1. Introduction

In the last two decades, there has been aroused the substantial interest in studying the propagation of laser beams through atmospheric turbulence due to many practical applications, such as the remote sensing, atmospheric optical communication, etc.[1,2] The influence of turbulence on the spreading of partially coherent beams was studied in Refs. [3]–[7]. The spectral properties of partially coherent beams propagating through atmospheric turbulence were investigated in Refs. [8] and [9]. The influence of turbulence on the correlation properties of partially coherent beams was studied in Refs. [10] and [11].

On the other hand, a unified theory of coherence and polarization has been developed, which makes it possible to study the changes not only in the degree of polarization, but also in the degree of coherence and the spectrum.[12] Correlation-induced changes in the degree of polarization, the degree of coherence and the spectrum of random electromagnetic beams propagating in free space were studied by using the unified theory of coherence and polarization.[13] Recently, the unified theory was applied to the investigation of the changes in the polarization of electromagnetic Gaussian Schell-model (EGSM) beams propagating through atmospheric turbulence.[14−16] The purpose of the present paper is to propose a method by which it is possible to study the changes not only in the degree of polarization, but also in the degree of coherence and the spectrum of partially coherent electromagnetic beams analytically. In this paper, based on the unified theory of coherence and polarization, turbulence-induced changes in the degree of coherence and the spectrum of the partially coherent electromagnetic beam are studied along with its changes in the degree of polarization. In order to obtain the analytical results with clear physical meaning, the quadratic approximation of Rytov's phase structure function is used. Some interesting results are obtained, which are illustrated by numerical examples and explained in physics.

2. Electric cross-spectral density matrix

In this paper, the EGSM beam is taken as a typical example of partially coherent electromagnetic beams. Assume that an EGSM beam propagates close to the z axis in atmospheric turbulence, which may be characterized by a 2×2 electric cross-spectral density matrix,[12,13] i.e.

$$\boldsymbol{W} \equiv W_{ij}(\boldsymbol{r}_1, \boldsymbol{r}_2, z, \omega)$$

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$$= \langle E_i^*(\boldsymbol{r}_1, z, \omega) E_j(\boldsymbol{r}_2, z, \omega) \rangle \quad (i = x, y; j = x, y), \quad (1)$$

where $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are transverse coordinates at the plane z, asterisk $*$ denotes the complex conjugate, $\langle\ \rangle$ represents the average over an ensemble of realizations of the electric field, ω is the angular frequency, E_x and E_y are electric field components which are taken to be along two mutually orthogonal directions perpendicular to the z-direction.

Using the extended Huygens–Fresnel principle, the element of the electric cross-spectral density matrix $W_{ij}(\boldsymbol{r}_1, \boldsymbol{r}_2, z, \omega)$ of an EGSM beam propagating through atmospheric turbulence reads[2]

$$\begin{aligned} & W_{ij}(\boldsymbol{r}_1, \boldsymbol{r}_2, z, \omega) \\ = & \left(\frac{k}{2\pi z}\right)^2 \iint \mathrm{d}\boldsymbol{r}'_1 \mathrm{d}\boldsymbol{r}'_2 W_{ij}^{(0)}(\boldsymbol{r}'_1, \boldsymbol{r}'_2, \omega) \\ & \times \exp\left\{ -\left(\frac{\mathrm{i}k}{2z}\right) \left[\left(\boldsymbol{r}'^2_1 - \boldsymbol{r}'^2_2\right) \right.\right. \\ & \left.\left. - 2\left(\boldsymbol{r}_1 \cdot \boldsymbol{r}'_1 - \boldsymbol{r}_2 \cdot \boldsymbol{r}'_2\right) + \left(\boldsymbol{r}_1^2 - \boldsymbol{r}_2^2\right) \right] \right\} \\ & \times \left\langle \exp\left[\psi(\boldsymbol{r}'_1, \boldsymbol{r}_1) + \psi^*(\boldsymbol{r}'_2, \boldsymbol{r}_2)\right] \right\rangle_m, \end{aligned} \quad (2)$$

where $W_{ij}^{(0)}(\boldsymbol{r}'_1, \boldsymbol{r}'_2, \omega)$ is the element of the electric cross-spectral density matrix at the source plane $z = 0$; k is the wave number related to the frequency ω by $k = \omega/c$, with c being the speed of light in vacuum; $\psi(\boldsymbol{r}', \boldsymbol{r})$ is the random phase factor which represents the effect of the turbulent atmosphere on a monochromatic spherical wave; $\langle\ \rangle_m$ denotes average over the ensemble of the turbulent medium, and it is expressed as[17]

