Incoherently coupled soliton pairs in nonlocal nonlinear media*

Lu Ke-Qing(卢克清)[a)†], Li Ke-Hao(李可昊)[a)c)], Zhao Wei(赵 卫)[a)], Zhang Yi-Qi(张贻齐)[a)], Zhang Mei-Zhi(张美志)[a)], Zhang Yu-Hong(张玉虹)[a)], Cheng Guang-Hua(程光华)[a)], and Zhang Yan-Peng(张彦鹏)[b)]

[a)] *State Key Laboratory of Transient Optics and Photonics, Xi'an Institute of Optics and Precision Mechanics, Chinese Academy of Sciences, Xi'an* 710119, *China*

[b)] *Department of Electronic Science and Technology, Xi'an Jiaotong University, Xi'an* 710049, *China*

[c)] *Graduate School of the Chinese Academy of Sciences, Beijing* 100049, *China*

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We show that incoherently coupled soliton pairs can exist in nonlocal Kerr-type nonlinear media. Such solitons can propagate in bright–bright, dark–dark, and gray–gray configurations. When the nonlocal nonlinearity is absent, these bright–bright and dark–dark soliton pairs are those observed previously in local Kerr-type nonlinear media. Our analysis indicates that for a self-focusing nonlinearity the intensity full width half maximum (FWHM) of the bright–bright pair components increases with the degree of nonlocality of the nonlinear response, whereas for a self-defocusing nonlinearity the intensity FWHM of the dark–dark and gray–gray pair components decreases with the increase in the degree of nonlocality of the nonlinear response. The stability of these soliton pairs has been investigated numerically and it has been found that they are stable.

Keywords: soliton pair, Kerr effect, nonlinear optics

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The nonlocality of the nonlinearity plays an important role in many nonlinear optical materials, such as thermo-optical materials,[1] nematic liquid crystals,[2] plasmas,[3] photorefractive crystals,[4−13] and Bose–Einstein condensates with long-range interparticle interactions.[14] Nonlocality means that the change of the refractive index in a particular point is determined by the light intensity not only in the same point but also in its vicinity. The nonlocal effect brings new features in the development of modulation instability[15] and azimuthal instability,[16] prevents catastrophic collapse of multidimensional solitons,[17] and stabilizes vortex[18] and two-dimensional (2D) fundamental solitons.[15] Recent work on nonlocal nonlinearity has shown that a large number of multi-hump multi-component vector solitons are the remarkable stabilization,[19] and stable gray solitons can be formed in defocusing nonlocal media.[20] In addition, vector solitons with dark–bright pairs in nonlocal Kerr-type nonlinear media have been studied.[21] It has been shown that stable bright solitons guided by dark soliton backgrounds can be formed in nonlocal self-defocusing system. However, can bright–bright and dark–dark soliton pairs exist in nonlocal Kerr-type nonlinear media? We should pay attention to this intriguing and challenging question.

In this paper, we show that incoherently coupled soliton pairs can exist in nonlocal Kerr-type nonlinear media. These solitons can exist in bright–bright, dark–dark, and gray–gray configurations. When the nonlocal nonlinearity is absent, bright–bright and dark–dark soliton pairs in nonlocal Kerr-type nonlinear media are those observed or studied previously in local Kerr-type nonlinear media. Our analysis indicates that for a self-focusing nonlinearity the intensity full width half maximum (FWHM) of the bright–bright pair components increases with the degree of nonlocality of the nonlinear response, and for a self-defocusing nonlinearity the intensity FWHM of the dark–dark and gray–gray pair components decreases with the increase in the degree of nonlocality of the nonlinear response. The stability of our solutions has been investigated numerically and it has been found that they are stable.

