Analytical results for the cluster size distribution in controlled deposition processes*

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(Received 13 March 2009; revised manuscript received 8 April 2009)

This paper proposes a controlled particle deposition model for cluster growth on the substrate surface and then presents exact results for the cluster (island) size distribution. In the system, at every time step a fixed number of particles are injected into the system and immediately deposited onto the substrate surface. It investigates the cluster size distribution by employing the generalized rate equation approach. The results exhibit that the evolution behaviour of the system depends crucially on the details of the adsorption rate kernel. The cluster size distribution can take the Poisson distribution or the conventional scaling form in some cases, while it is of a quite complex form in other cases.

Keywords: cluster growth, deposition, kinetic behaviour

PACC: 6810J, 8220M, 8115

1. Introduction

Cluster growth is a fundamental and important kinetic phenomenon which underlies a wide variety of fields in science and technology.[1−5] In recent decades, the kinetics of cluster growth have been thoroughly studied and well understood by theoretical computations and Monte Carlo simulations.[6−13] The clusters can grow through the binary coagulation scheme, $A_i + A_j \longrightarrow A_{i+j}$, where A_i denotes a cluster containing i monomers. Practically, there also exist some other classes of cluster growth mechanisms in physics and in social science. For example, Ispolatov *et al.*[14] and Leyvraz *et al.*[15] proposed a migration-driven growth mechanism, $A_i + A_j \to A_{i-1} + A_{j+1}$ $(i \leq j)$. That is, one monomer leaves a smaller aggregate and joins in a larger one. In order to mimic some realistic situations, in which the clusters are chemically stable under normal conditions and the coagulation of clusters occurs only in the presence of the catalyst, we also proposed a three-molecule catalysis-driven aggregation mechanism, $A_i + A_j + C \to A_{i+j} + C^*$, where C represents the catalyst.[16,17] Considerable research works have shown that distinct mechanisms may give rise to rich nonequilibrium kinetic behaviours of cluster growth.

Additionally, a deposit growth system may evolve according to another universal mechanism, namely, random sequential adsorption (RSA).[18−23] In the system, the particles are irreversibly deposited onto the substrate surface from a gas or solution, and they are then clamped in their positions. That is, the deposited particles can neither move on the surface, nor desorb back into the fluid phase. Moreover, if the particle interacts with the site which is already occupied by a particle, it will return instantaneously to the bulk, namely, the deposition overlap of particles is not permitted. Ben-Naim and Krapivsky studied the statistical properties of the Kolmogorov–Avrami–Johnson–Mehl nucleation and growth model in one dimension, in which the nucleation of stable phases is incorporated with ballistic growth.[22] Such an adsorption and deposition model can be used to mimic some physical situations in which the diffusion and desorption of deposited particles can be neglected in comparison with particle adsorption. In recent years, many self-assembly techniques, including glancing angle deposition (GLAD), have been successfully employed to fabricate three-dimensional nano-structured thin films on planar substrates, such as nano-columnar films and nanorod arrays.[24−32] In GLAD processes, the deposition overlap of particles is allowed to occur, even preferentially in some situations. Moreover, many experimental results showed that the films with three-dimensional microstructure can exhibit a variety of intriguing properties which are different from conven-

*Project supported by the National Natural Science Foundation of China (Grant Nos. 10775104, 10875086 and 10305009).

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tional films.[33−35] It is well known that the physical and chemical properties of three-dimensional films might have relation with the microstructure topology (such as the width and length of nanocolumns and their size distributions). Karabacak *et al.* studied the shadowing growth of isolated nanocolumnar structures on Si substrates through GLAD and observed that the width of isolated columns grows as a power law in the length.[36] Enomoto and Taguchi investigated the columnar growth in continuum ballistic deposition by computer simulations and found that morphological transitions of the growing surface can occur due to varying phenomenological parameters.[37] While in realistic scanning electron micrographs of isolated nano-columnar structures, one can clearly observe that long nanocolumns coexist with short ones.[25,36]