$$\begin{aligned} & \left\langle \exp\left[\psi(\boldsymbol{r}'_1, \boldsymbol{r}_1) + \psi^*(\boldsymbol{r}'_2, \boldsymbol{r}_2)\right] \right\rangle_m \\ = & \exp\left[-0.5 D_\Psi(\boldsymbol{r}'_d, \boldsymbol{r}_d)\right] \end{aligned} \quad (3)$$

with $\boldsymbol{r}'_d = \boldsymbol{r}'_1 - \boldsymbol{r}'_2$, $\boldsymbol{r}_d = \boldsymbol{r}_1 - \boldsymbol{r}_2$, and $D_\Psi(\boldsymbol{r}'_d, \boldsymbol{r}_d)$ being the wave structure function. It is noted that Rytov method is known to be applicable to the weak turbulence regime especially when fourth-order moments such as scintillations are considered. Generally, the weak turbulence is associated with Rytov log amplitude variance $0.307 C_n^2 k^{7/6} z^{11/6}$ that is quite smaller than unity. However, in this paper we are concerned with the second-order moment studied by using the wave structure function which is approximated by the phase structure function. The Rytov's phase structure function is known to hold true not only for the case of "weak fluctuations", but also for the case of "strong fluctuations", i.e. if $0.307 C_n^2 k^{7/6} z^{11/6} > 0.5$.[18]

With the assumption of a locally homogeneous turbulent medium and Kolmogrov spectrum, it has been shown that when the transverse dimension of the source and the separation $\boldsymbol{r}_d$ between the observation points are much less than the first Fresnel zone, then[17]

$$\begin{aligned} & D_\Psi(\boldsymbol{r}'_d, \boldsymbol{r}_d) \\ = & \ 2.91 k^2 z \int_0^1 C_n^2(\eta z) \left|\eta \boldsymbol{r}_d + (1-\eta)\boldsymbol{r}'_d\right|^{5/3} \mathrm{d}\eta, \end{aligned} \quad (4)$$

where $C_n^2(z)$ is the refraction index structure constant which describes how strong the turbulence is. In general, the analytical expression for $D_\Psi(\boldsymbol{r}'_d, \boldsymbol{r}_d)$ cannot be obtained. However, if we use a quadratic function instead of a power of 5/3 in Eq. (4), equation (4) can be solved analytically. It is known that power of five thirds takes into account not only tip-tilt phase fluctuations but also the effects of defocusing, astigmatism, etc. A quadratic phase structure function implies that only wave tilts are induced on the wave by turbulence. However, approximating five thirds to quadratic will not cause appreciable deviations from the experimentally observed measurements,[19] and this quadratic approximation has been used widely.[16–20] Therefore, for the case of homogeneous turbulence, in the quadratic approximation of Rytov's phase structure function, equation (3) is written as[17]

$$\begin{aligned} & \left\langle \exp\left[\psi(\boldsymbol{r}'_1, \boldsymbol{r}_1) + \psi^*(\boldsymbol{r}'_2, \boldsymbol{r}_2)\right] \right\rangle_m \\ \cong & \exp\left[-\frac{\boldsymbol{r}'^2_d + \boldsymbol{r}'_d \cdot \boldsymbol{r}_d + r_d^2}{\rho_0^2}\right], \end{aligned} \quad (5)$$

where $\rho_0 = \left(0.545 C_n^2 k^2 z\right)^{-3/5}$ is the coherence length of a spherical wave propagating in turbulence, whose behaviour is described by the Kolmogorov model.

To simplify the analysis we assume that x and y components of the electric field are uncorrelated at each source point, i.e.

$$\begin{aligned} & W_{ij}^{(0)}(x'_1, x'_2, y'_1, y'_2, \omega) \\ = & \ W_{ji}^{(0)}(x'_1, x'_2, y'_1, y'_2, \omega) \\ = & \ 0, \quad (i \neq j). \end{aligned} \quad (6)$$