Let us consider two mutually incoherent optical beams that propagate in a nonlocal Kerr-type non-

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†Corresponding author. E-mail: kqinglu@opt.ac.cn

linear medium along the z axis and are allowed to diffract only along the x direction. At this point, the refractive index change Δn along the x axis is given by[14−24]

$$\Delta n\left(I\right)=\pm\int_{-\infty}^{\infty}R\left(x'-x\right)I\left(x',z\right)\mathrm{d}x', \tag{1}$$

where $I=I\left(x,z\right)$ is total power density of the two optical beams, the response function $R\left(x\right)$ characterizes the nonlocal response of the medium and is assumed to be real, localized, and symmetric, and the upper (lower) sign corresponds to a focusing (defocusing) nonlinearity. Moreover, let us assume that the response function $R\left(x\right)$ is narrow compared to the extent of the two optical beams, i.e., the nonlocality is weak. In this case, equation (1) takes the form[22−24]

$$\Delta n\left(I\right)=\pm\left(I+\delta\frac{\partial^2 I}{\partial x^2}\right), \tag{2}$$

where parameter δ stands for the degree of nonlocality of the nonlinear response. By associating slowly varying envelopes $\phi\left(x,z\right)$ and $\varphi\left(x,z\right)$, then one quickly finds the following set of paraxial wave equations[21,23]

$$\mathrm{i}\frac{\partial\phi}{\partial z}+\frac{1}{2}\frac{\partial^2\phi}{\partial x^2}\pm\left[\left|\phi\right|^2+\left|\varphi\right|^2+\delta\frac{\partial^2\left(\left|\phi\right|^2+\left|\varphi\right|^2\right)}{\partial x^2}\right]\phi=0, \tag{3}$$

$$\mathrm{i}\frac{\partial\varphi}{\partial z}+\frac{1}{2}\frac{\partial^2\varphi}{\partial x^2}\pm\left[\left|\phi\right|^2+\left|\varphi\right|^2+\delta\frac{\partial^2\left(\left|\phi\right|^2+\left|\varphi\right|^2\right)}{\partial x^2}\right]\varphi=0, \tag{4}$$

where $I=\left|\phi\right|^2+\left|\varphi\right|^2$ is used because the two optical beams are mutually incoherent. In what follows, we will discuss the possible soliton pair solutions of Eqs. (3) and (4). Their characteristics and stability properties will also be considered.

Let us first consider a self-focusing nonlinearity, which corresponds to the upper sign in Eqs. (3) and (4). We search for bright–bright soliton pair solutions to Eqs. (3) and (4) of the form[15]

$$\phi\left(x,z\right)=u\left(x\right)\cos\theta\exp\left(\mathrm{i}\Gamma z\right), \tag{5}$$

$$\varphi\left(x,z\right)=u\left(x\right)\sin\theta\exp\left(\mathrm{i}\Gamma z\right), \tag{6}$$

where the profile $u\left(x\right)$ is real, symmetric, and exponentially localized, Γ is the propagation constant, and the parameter θ is an arbitrary projection angle. In the limit of the self-focusing nonlinearity, substitution of Eqs. (5) and (6) into Eqs. (3) and (4) leads to the following differential equation

$$\frac{\mathrm{d}^2u}{\mathrm{d}x^2}+2\left(u^2-\Gamma\right)u+2\delta u\frac{\mathrm{d}^2u^2}{\mathrm{d}x^2}=0. \tag{7}$$

For bright solitons one requires the boundary conditions: $u\left(\infty\right)=u'\left(\infty\right)=0$, $u'\left(0\right)=0$, and $u\left(0\right)=u_0$. When $\mathrm{d}u/\mathrm{d}x$ is multiplied by Eq. (7), we have

$$4\delta\left[\frac{\mathrm{d}^2u}{\mathrm{d}x^2}u+\left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)^2\right]\frac{\mathrm{d}u}{\mathrm{d}x}u+2\frac{\mathrm{d}u}{\mathrm{d}x}u^3-2\Gamma\frac{\mathrm{d}u}{\mathrm{d}x}u+\frac{\mathrm{d}^2u}{\mathrm{d}x^2}\frac{\mathrm{d}u}{\mathrm{d}x}=0, \tag{8}$$

of which the first integral is given by

$$\left(4\delta u^2+1\right)\left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)^2+\left(u^2-2\Gamma\right)u^2=C. \tag{9}$$