In this work, motivated by these works on the RSA and GLAD processes, we propose a controlled particle deposition model for three-dimensional cluster (equivalently, island or column) growth on the substrate surface and then present the exact results for the cluster size distribution. In our system, the substrate is laid in a chamber into which particles are continuously injected at a fixed rate. And the particle deposition can proceed at a constant rate p throughout the process. Without any loss of generality, we set $p = 1$. Such a controlled particle deposition may be realized by the method of pulsed laser deposition. Moreover, the diffusion between two different clusters and the desorption of deposited particles are negligible in comparison with the adsorption of particles. The deposition process is dominated by the adsorption reaction, and thus, it can be described by the monomer adsorption scheme, $A_1 + A_k \xrightarrow{I(k)} A_{k+1}$. Here, A_k denotes a pre-existing cluster of size k at a given surface site onto which the newly introduced particle will be adsorbed and then deposited, and $I(k)$ is the deposition (or equivalently, preferential adsorption) rate at which a new particle is adsorbed by a cluster of size k. For $k \geq 1$, the above reaction scheme can also be called as overlapping adsorption. In this model, the overlapping deposition onto a site or a cluster is permitted, even preferentially in the late stage of the deposition process. Since the deposits (i.e., the clusters) can be three-dimensional, the adsorption ability of a deposit may have relation with its effective surface area. In other words, the rate kernel $I(k)$ may depend on the cluster size k. It is believed that our model may mimic some realistic processes such as the deposit growth by GLAD in semiconductor and ceramic materials. On the other hand, this model is also of interest on its own right. The results show that the cluster (island) size distribution approaches the conventional scaling form in some cases, while it takes the famous Poisson distribution form in other cases.

The rest of the paper is organized as follows. In Section 2, we propose a deposition-driven cluster growth model. We then determine the analytic solution of the cluster size distribution by means of the rate equation approach. Finally, a brief summary is given in Section 3.

2. Kinetic behaviour of sequential particle deposition processes

In this work, we investigate analytically the kinetics of the controlled deposition model by employing the approach of generalized rate equation. At time t, the concentration of the clusters A_k consisting of k particles is denoted as $a_k(t)$. The generalized rate equations for our model can then be given as follows:

$$\begin{aligned}\frac{\mathrm{d}a_1}{\mathrm{d}t} &= \frac{I(0)(1-\sum_{k\geq 1} a_k) - I(1)a_1}{I(0)(1-\sum_{k\geq 1} a_k) + \sum_{k\geq 1} I(k)a_k},\\ \frac{\mathrm{d}a_j}{\mathrm{d}t} &= \frac{I(j-1)a_{j-1} - I(j)a_j}{I(0)(1-\sum_{k\geq 1} a_k) + \sum_{k\geq 1} I(k)a_k} \ (j>1),\end{aligned} \tag{1}$$

where the term $1 - \sum_{k\geq 1} a_k(t)$ denotes the number of the uncovered surface sites of the substrate at time t and $I(0)$ denotes the rate at which the newly-introduced particle is deposited onto the unoccupied surface site. It should be pointed out that in our investigation the total area of the substrate is normalized to unity.

Multiplying Eqs. (1) with j and summing them up, we obtain $\dot{M}(t) = \sum_{k\geq 1} k\dot{a}_k(t) = 1$, which directly yields the expression of the total size of clusters,

$$M(t) = M(0) + t. \tag{2}$$

Equation (2) indicates that the total size of clusters always grows linearly with time. It is not surprising because one and only one particle is deposited onto the substrate surface at each time step. We then determine the analytic solutions of Eqs. (1) in several distinct cases with special rate kernels.

2.1. The system with constant rate kernel

We first consider the case in which the adsorption rate kernel is independent of the size of matrix cluster. Set $I(0) = 1$ and $I(k) = 1 + \mu$ for all $k = 1, 2, 3, \ldots$. Such a kernel is sound for random deposition systems. For example, each cluster only has one tip onto which the newly-introduced particle can be adsorbed. Obviously, the physically meaningful value of μ must be greater than or equal to -1.

When $\mu = -1$, equation (1) reduces to

$$\frac{\mathrm{d}a_1}{\mathrm{d}t} = 1, \quad \frac{\mathrm{d}a_j}{\mathrm{d}t} = 0 \quad (j > 1). \tag{3}$$

From Eqs. (3) one readily obtains $a_1(t) = a_1(0) + t$ and $a_j(t) = a_j(0)$ $(j > 1)$. On the other hand, it is obvious that $\sum_{k\geq 1} a_k(t) \leq 1$. Thus, for such a system, the deposition process will cease at a cutoff time $t_\mathrm{c} = 1 - \sum_{k\geq 1} a_k(0)$, because at time t_c all the sites have been occupied by the newly deposited particles as well as those old clusters.