We consider an EGSM beam, whose diagonal elements of the electric cross-spectral density matrix at the source plane $z = 0$ in the rectangular coordinate system is expressed as

$$\begin{aligned} & W_{ii}^{(0)}(x'_1, x'_2, y'_1, y'_2, \omega) \\ = & \ I_i S_i^{(0)}(\omega) \exp\left(-\frac{x'^2_1}{w_{0i}^2}\right) \exp\left(-\frac{x'^2_2}{w_{0i}^2}\right) \\ & \times \exp\left(-\frac{(x'_1 - x'_2)^2}{2\sigma_{0i}^2}\right) \exp\left(-\frac{y'^2_1}{w_{0i}^2}\right) \end{aligned}$$

$$\times \exp\left(-\frac{y'^2_2}{w_{0i}^2}\right)\exp\left(-\frac{(y'_1-y'_2)^2}{2\sigma_{0i}^2}\right), \quad (7)$$

where the coefficient I_i is a constant, w_{0i} is the waist width, σ_{0i} is the spatial correlation length of the source at the plane $z=0$, and $S_i^{(0)}(\omega)$ is the normalized source spectrum. I_i, w_{0i} and σ_{0i} are independent of position but may depend on frequency. We assume, for simplicity, that the values of waist width w_{0i} for the x- and y-polarizations in the source are the same and independent of frequency ω, i.e. $w_{0x}=w_{0y}=w_0$. The spectra of the x- and the y-polarizations in the source are the same, and the Gaussian line is adopted, i.e.

$$S_x^{(0)}(\omega)=S_y^{(0)}(\omega)=S^{(0)}(\omega)=\exp\left[-\frac{(\omega-\omega_0)^2}{2\Gamma^2}\right], \quad (8)$$

where ω_0 is the central frequency of the source spectrum, and Γ is the bandwidth (HWFM).

Substituting Eqs. (5) and (7) into Eq. (2) and performing very tedious integrations, we find that $W_{ii}(x_1,x_2,y_1,y_2,z,\omega)$ in the rectangular coordinate system is given by

$$\begin{aligned}
&W_{ii}(x_1,x_2,y_1,y_2,z,\omega)\\
&=I_iS^{(0)}(\omega)\frac{k^2}{4z^2\beta_i^2\theta_i^2}\exp\left\{\frac{\mathrm{i}k}{2z}(x_1^2-x_2^2)-\frac{(x_1-x_2)^2}{\rho_0^2}+\left[\frac{(x_1-x_2)}{2\rho_0^2\beta_i}+\frac{\mathrm{i}k}{2z\beta_i}x_1\right]^2\right.\\
&\left.+\left[\frac{(1-\eta_i)(x_1-x_2)}{2\rho_0^2\theta_i}+\frac{\mathrm{i}k(x_2-\eta_ix_1)}{2z\theta_i}\right]^2\right\}\exp\left\{\frac{\mathrm{i}k}{2z}(y_1^2-y_2^2)-\frac{(y_1-y_2)^2}{\rho_0^2}+\left[\frac{(y_1-y_2)}{2\rho_0^2\beta_i}+\frac{\mathrm{i}k}{2z\beta_i}y_1\right]^2\right.\\
&\left.+\left[\frac{(1-\eta_i)(y_1-y_2)}{2\rho_0^2\theta_i}+\frac{\mathrm{i}k(y_2-\eta_iy_1)}{2z\theta_i}\right]^2\right\}, \quad (9)
\end{aligned}$$

where

$$\beta_i^2=\frac{1}{w_0^2}+\frac{1}{2w_0^2\alpha_i^2}+\frac{1}{\rho_0^2}-\mathrm{i}\frac{k}{2z}, \quad (10)$$

$$\theta_i^2=\frac{1}{w_0^2}+\frac{1}{2w_0^2\alpha_i^2}+\frac{1}{\rho_0^2}+\mathrm{i}\frac{k}{2z}-\eta_i^2, \quad (11)$$

$$\eta_i=\frac{1}{2\beta_i}\left(\frac{1}{w_0^2\alpha_i^2}+\frac{2}{\rho_0^2}\right) \quad (12)$$

with $\alpha_i=\sigma_{0i}/w_0$ being the global degree of spatial coherence, or simply, the coherence parameter,[21] $0<\alpha_i<+\infty$. The smaller α_i is, the worse the spatial correlation of the EGSM beam will be.

Equation (9) is the formula for the element of the electric cross-spectral density matrix of an EGSM beam in atmospheric turbulence, from which the closed-form expressions for the degree of polarization, the degree of coherence and the spectrum of an EGSM beam propagating through atmospheric turbulence can be derived.

3. Degree of polarization

The degree of polarization of an electromagnetic beam is given by[12,13]

$$P(x,y,z,\omega)=\sqrt{1-\frac{4\ \mathrm{Det}\ \boldsymbol{W}(x,y,z,\omega)}{\left[\mathrm{Tr}\ \boldsymbol{W}(x,y,z,\omega)\right]^2}}, \quad (13)$$

where Tr is the trace of the cross-spectral density matrix.