Substituting $u\left(\infty\right)=u'\left(\infty\right)=0$ and $x\to\infty$ into Eq. (9) leads to $C=0$. On the other hand, substitution of $u'\left(0\right)=0$, $u\left(0\right)=u_0$, $C=0$, and $x=0$ into Eq. (9) yields $u_0^2=2\Gamma$. Thus we can rewrite Eq. (9) as

$$\pm\frac{\mathrm{d}x}{\mathrm{d}u}=\frac{4\delta u}{u_0^2-u^2}\sqrt{\frac{u_0^2-u^2}{4\delta u^2+1}}+\frac{1}{u\left(u_0^2-u^2\right)}\sqrt{\frac{u_0^2-u^2}{4\delta u^2+1}}. \tag{10}$$

By integrating Eq. (10) once and employing $u\left(0\right)=u_0$, we find that

$$\pm x=\frac{1}{u_0}\tanh^{-1}\left\{\left[\frac{u_0^2-u^2}{u_0^2\left(1+4\delta u^2\right)}\right]^{1/2}\right\}+\left(4\delta\right)^{1/2}\tan^{-1}\left\{\left[\frac{4\delta\left(u_0^2-u^2\right)}{1+4\delta u^2}\right]^{1/2}\right\}, \tag{11}$$

which is known to describe single beam bright solitons.[24] The soliton pair components can be considered as the θ projections of the fundamental bright soliton envelope. Figure 1 shows the normalized intensity profiles of the soliton pair and the associated refractive index change when $u_0=1$, $\theta=30^\circ$, and $\delta=0$, 0.25, and 0.5. Figure 1 also illustrates that the intensity FWHM of the two components increases with δ.

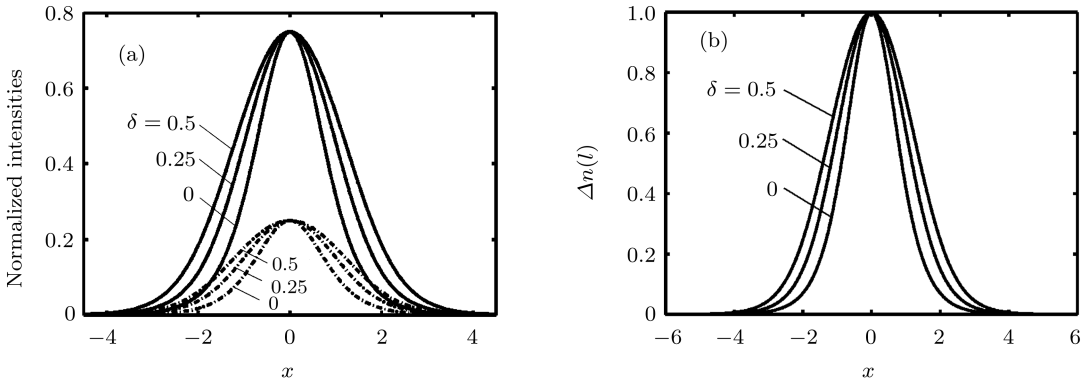


Fig. 1. Components of (a) bright–bright soliton pairs, $|\phi|^2$ (solid curve) and $|\varphi|^2$ (dot-dashed curve), and corresponding (b) refractive index change when $u_0 = 1$, $\theta = 30°$, and $\delta = 0$, 0.25, and 0.5.