We then investigate the more interesting system with $\mu > -1$. Equations (1) can then be rewritten as

$$\begin{aligned} \frac{\mathrm{d}a_1}{\mathrm{d}t} &= \frac{(1-N) - (1+\mu)a_1}{1+\mu N}, \\ \frac{\mathrm{d}a_j}{\mathrm{d}t} &= \frac{(1+\mu)(a_{j-1} - a_j)}{1+\mu N} \quad (j > 1). \end{aligned} \tag{4}$$

Summing up Eqs. (4), we obtain

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{(1-N)}{1+\mu N}, \tag{5}$$

which is readily solved to give the implicit solution

$$(1+\mu)\ln\left[\frac{1-N(0)}{1-N(t)}\right] - \mu[N(t) - N(0)] = t. \tag{6}$$

For the special case $\mu = 0$ (namely, completely random deposition), equation (6) reduces to the explicit solution

$$N(t) = 1 - [1 - N(0)]\,\mathrm{e}^{-t}. \tag{7}$$

Substituting Eq. (7) into Eqs. (4), we readily deduce the exact solution of $a_k(t)$ as follows:

$$\begin{aligned} &a_k(t) \\ &= \mathrm{e}^{-t}\left\{a_k(0) + \sum_{i=1}^{k-1}\frac{a_i(0)}{(k-i)!}t^{k-i} + [1-N(0)]\frac{t^k}{k!}\right\}. \end{aligned} \tag{8}$$

Under the simplest initial condition, $a_k(0) = 0$, it follows from Eq. (8) that the cluster size distribution takes the famous Poisson distribution

$$a_k(t) = \frac{t^k}{k!}\,\mathrm{e}^{-t}. \tag{9}$$

In the long-time limit, we obtain $\langle a_k(t)\rangle = t$. So, the average size of clusters grows linearly with time.

Then we discuss the general case $\mu > -1$ and $\mu \neq 0$. In the long-time limit, equation (6) can be asymptotically rewritten as

$$N(t) \simeq 1 - C_1\,\mathrm{e}^{-t/(1+\mu)}, \tag{10}$$

where $C_1 = [1 - N(0)]\exp\{-\mu[1 - N(0)]/(1+\mu)\}$. Moreover, introducing a new time function, $E(t) = (1+\mu)\int_0^t [1+\mu N(t')]^{-1}\,\mathrm{d}t'$, we solve Eq. (4) to obtain the implicit solution

$$\begin{aligned} a_k(t) = {} & \frac{(1+\mu)^{k-1}}{\mu^k}[1-N(t)] + \sum_{i=1}^{k-1}\left[a_i(0) - \frac{(1+\mu)^{i-1}}{\mu^i}[1-N(0)]\right]\frac{[E(t)]^{k-i}}{(k-i)!}\,\mathrm{e}^{-E(t)} \\ & + \left\{a_k(0) - \frac{(1+\mu)^{k-1}}{\mu^k}[1-N(0)]\right\}\mathrm{e}^{-E(t)}. \end{aligned} \tag{11}$$

When $-1 < \mu < 0$ (namely, the deposited clusters have a little anti-affinity for particles of their kind), equation (11) can be asymptotically rewritten as the following equation at large times

$$a_k(t) \simeq \left\{a_k(0) - \frac{(1+\mu)^{k-1}}{\mu^k}[1-N(0)] + \sum_{i=1}^{k-1}\left[a_i(0) - \frac{(1+\mu)^{i-1}}{\mu^i}[1-N(0)]\right]\frac{t^{k-i}}{(k-i)!}\right\}\mathrm{e}^{-t}. \tag{12}$$

Especially, the size distribution of the clusters with $k \leq |\mu(1+\mu)^{-1}|t$ asymptotically takes the Poisson distribution form

$$a_k(t) \sim \left[a_1(0) - \frac{1-N(0)}{\mu}\right]\frac{t^{k-1}}{(k-1)!}\,\mathrm{e}^{-t}. \tag{13}$$

When $\mu > 0$, we find, by using Eqs. (10) and (11), that the cluster size distribution takes the exponential form at large times,

$$a_k(t) \simeq \frac{C_1}{1+\mu}\left(1+\frac{1}{\mu}\right)^k \mathrm{e}^{-t/(1+\mu)} \propto \exp[k\ln(1+\mu^{-1}) - (1+\mu)^{-1}t], \tag{14}$$

which is quite different from the above Poisson distribution in the case $-1 < \mu < 0$.