Substituting Eq. (7) into Eq. (13) and considering Eq. (6) yield

$$P_0=P(x,y,0,\omega)=\frac{|I_y-I_x|}{I_y+I_x}. \quad (14)$$

Equation (14) indicates that the polarization across the source is uniform.

Substituting Eq. (9) into Eq. (13) and considering Eq. (6), we obtain

$$P(x,y,z,\omega)=\frac{\left|\frac{I_x}{\beta_x^2\theta_x^2}\exp\left[-\left(\frac{k^2}{2z^2\beta_x^2\theta_x^2}\right)\left(\frac{x^2+y^2}{w_0^2}\right)\right]-\frac{I_y}{\beta_y^2\theta_y^2}\exp\left[-\left(\frac{k^2}{2z^2\beta_y^2\theta_y^2}\right)\left(\frac{x^2+y^2}{w_0^2}\right)\right]\right|}{\frac{I_x}{\beta_x^2\theta_x^2}\exp\left[-\left(\frac{k^2}{2z^2\beta_x^2\theta_x^2}\right)\left(\frac{x^2+y^2}{w_0^2}\right)\right]+\frac{I_y}{\beta_y^2\theta_y^2}\exp\left[-\left(\frac{k^2}{2z^2\beta_y^2\theta_y^2}\right)\left(\frac{x^2+y^2}{w_0^2}\right)\right]}. \quad (15)$$

Equation (15) shows that the degree of polarization of an EGSM beam propagating through atmospheric turbulence at the observation point (x, y, z) is dependent on C_n^2, α_i and I_i, but independent of Γ.

The following two special cases of Eq. (15) are of interest:

(a) For $C_n^2 = 0$, we have

$$P_\infty = \lim_{z\to\infty} P(x, y, z, \omega) = \frac{\left|I_y(1/\alpha_x^2 + 1) - I_x(1/\alpha_y^2 + 1)\right|}{I_y(1/\alpha_x^2 + 1) + I_x(1/\alpha_y^2 + 1)}. \quad (16)$$

Equation (16) indicates that in the limit of long-propagation distance, the degree of polarization P_∞ in free space tends to the value which depends on the correlation properties of the source.

(b) For $C_n^2 \neq 0$, we have

$$P_\infty = \lim_{z\to\infty} P(x, y, z, \omega) = \frac{|I_y - I_x|}{I_y + I_x} = P_0. \quad (17)$$

Equation (17) shows that P_∞ tends to the value at the source plane due to turbulence, which is the main result in Refs. [14]–[16].

Numerical calculations are performed by using Eq. (15) to illustrate the influence of turbulence on the degree of polarization. Figure 1 gives the contours of the degree of polarization $P(x, 0, z, \omega)$. The contours of $P(0, y, z, \omega)$ are similar to those of $P(x, 0, z, \omega)$, and omitted here. Figure 1 shows the curves for degree of polarization versus propagation distance z and relative transverse coordinate x/w_0 both in free space and in atmospheric turbulence. Figure 2 gives the curves for degree of polarization $P(x, y, z, \omega)$ versus propagation distance z for different values of the relative transverse coordinates x/w_0 and y/w_0. Figure 2(a) shows that after a sufficiently long propagation distance for different off-axis points the values of degree of polarization reach the same value of asymptotical value P_∞, which is different from the value at the source plane and determined by Eq. (16). Whereas, figure 2(b) indicates that in atmospheric turbulence, for different off-axis points the values of degree of polarization tend to the same value at the source plane in the limit of long-propagation distance, which is determined by Eq. (17). In addition, it can be seen that the degree of polarization across the xoy plane becomes uniform again after a sufficient long propagation distance both in free space and in atmospheric turbulence.

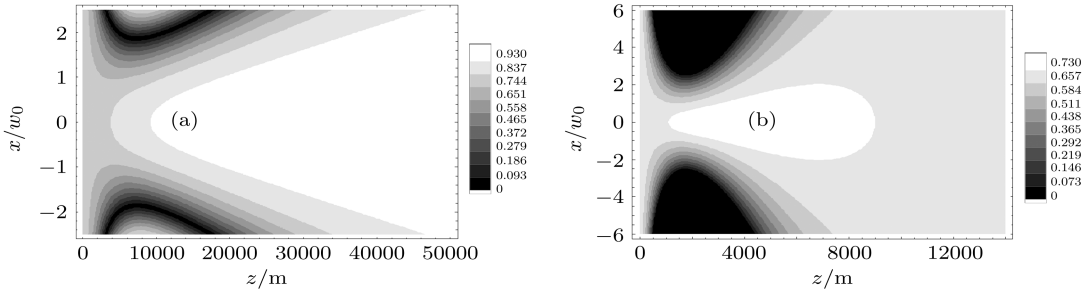


Fig. 1. Contours of the degree of polarization $P(x, 0, z, \omega)$, where calculation parameters are $w_0 = 0.05$ m, $\omega_0 = 3.2 \times 10^{15}$ s^{-1}, $I_y/I_x = 6$, $\alpha_x = 1.6$, $\alpha_y = 3$ and $C_n^2 = 0$ (a) and 10^{-14} m$^{-2/3}$ (b).