For a self-defocusing nonlinearity, the sign of the refractive index change is negative, which corresponds to the lower sign in Eqs. (3) and (4). We introduce a spatial variable $\xi = x - Vz$, with V being the soliton transverse velocity, and look for soliton pair solutions of the form[23]

$$\phi(x,z) = \sqrt{\sigma(\xi)}\cos\theta\exp\left[\mathrm{i}\Gamma z + \mathrm{i}\vartheta(\xi)\right], \quad (12)$$

$$\varphi(x,z) = \sqrt{\sigma(\xi)}\sin\theta\exp\left[\mathrm{i}\Gamma z + \mathrm{i}\vartheta(\xi)\right], \quad (13)$$

where $\sigma(\xi)$ is the soliton intensity and $\vartheta(\xi)$ is the soliton phase. In the limit of the self-defocusing nonlinearity, substituting Eqs. (12) and (13) into Eqs. (3) and (4) leads to

$$2\sigma(1-4\delta\sigma)\frac{\mathrm{d}^2\sigma}{\mathrm{d}\xi^2} - \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\xi}\right)^2 - 4\sigma^2\left(\frac{\mathrm{d}\vartheta}{\mathrm{d}\xi}\right)^2 + 8V\sigma^2\frac{\mathrm{d}\vartheta}{\mathrm{d}\xi} - 8\sigma(\Gamma+\sigma) = 0, \quad (14)$$

$$\frac{\mathrm{d}}{\mathrm{d}\xi}\left[\sigma\left(\frac{\mathrm{d}\vartheta}{\mathrm{d}\xi} - V\right)\right] = 0. \quad (15)$$

For gray solitons one requires the boundary conditions: $\sigma(0) = \sigma_1$, $\sigma(\infty) = \sigma_0$, $\sigma'(\infty) = \sigma''(\infty) = 0$, and $\sigma'(0) = 0$. Here, σ_0 is defined as the soliton background intensity. When $\xi \to \pm\infty$, the phase of gray soliton is constant.[25] Thus one also requires $\vartheta'(\infty) = 0$. Substituting $\sigma'(\infty) = \sigma''(\infty) = 0$, $\vartheta'(\infty) = 0$, $\sigma(\infty) = \sigma_0$, and $\xi \to \infty$ into Eq. (14) leads to $\Gamma = -\sigma_0$. Substitution of $\sigma(\xi) = u^2(\xi)$ into Eq. (14) and posterior integration yields

$$\left(\frac{\mathrm{d}u}{\mathrm{d}\xi}\right)^2 = \frac{C_1 + \left[u^4 - \left(V^2 - 2\Gamma\right) - \dfrac{C}{u^2}\right]}{1 - 4\delta u^2}, \quad (16)$$

where C_1 is an integration constant and $C = [\sigma(\mathrm{d}\vartheta/\mathrm{d}\xi - V)]^2$ is a constant. Here we have used Eq. (15). Substituting $\sigma(\xi) = u^2(\xi)$ and $(\mathrm{d}u/\mathrm{d}\xi)^2 = (1/4\sigma)(\mathrm{d}\sigma/\mathrm{d}\xi)^2$ into Eq. (16) leads to

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\xi}\right)^2 = 4\frac{\sigma^3 - \left(V^2 - 2\Gamma\right)\sigma + C_1\sigma - C}{1 - 4\delta\sigma}. \quad (17)$$

Under the conditions $\sigma(\infty) = \sigma_0$, $\sigma'(\infty) = 0$, $\sigma(0) = \sigma_1$, and $\sigma'(0) = 0$, we have

$$\sigma_0^3 - \left(V^2 - 2\Gamma\right)\sigma_0^2 + C_1\sigma_0 - C = 0, \quad (18)$$

$$\sigma_1^3 - \left(V^2 - 2\Gamma\right)\sigma_1^2 + C_1\sigma_1 - C = 0. \quad (19)$$