2.2. The system with the shifted linear rate kernel

We then turn to the system in which the adsorption rate is linear in the size of matrix cluster, i.e., $I(i) = i + \lambda$, $i = 0, 1, 2\ldots$. Here, λ denotes the initial attractiveness of the uncovered surface sites of substrate. If $\lambda = 0$, the surface sites that have not adsorbed any particles initially can keep uncovered for ever according to Eq. (1). However, in some realistic systems, every surface site of the substrate has a certain chance to adsorb the dissociating particles even if it is uncovered in the beginning of deposition process. Thus, we have $\lambda > 0$. Equation (1) then reduces to

$$\begin{aligned} \frac{\mathrm{d}a_1}{\mathrm{d}t} &= \frac{\lambda(1-N) - (1+\lambda)a_1}{\lambda + M}, \\ \frac{\mathrm{d}a_j}{\mathrm{d}t} &= \frac{(j-1+\lambda)a_{j-1} - (j+\lambda)a_j}{\lambda + M} \quad (j > 1) \end{aligned} \tag{15}$$

with the shorthand notation $M(t) = \sum_{k\geq 1} k a_k(t)$. Summing up Eq. (15), we obtain

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{\lambda(1-N)}{\lambda + M(0) + t}. \tag{16}$$

Equation (16) can be readily solved to yield

$$N(t) = 1 - C_3(C_2 + t)^{-\lambda}, \tag{17}$$

where $C_2 = \lambda + M(0)$ and $C_3 = [1 - N(0)]C_2^{\lambda}$. Substituting Eqs. (2) and (17) into Eq. (15), we determine the exact solution of $a_k(t)$ as follows:

$$\begin{aligned} a_k(t) = \Bigg[& a_k(0)C_2^{k+\lambda} + \sum_{i=1}^{k-1} \frac{a_i(0)C_2^{i+\lambda}\Gamma(k+\lambda)}{\Gamma(i+\lambda)\Gamma(k-i+1)} t^{k-i} \\ & + C_3 \frac{\Gamma(k+\lambda)}{\Gamma(\lambda)\Gamma(k+1)} t^k \Bigg] (C_2 + t)^{-k-\lambda}. \end{aligned} \tag{18}$$

Under the initial condition of plain substrate, $a_k(0) = 0$, equation (18) reduces to

$$a_k(t) = \lambda^{\lambda} \frac{\Gamma(k+\lambda)}{\Gamma(\lambda)\Gamma(k+1)} t^k (\lambda + t)^{-k-\lambda}. \tag{19}$$

In the region of $t \gg \lambda$ and $k \gg 1$, equation (19) can be asymptotically rewritten as

$$a_k(t) \simeq \frac{\lambda^{\lambda}}{\Gamma(\lambda)} k^{\lambda-1} t^{-\lambda} \exp(-\lambda k t^{-1}). \tag{20}$$

Equation (20) shows that at a given time, the size distribution of relatively small clusters ($k \ll t$) can asymptotically take a power law in size k while the large clusters with $k \gg t$ can hardly be fabricated. Most interestingly, the large-time size distribution of large clusters with size $k \gg 1$ approaches the conventional scaling form[8]

$$a_k(t) \sim k^{-\tau} t^{-w} \Phi[-k/S(t)], \quad S(t) \propto t^z. \tag{21}$$

Here, $S(t)$ denotes the typical size of clusters which plays a role analogous to that of the correlation length in ordinary critical phenomena. For this case, the typical size grows linearly with time.

2.3. The system with the general size-dependent rate kernel

Next, we discuss the general adsorption rate kernel $I(i) = i^{\gamma} + u$ ($i = 0, 1, 2\ldots$), with $\gamma > 0$ denoting the tendency of preferential adsorption onto large clusters ($\gamma \neq 1$). The γ is such a geometric index which characterizes the effective adsorption surface area of a cluster. Generally, γ may have relation with the fractal dimension of clusters. It is difficult to derive the exact solution of $a_k(t)$ from Eq. (1). We then turn to determine the asymptotic solution of $a_k(t)$ at large times. We also consider the system under the initial condition of plain substrate. It follows from Eqs. (10) and (17) that for both the cases $\gamma = 0$ and $\gamma = 1$, $N(t) \simeq 1$ in the long-time limit. Moreover, $M(t) \propto t$ at $t \gg 1$. Thus, we can assume that at $t \gg 1$,

$$\sum\nolimits_k k^{\gamma} a_k(t) \simeq v t^{\theta}, \tag{22}$$

where v and θ are two undetermined constants ($\theta > 0$). Summing up Eq. (1), we obtain the following asymptotic differential equation at large times:

$$\frac{\mathrm{d}N}{\mathrm{d}t} \simeq \frac{u(1-N)}{u + vt^{\theta}}. \tag{23}$$

From Eq. (23) one can then derive the asymptotic large-time solution of $N(t)$,

$$N(t) \simeq 1 - C_5 \mathrm{e}^{-C_4 t^{1-\theta}}, \tag{24}$$

where $C_4 = u/v(1-\theta)$ and C_5 is an integration constant.