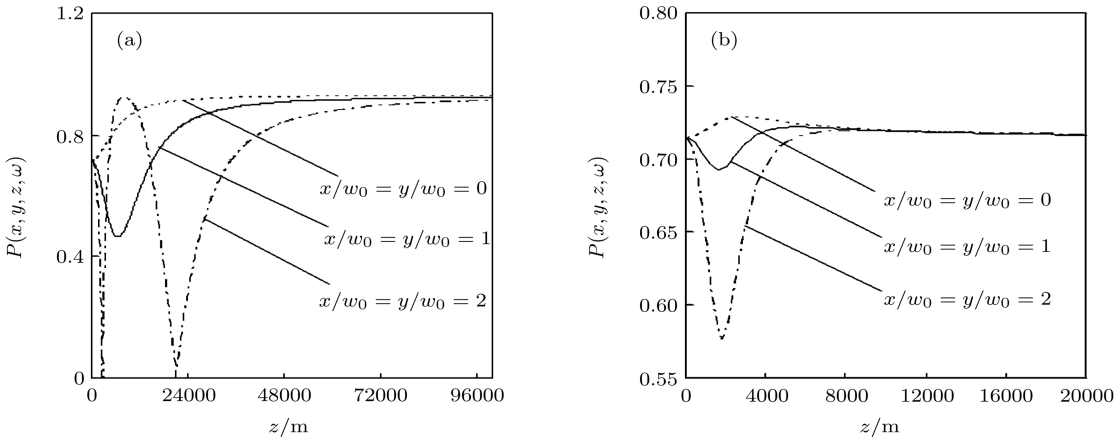


Fig. 2. Curves for degree of polarization $P(x, y, z, \omega)$ versus propagation distance z for different off-axis points, where the calculation parameters are the same as those in Fig. 1, but $C_n^2 = 0$ (a), and 10^{-14} m$^{-2/3}$ (b).

Generally, the degree of polarization of a partially coherent beam propagating through atmospheric turbulence is affected by two mechanisms. One is associated with the correlation properties of the source, which may be referred to as 'correlation-induced' change, and the other is due to the atmospheric turbulence, which is 'turbulence-induced' change. The interplay between them is rather complicated. In free space, the degree of polarization depends on the correlation properties of the source (see Eqs. (15) and (16)). The effect of the atmospheric turbulence becomes strong with the propagation distance increasing and 'washes out' all the information about the correlation properties of the source. This is the physical reason why the degree of polarization acquires its original value (see Eq. (17)).

4. Degree of coherence

A quantitative measure of the field correlation strength at a pair of points, $P_1(x_1, y_1, z)$ and $P_2(x_2, y_2, z)$, is given by the degree of coherence, which is defined as[12,13]

$$\mu(x_1, x_2, y_1, y_2, z, \omega) = \frac{\mathrm{Tr}\, \boldsymbol{W}(x_1, x_2, y_1, y_2, z, \omega)}{[\mathrm{Tr}\, \boldsymbol{W}(x_1, y_1, z, \omega)\mathrm{Tr}\, \boldsymbol{W}(x_2, y_2, z, \omega)]^{1/2}}. \quad (18)$$

Substituting Eq. (9) into Eq. (18) and considering a pair of points, $P_1(-x, -y, z)$ and $P_2(x, y, z)$, symmetric about the z axis, yield