Substitution of $\sigma(\infty) = \sigma_0$, $\sigma'(\infty) = \sigma''(\infty) = 0$, and $C = [\sigma(\mathrm{d}\vartheta/\mathrm{d}\xi - V)]^2$ into Eq. (14) yields

$$\left(V^2 - 2\Gamma\right)\sigma_0^2 - 2\sigma_0^3 - C = 0. \quad (20)$$

From Eqs. (18)–(20) and $\Gamma = -\sigma_0$, one can obtain that $V^2 = \sigma_1$, $C_1 = \sigma_0(\sigma_0 + 2\sigma_1)$, and $C = \sigma_0\sigma_1$. Thus we can rewrite Eq. (17) as

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\xi}\right)^2 = 4\frac{(\sigma-\sigma_1)(\sigma-\sigma_0)^2}{1-4\delta\sigma}. \quad (21)$$

Integration of Eq. (21) leads to

$$\pm\xi = \sqrt{\frac{1-4\delta\sigma_0}{\sigma_0 - V^2}}\tanh^{-1}\left(\sqrt{\frac{(\sigma-V^2)(1-4\delta\sigma_0)}{(1-4\delta\sigma)(\sigma_0-V^2)}}\right) + \sqrt{4\delta}\tan^{-1}\left(\sqrt{\frac{4\delta(\sigma-V^2)}{1-4\delta\sigma}}\right). \quad (22)$$

For the soliton phase we obtain

$$\pm\vartheta = \tan^{-1}\left(\sqrt{\frac{\sigma - V^2}{V(1-4\delta\sigma)}}\right)$$

$$- V\sqrt{4\delta}\tan^{-1}\left(\sqrt{\frac{4\delta\left(\sigma - V^2\right)}{1 - 4\delta\sigma}}\right). \quad (23)$$

Notice that a constant phase contribution (phase in the centre) has been removed in Eq. (23). As in the previous case, equation (22) is known to describe single beam dark solitons for $V = 0$ and single beam gray solitons for $V \neq 0$. The soliton pair components can then be simply obtained through a θ projection.

Figure 2 shows the normalized intensity profiles of the dark–dark soliton pair and the associated refractive index change when $\sigma_0 = 1$, $\theta = 30°$, and $\delta = 0$, 0.1, and 0.2. On the other hand, the gray–gray soliton pairs obtained at $\sigma_0 = 1$, $\theta = 30°$, $V = 0.3$, and $\delta = 0$, 0.1, and 0.2 is shown in Fig. 3. A close examination of Figs. 2 and 3 also reveals that the intensity FWHM of the dark–dark and gray–gray pair components decreases with the increase in δ.

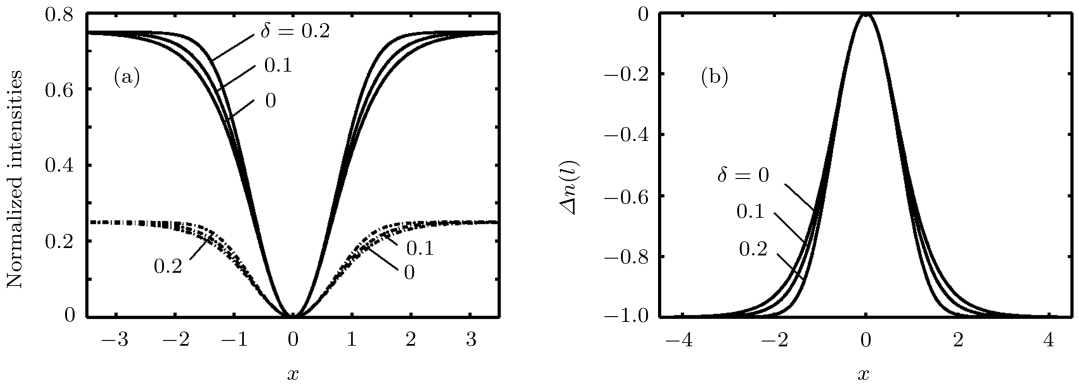


Fig. 2. Components of (a) dark–dark soliton pairs, $|\phi|^2$ (solid curve) and $|\varphi|^2$ (dot-dashed curve), and corresponding (b) refractive index change when $\sigma_0 = 1$, $\theta = 30°$, and $\delta = 0$, 0.1, and 0.2.