By using Eq. (24) we can asymptotically rewrite Eq. (1) as

$$\begin{aligned}&\frac{\mathrm{d}a_1}{\mathrm{d}t} \simeq v^{-1}t^{-\theta}[uC_5\,\mathrm{e}^{-C_4t^{1-\theta}} - (u+1)a_1],\\ &\frac{\mathrm{d}a_j}{\mathrm{d}t} \simeq v^{-1}t^{-\theta}\{[u+(j-1)^\gamma]a_{j-1} - (u+j^\gamma)a_j\}\\ &(j>1).\end{aligned} \tag{25}$$

Under the initial condition of $a_k(0) = 0$, equations (25) are solved to give the large-time asymptotic solution of $a_k(t)$ as follows:

$$a_k(t) \simeq \frac{C_5u}{k^\gamma}\prod_{j=1}^{k-1}(uj^{-\gamma}+1)\,\mathrm{e}^{-C_4t^{1-\theta}}. \tag{26}$$

For $k \gg 1$, equation (26) can be rewritten as

$$a_k(t) \simeq \frac{C_5u}{k^\gamma}\,\mathrm{e}^{vC_4k^{1-\gamma}}\,\mathrm{e}^{-C_4t^{1-\theta}}. \tag{27}$$

One can use the equations $\sum_{j=1}^{k_{\max}} ja_j(t) \simeq t$ and $\sum_{j=1}^{k_{\max}} a_j(t) \simeq 1$ to estimate the maximum size $k_{\max}$ of clusters at time t. By using Eq. (27) we find $k_{\max} \sim t$. On the other hand, inserting Eq. (27) into Eq. (22), we can deduce $\theta = \gamma$ and $v = 1$. Thus, the large-time solution of $a_k(t)$ for $k \gg 1$ is determined as follows:

$$a_k(t) \simeq C_5uk^{-\gamma}\exp[C_4(k^{1-\gamma} - t^{1-\gamma})]. \tag{28}$$

For $\gamma > 1$, the cluster size distribution evolves to a steady state, and it asymptotically exhibits a power-law tail in size, $a_k(t) \sim k^{-\gamma}$; while for $\gamma < 1$, the cluster size distribution takes the stretched exponential form, $a_k(t) \sim \exp[C_4k^{1-\gamma} - \gamma\ln k]$.

2.4. The system with the special rate kernel $I(0) = 0$ and $I(k) = k^\gamma$

Finally, we discuss another interesting case in which the particles cannot be adsorbed onto the empty area of the surface. In the late stage of GLAD process, the incident deposition flux is preferentially deposited on top of the clusters (islands) and particles are difficult to reach the valleys between clusters due to the shadowing effect, and thus, the initially shadowy areas may be left unoccupied for ever.[36] Moreover, the tall columns (i.e., large-size clusters) may have more probability of capturing particles from the deposition flux and get taller. So, the adsorption rate kernel may depend on the size of matrix cluster. Besides, this model can also be used to mimic the deposition-driven growth of suspended clusters in a solution with initial seeds. Set $I(0) = 0$ and $I(k) = k^\gamma$. Then the governing rate equation can be written as

$$\frac{\mathrm{d}a_k}{\mathrm{d}t} = \frac{I(k-1)a_{k-1} - I(k)a_k}{\sum_{j=1}^{\infty} I(j)a_j}, \tag{29}$$

where we impose the boundary condition $a_0(t) \equiv 0$. Summing up Eq. (29) over all k, we obtain $\dot{N}(t) = \sum_{k=1}^{\infty}\dot{a}_k(t) = 0$, which yields $N(t) \equiv N(0)$. So, the total number of clusters is conserved by the dynamics of this deposition model.