$$\begin{aligned}
&\mu(-x, x, -y, y, z, \omega) \\
&= \frac{\dfrac{I_x}{\beta_x^2\theta_x^2}\exp\left\{-\left[\dfrac{4w_0^2}{\rho_0^2}+\dfrac{w_0^2}{\beta_x^2\theta_x^2}\left(\dfrac{k^2}{2z^2w_0^2\alpha_x^2}+\dfrac{2k^2}{z^2\rho_0^2}-\dfrac{k^2}{2z^2w_0^2}-\dfrac{2}{w_0^2\rho_0^4}\right)\right]\left(\dfrac{x^2+y^2}{w_0^2}\right)\right\}}{\dfrac{I_x}{\beta_x^2\theta_x^2}\exp\left[-\left(\dfrac{k^2}{2z^2\beta_x^2\theta_x^2}\right)\left(\dfrac{x^2+y^2}{w_0^2}\right)\right]+\dfrac{I_y}{\beta_y^2\theta_y^2}\exp\left[-\left(\dfrac{k^2}{2z^2\beta_y^2\theta_y^2}\right)\left(\dfrac{x^2+y^2}{w_0^2}\right)\right]} \\
&+ \frac{\dfrac{I_y}{\beta_y^2\theta_y^2}\exp\left\{-\left[\dfrac{4w_0^2}{\rho_0^2}+\dfrac{w_0^2}{\beta_y^2\theta_y^2}\left(\dfrac{k^2}{2z^2w_0^2\alpha_y^2}+\dfrac{2k^2}{z^2\rho_0^2}-\dfrac{k^2}{2z^2w_0^2}-\dfrac{2}{w_0^2\rho_0^4}\right)\right]\left(\dfrac{x^2+y^2}{w_0^2}\right)\right\}}{\dfrac{I_x}{\beta_x^2\theta_x^2}\exp\left[-\left(\dfrac{k^2}{2z^2\beta_x^2\theta_x^2}\right)\left(\dfrac{x^2+y^2}{w_0^2}\right)\right]+\dfrac{I_y}{\beta_y^2\theta_y^2}\exp\left[-\left(\dfrac{k^2}{2z^2\beta_y^2\theta_y^2}\right)\left(\dfrac{x^2+y^2}{w_0^2}\right)\right]}.
\end{aligned} \quad (19)$$

Equation (19) indicates that $\mu(-x, x, -y, y, z, \omega)$ depends on the structure constant of the refractive index C_n^2, the coherence parameter α_i and position parameters of observation points $P_1(-x, -y, z)$ and $P_2(x, y, z)$, but the bandwidth Γ has no effect on the degree of coherence. In addition, it is clear from Eq. (19) that $\mu(-x, x, -y, y, z, \omega)$ is strictly real function and a Gaussian profile at the z plane.

Numerical examples are given by using Eq. (19) to show the changes in the degree of coherence of EGSM beams propagating through atmospheric turbulence. Contours of the degree of coherence $\mu(-x, x, 0, 0, z, \omega)$ are shown in Fig. 3. As can be seen, in free space (see Fig. 3(a)) the width of the degree of coherence becomes large as the propagation distance z increases. However, in atmospheric turbulence there exists a maximum value of the width of the degree of coherence for an EGSM beam with small coherence parameters (see Fig. 3(b), $\alpha_x = 0.04$, $\alpha_y = 0.2$), which is in agreement with the result obtained in Ref. [10]. On the other hand, in atmospheric turbulence for an EGSM beam with large coherence parameters (see Fig. 3(c), $\alpha_x = 1$, $\alpha_y = 0.4$) in the beginning the width of the degree of coherence decreases with z increasing, and then it changes slowly, which is a new result obtained in this paper.

It is known that the degree of coherence of partially coherent beams propagating through atmospheric turbulence is affected by two mechanisms. One is associated with the correlation properties of the source, and the other is due to the atmospheric turbulence. There exists substantial original wave front randomness of partially coherent beams, and a small coherence parameter means large substantial original wavefront randomness. The turbulence produces additional phase randomness that adds to any one that is already there. In physics, the existence of substantial original randomness on the wave front will reduce the influence of atmospheric turbulence. In other words, the larger the substantial original randomness is, the less the turbulence produces additional phase randomness. Therefore, for small coherence parameter the effect of turbulence on the degree of coherence is small, and the width of the degree of coherence still increases

as the propagation distance increases in the beginning because there is an increase in the width of the degree of coherence in free space, and with the increase of propagation distance it reaches its maximum due to the effect of turbulence on the degree of coherence increasing. For a large coherence parameter the turbulence dominates the width of the degree of coherence, which results in a decrease in the width of the degree of coherence in the beginning, and then it changes slowly because of the increase in wave front randomness.