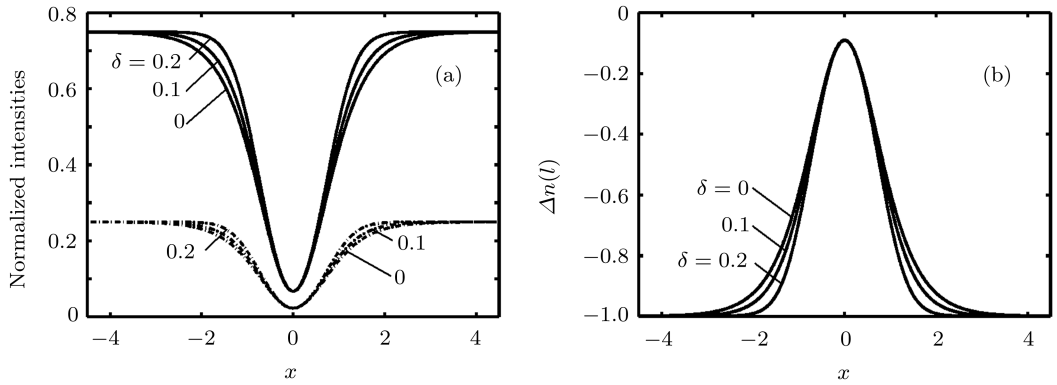


Fig. 3. Components of (a) gray–gray soliton pairs, $|\phi|^2$ (solid curve) and $|\varphi|^2$ (dot-dashed curve), and corresponding (b) refractive index change when $\sigma_0 = 1$, $\theta = 30°$, $V = 0.3$, and $\delta = 0$, 0.1, and 0.2.

It is also very instructive to discuss our results in the absence of the nonlocal nonlinearity. When δ is zero, our physical system becomes the physical system of incoherently coupled soliton pairs in local Kerr-type nonlinear media. In this case, equation (11) reduces to $u(x) = u_0 \operatorname{sech}(u_0 x)$. Substitution of the expression we have just found for $u(x)$ into Eqs. (5) and (6) yields

$$|\phi| = u_0 \operatorname{sech}(u_0 x)\cos\theta, \quad (24)$$

$$|\varphi| = u_0 \operatorname{sech}(u_0 x)\sin\theta. \quad (25)$$

Equations (24) and (25) have the same form as Eq. (8) in Ref. [26], in which bright–bright soliton pairs were discussed in local Kerr-type nonlinear media. On the other hand, when $\delta = 0$ equation (22) reduces to

$$\sigma^{1/2} = \sigma_0^{1/2}\left\{1 - \left(1 - \frac{V^2}{\sigma_0}\right)\operatorname{sech}^2\left[\left(\sigma_0 - V^2\right)^{1/2}\xi\right]\right\}^{1/2}.$$

Substitution of this form of $\sigma(\xi)$ into Eqs. (12) and (13) yields

$$|\phi| = \sigma_0^{1/2}\left\{1-\left(1-\frac{V^2}{\sigma_0}\right)\mathrm{sech}^2\left[\left(\sigma_0-V^2\right)^{1/2}\xi\right]\right\}^{1/2}\cos\theta, \tag{26}$$

$$|\varphi| = \sigma_0^{1/2}\left\{1-\left(1-\frac{V^2}{\sigma_0}\right)\mathrm{sech}^2\left[\left(\sigma_0-V^2\right)^{1/2}\xi\right]\right\}^{1/2}\sin\theta. \tag{27}$$

Of particular interest is that when $V = 0$ equations (26) and (27) reduce to Eq. (19) without the linear coupling in Ref. [27], in which dark–dark soliton pairs were discussed in local Kerr-type nonlinear media.