The γ-th moment of the cluster size distribution is set as $M_\gamma(t) = \sum_{j=1}^{\infty} j^\gamma a_j(t)$. Introducing a new time variable, $T(t) = \int_0^t p[M_\gamma(t')]^{-1}\,\mathrm{d}t'$, we recast Eq. (29) to

$$\frac{\mathrm{d}a_k(T)}{\mathrm{d}T} = (k-1)^\gamma a_{k-1}(T) - k^\gamma a_k(T). \tag{30}$$

Equation (30) can be exactly solved for arbitrary conditions. Here, we only consider the important monodisperse initial case, namely, $a_k(0) = \delta_{k1}$. By solving Eq. (30) we then obtain $a_1(T) = \mathrm{e}^{-T}$ and the asymptotic solution of $a_k(T)$ $(k > 1)$ at large times,

$$a_k(T) = \begin{cases}\displaystyle\prod_{j=2}^{k}\frac{(j-1)^\gamma}{j^\gamma-1}\,\mathrm{e}^{-T} + \sum_{l=2}^{k} C_{lk}\,\mathrm{e}^{-l^\gamma T} & \text{if } \gamma \neq 0,\\ \dfrac{1}{(k-1)!}T^{k-1}\,\mathrm{e}^{-T} & \text{if } \gamma = 0,\end{cases} \tag{31}$$

where C_{lk} $(l = 2, 3, \ldots, k)$ are a set of integration constants. Moreover, all the constants C_{lk} are related as $\sum_{l=2}^{k} C_{lk} = -\prod_{j=2}^{k}(j-1)^\gamma/(j^\gamma-1)$ and $(k-1)^\gamma C_{l,k-1} = (k^\gamma - l^\gamma)C_{lk}$, where $l = 2, 3, \ldots, k$ and $k = 2, 3, 4, \ldots$. Obviously, all the constants C_{lk} can be determined by the above two expressions. Now the problem is reduced to determine the time function $T(t)$. Unfortunately, it is difficult to deduce the exact expression of $T(t)$. We then use the definition of $T(t)$ to estimate it and obtain the following asymptotic expression:

$$T(t) \simeq \begin{cases} C_6 + C_7t^{1-\gamma} & \text{if } \gamma \neq 1,\\ \ln(1+t) & \text{if } \gamma = 1,\end{cases} \tag{32}$$

where C_6 and C_7 are two constants. In the long-time limit, the size distribution of large clusters $(k \gg 1)$ can be asymptotically rewritten as

$$a_k(t) \simeq \begin{cases} k^{-\gamma}\,\mathrm{e}^{-C_6} + \displaystyle\sum_{l=2}^{k} C_{lk}\,\mathrm{e}^{-l^\gamma C_6} & \text{if } \gamma > 1,\\ \mathrm{e}^{k^{1-\gamma}}\,\mathrm{e}^{-C_7t^{1-\gamma}} & \text{if } 0 < \gamma < 1.\end{cases} \tag{33}$$

In the special case of $\gamma = 0$ (namely, completely random deposition case), the cluster size distribution takes the Poisson distribution form

$$a_k(t) = \frac{t^{k-1}}{(k-1)!}\exp(-t). \tag{34}$$

And in the case $\gamma = 1$, from Eq. (30) one can readily derive the exact solution

$$a_k(t) = (1+t)^{-1}[1-(1+t)^{-1}]^{k-1}. \tag{35}$$

In the region of $k \gg 1$ and $t \gg 1$, equation (35) can be further rewritten as

$$a_k(t) \simeq t^{-1}\exp(-kt^{-1}), \tag{36}$$

which approaches the conventional scaling form of Eq. (21) with the universal exponents $\tau = 0$ and $w = z = 1$.

The above results show that for this system the cluster size distribution cannot be scaled in most cases, except for the special case of $\gamma = 1$.

3. Summary

We have proposed a simple cluster growth model, in which, at every time step, one and only one new particle is injected into the system and then immediately deposited onto the substrate. We have investigated the size distribution of clusters (islands) on the substrate surface by using the generalized rate equation. The results show that the evolution behaviour of the system depends strongly on the details of the adsorption rate kernel. The cluster size distribution $a_k(t)$ takes the Poisson distribution if the adsorption rate kernel is independent of cluster size; while it can approach the conventional scaling form if the adsorption rate kernel is linear in cluster size. In most cases with the general rate kernel, the cluster size distribution is of a quite complex form. This model is expected to provide some qualitative predictions of the cluster size distribution in realistic processes such as the epitaxial growth by controlled ion implantation.

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