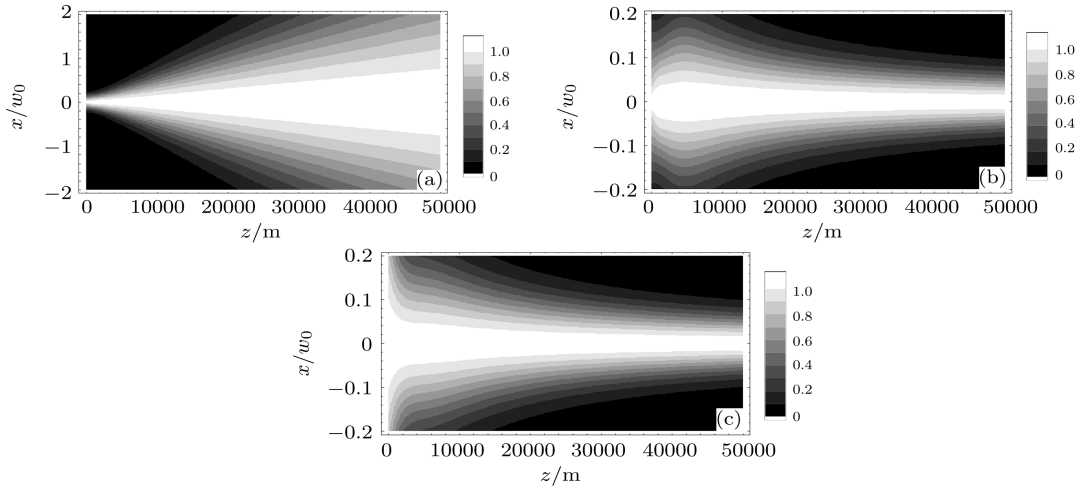


Fig. 3. Contours of the degree of coherence $\mu(-x, x, 0, 0, z, \omega)$, where the calculation parameters are $w_0 = 0.05$ m, $\omega_0 = 3.2 \times 10^{15}$ s^{-1}, $I_y/I_x = 5$, and the other parameters are (a) $\alpha_x = 0.04$, $\alpha_y = 0.2$, $C_n^2 = 0$; (b) $\alpha_x = 0.04$, $\alpha_y = 0.2$, $C_n^2 = 2 \times 10^{-15}$ m$^{-2/3}$; (c) $\alpha_x = 1$, $\alpha_y = 0.4$, $C_n^2 = 2 \times 10^{-15}$ m$^{-2/3}$.

5. Spectrum

The spectrum of an electromagnetic beam is given by the expression[12,13]

$$S(x, y, z, \omega) = \mathrm{Tr}\, \boldsymbol{W}(x, y, z, \omega). \tag{20}$$

The substitution of Eq. (9) into Eq. (20) yields

$$S(x, y, z, \omega) = S^{(0)}(\omega) M(x, y, z, \omega), \tag{21}$$

where

$$\begin{aligned} & M(x, y, z, \omega) \\ = & \frac{k^2}{4z^2} \left\{ \frac{I_x}{\beta_x^2 \theta_x^2} \exp\left[-\left(\frac{k^2}{2z^2 \beta_x^2 \theta_x^2} \right) \left(\frac{x^2 + y^2}{w_0^2} \right) \right] \right. \\ & \left. + \frac{I_y}{\beta_y^2 \theta_y^2} \exp\left[-\left(\frac{k^2}{2z^2 \beta_y^2 \theta_y^2} \right) \left(\frac{x^2 + y^2}{w_0^2} \right) \right] \right\} \end{aligned} \tag{22}$$

is called the spectral modifier[22] that describes how the turbulence modifies the source spectrum $S^{(0)}(\omega)$. $M(x, y, z, \omega)$ at the observation point (x, y, z) is dependent on C_n^2, α_i and I_i but independent of Γ.

Contours of the normalized spectrum $S(x, 0, z, \omega)/S_{\max}$ are shown in Fig. 4, where $S_{\max}$ is the maximum value of $S(x, 0, z, \omega)$. Figure 4(a) indicates that in free space the on-axis spectrum is blue-shifted, and the spectrum becomes red-shifted with x/w_0 increasing. However, the spectrum shift decreases due to turbulence (see Fig. 4(b)). In the previous work, the influence of turbulence on the spreading was studied only for monochromatic or quasi-monochromatic laser beams.[3–7] From Fig. 4(b) we can see that the turbulence results in a spreading of the spectrum of EGSM beams.

The physical explanation of on-axis spectral shift in Fig. 4 is given in Fig. 5. For simplicity, the normalized spectral modifier is taken as $M(\omega) = M(x, y, z, \omega)/M_{\max}$, where $M_{\max}$ is the maximum of $M(x, y, z, \omega)$. From Fig. 5 it can be seen that in free space, $M(\omega)$ increases with ω increasing, which results in blue shift. However, the change in $M(\omega)$ decreases with C_n^2 increasing. For $C_n^2 = 10^{-14}$ m$^{-2/3}$, $M(\omega)$ remains nearly unchanged as ω varies, i.e. the effect of $M(\omega)$ on $S^{(0)}(\omega)$ is very weak. Therefore, spectral shift decreases due to turbulence. The physical explanation of the off-axis spectral shift in Fig. 4 can be made in a similar way, and is omitted here.