Finally, we discuss the stability properties of these soliton pairs. In particular, their stability is investigated here using numerical techniques such as beam propagation methods. For bright–bright soliton pairs the normalized envelopes given by Eqs. (5) and (6) have been used as the input beam profiles. As an example, let us assume that $u_0 = 1$, $\theta = 30°$, and $\delta = 0.5$ (the example given in Fig. 1). Figure 4 depicts the evolution of the pair components when the high intensity beam has been perturbed by 10% in its amplitude. The pair does not break up and oscillates around the soliton solution. For dark–dark and gray–gray soliton pairs the normalized envelopes given by Eqs. (12) and (13) have been used as the input beam profiles. Let us consider a gray–gray soliton pair obtained earlier at $\sigma_0 = 1$, $\theta = 30°$, $V = 0.3$, and $\delta = 0.1$. Figure 5 shows the evolution of the gray–gray soliton pair components when the high intensity beam has been perturbed by 10% in its amplitude. Evidently, the gray–gray soliton pair is stable. Similar plots are obtained also for dark–dark soliton pairs. However, for background intensities larger than $1/|4\delta|$ the plane-wave solutions become modulationally unstable.[24] We conclude that incoherently coupled soliton pairs in nonlocal Kerr-type nonlinear media are stable when $\sigma_0 < 1/|4\delta|$.

In conclusion, we have shown that incoherently coupled soliton pairs can exist in nonlocal Kerr-type nonlinear media. Such soliton pairs can exist in bright–bright, dark–dark, and gray–gray configurations. When the nonlocal nonlinearity is absent, these bright–bright and dark–dark soliton pairs are those observed previously in local Kerr-type nonlinear media. Our analysis indicates that for a self-focusing nonlinearity the intensity FWHM of the bright–bright pair components increases with δ, whereas for a self-defocusing nonlinearity the intensity FWHM of the dark–dark and gray–gray pair components decreases with the increase in δ. The stability of these soliton pairs has been investigated numerically and we have found that they are stable.

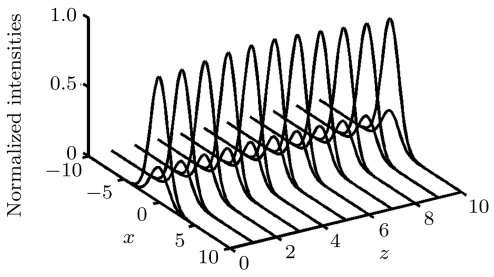


Fig. 4. Stable propagation of a ($u_0 = 1$, $\theta = 30°$, $\delta = 0.5$) bright–bright soliton pair when its larger intensity component is perturbed by 10% at the input.

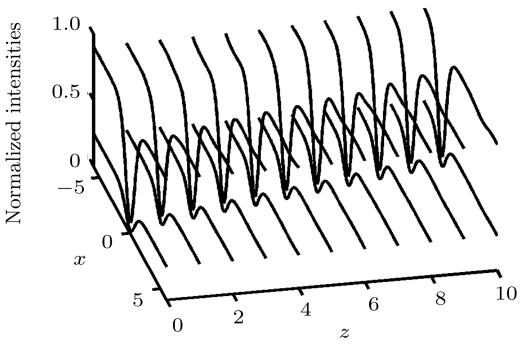


Fig. 5. Stable propagation of a ($\sigma_0 = 1$, $\theta = 30°$, $V = 0.3$, $\delta = 0.1$) gray–gray soliton pair when its larger intensity component is perturbed by 10% at the input.