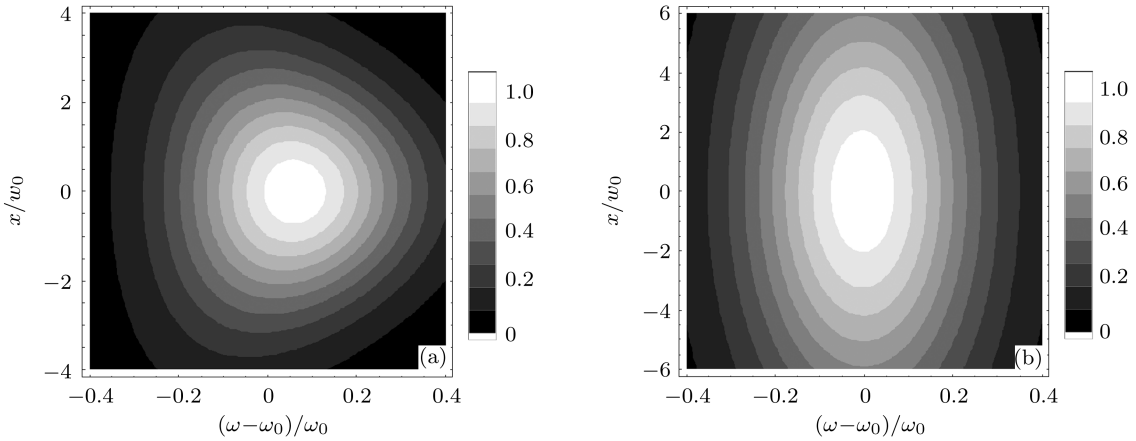


Fig. 4. Contours of the normalized spectrum $S(x,0,z,\omega)/S_{\max}$, where the calculation parameters are $w_0 = 0.05$ m, $\omega_0 = 3.2\times10^{15}$ s^{-1}, $\Gamma = 0.6\times10^{15}$ s^{-1}, $z = 10000$ m, $I_y/I_x = 1$, $\alpha_x = \alpha_y = 0.2$ and $C_n^2 = 0$ (a) and 10^{-14} m$^{-2/3}$ (b).

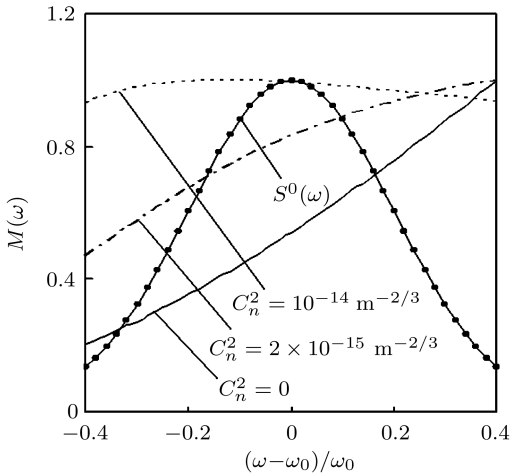


Fig. 5. Physical explanation of on-axis spectral shift in Fig. 4. The calculation parameters are the same as those in Fig. 4.

6. Concluding remarks

In this paper, based on the unified theory of coherence and polarization, turbulence-induced changes not only in the polarization, but also in the coherence and the spectrum of partially coherent electromagnetic beams on propagation are studied. The EGSM beam is taken as a typical example of partially coherent electromagnetic beams, the closed-form expressions for the degree of polarization, the degree of coherence and the spectrum of EGSM beams propagating through atmospheric turbulence are derived in the quadratic approximation of Rytov's phase structure function. In comparison with previous results, the results obtained in this paper are more general. For example, in Ref. [10] Lu *et al.* has pointed out that when the beam propagates a particular distance the width of the degree of coherence of an EGSM beam propagating through atmospheric turbulence reaches its maximum value. In the present paper, it is shown that in atmospheric turbulence our result reduces into that in Ref. [10] for the EGSM beam with small coherence parameters; whereas for the EGSM beam with large coherence parameters the width of degree of coherence decreases with the increase of propagation distance, and then it changes slowly. In addition, the bandwidth has an effect neither on the degree of polarization nor on the degree of coherence of EGSM beams. The method described in this paper would be useful for studying the changes in the polarization, the coherence and the spectrum of other types of partially coherent electromagnetic beams propagating through atmospheric turbulence.

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