References

[1] Rotschild C, Cohen O, Manela O, Segev M and Carmon T 2005 *Phys. Rev. Lett.* **95** 213904

[2] Conti C, Peccianti M and Assanto G 2003 *Phys. Rev. Lett.* **91** 073901

[3] Litvak A G, Mironov V A, Fraiman G M and Yunakovskii A D 1975 *Sov. J. Plasma Phys.* **1** 31

[4] Mamaev A V, Zozulya A A, Mezentsev V K, Anderson D Z and Saffman M *Phys. Rev.* A **56** R1110
[5] Hou C F, Du C G, Abdurusul and Li S Q 2001 *Chin. Phys. Lett.* **18** 1607
[6] She W L, Zheng X G, Ni X H and Guo H 2001 *Chin. Phys. Lett.* **18** 1346
[7] Guo R, Ling Z F, Chen X H, Zhang G Q, Zhang X Z, Wen H D, Jiang Y and Liu S M 2000 *Chin. Phys. Lett.* **17** 804
[8] Wang X S, Ouyang S G and She W L 2002 *Acta Phys. Sin.* **52** 377 (in Chinese)
She W L 2005 *Chin. Phys.* **14** 2514
[9] Hou C F, Yao F F, Zhou Z X, Pei Y B and Sun X D 2004 *Chin. Phys.* **13** 1738
[10] Lu K Q, Zhao W, Yang Y L, Zhu X P, Li J P and Zhang Y P 2004 *Chin. Phys.* **13** 2077
Lu K Q, Zhao W, Yang Y L, Zhang M Z, Li J P and Liu H J 2006 *Chin. Phys.* **15** 403
[11] Lu K Q, Yang Y L, Zhao W, Zhang M Z, Yang Y, Zhang L, Liu X M and Zhang Y P 2007 *Chin. Phys.* **16** 3423
Zhang L, Lu K Q, Zhang M Z, Liu X M and Zhang Y P 2008 *Chin. Phys.* B **17** 1674
[12] Zhang Y Q, Lu K Q, Zhang L, Zhang M Z and Li K H 2008 *Acta Phys. Sin.* **57** 6354 (in Chinese)
[13] Lu K Q and Tang T T 1999 *Acta Phys. Sin.* **48** 117 (in Chinese)
[14] Santos L, Shlyapnikov G V, Zoller P and Lewenstein M 2000 *Phys. Rev. Lett.* **85** 1791
[15] Kr'olikowski W, Bang O, Nikolov N I, Neshev D, Wyller J, J J Rasmussen and Edmundson D 2004 *J. Opt.* B: *Quant. Semiclassical Opt.* **6** S288
[16] Lopez-Aguayo S, Desyatnikov A S and Kivshar Yu S 2006 *Opt. Express* **14** 7903
[17] Bang O, Krolikowski W, Wyller J and Rasmussen J J 2002 *Phys. Rev.* E **66** 046619
[18] Rotschild C, Cohen O, Manela O, Segev M and Carmon T 2005 *Phys. Rev. Lett.* **95** 213904
[19] Xu Z, Kartashov Y V and Torner L 2006 *Phys. Rev.* E **73** 055601
[20] Kartashov Y V and Torner L 2007 *Opt. Lett.* **32** 946
[21] Lin Y Y and Lee R K 2007 *Opt. Express* **15** 8781
[22] Krolikowski W, Bang O, Rasmussen J J and Wyller J 2001 *Phys. Rev.* E **64** 016612
[23] Christodoulides D N, Singh S R, Carvalho M I and Segev M 1996 *Appl. Phys. Lett.* **68** 1763
[24] Krolikowski W and Bang O 2000 *Phys. Rev.* E **63** 016610
[25] Grandpierre A G, Christodoulides D N, Coskun T H, Segev M and Kivshar Y S 2001 *J. Opt. Soc. Am.* B **18** 55
[26] Christodoulides D N and Joseph R I 1988 *Opt. Lett.* **13** 53
[27] Kivshar Y S and Turitsyn S K 1993 *Opt. Lett.* **18** 337