Effect of mode–mode competition on atom–atom entanglement*

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A system consisting of two atoms interacting with a two-mode vacuum is considered, where each atom is resonant with the two cavity modes through two different competing transitions. The effect of mode–mode competition on the atom–atom entanglement is investigated. We find that the entanglement between the two atoms can be induced by the mode–mode competition. For the initial atomic state $|\Psi(0)\rangle$, whether the atoms are initially separated or entangled, a large or even maximal entanglement between them can be obtained periodically by introducing the mode–mode competition. For the initial atomic state $|\Phi(0)\rangle$, the strong mode–mode competition can prevent the two atoms entangled initially from suffering entanglement sudden death; besides, it makes them in a more stable and longer-lived entanglement than in the non-competition case.

Keywords: mode–mode competition, concurrence, entanglement sudden death

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1. Introduction

Entanglement is a core resource for quantum information processing (QIP).[1] Various kinds of QIP rely on having entangled states of two qubits, such as quantum computation,[2] quantum cryptography,[3−8] and quantum teleportation.[9] Ideally, one hopes that entanglement needed for QIP can be maintained for sufficiently long time to fulfill the designed tasks. However, entanglement is easily degraded when it is under the influence of environmental noises. Therefore, creating long-lived entanglement and avoiding disentanglement in a practical way are what count in realistic QIP protocols. Recently, Yönac *et al.*[10,11] have shown that entanglement may decrease abruptly and non-smoothly to zero in a finite time due to the influence of quantum noise. This non-smooth finite-time decay, termed as entanglement sudden death (ESD), has attracted increasing attention for the last years. Lately, it has been found that ESD is present in different scenarios for certain classes of states.[12−26] Quite recently, this phenomenon was demonstrated in experiments.[27,28] Clearly, ESD is a serious limiting factor for the use of entangled qubits in QIP. It would seem important to stabilize quantum systems against this unwanted phenomenon.

However, in the above studies, a given mode is assumed to act only on one optical transition, i.e., the two modes do not act simultaneously on one transition. From the semiclassical point of view, if an atom interacts with different fields, several nonlinear processes can take place at the same time, which leads to many new dynamical aspects as a result of mode–mode (MM) competition,[29−31] such as Stokes and anti-Stokes processes.[32] In Ref. [33], Iyotsna and Agarwal found that the quantum effects were rather dominant in the MM competition model. The authors have studied the atomic dynamical properties, the statistical properties of the photon and the entanglement dynamics in this model.[34−40] In the present paper, we shall extend the MM competition model to a more complicated case with consists of two two-level atoms, each interacting with the two cavity modes through two different competing transitions, and study the effect of MM competition on the atom–atom entanglement. It shows that the two separated atoms can be entangled by the MM competition. Moreover, the

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MM competition can be used efficiently to prevent the atom–atom entanglement from undergoing ESD. For the case of strong MM competition, we show that long-lived high values of atom–atom entanglement can be achieved.

The rest of this paper is organized as follows: In Section 2, we describe our model and present the Hamiltonian of the system. In Sections 3 and 4, the effect of MM competition on the atom–atom entanglement is discussed when the two atoms are initially in

$$|\Psi(0)\rangle = \cos\left(\frac{\theta}{2}\right)|eg\rangle + \sin\left(\frac{\theta}{2}\right)|ge\rangle$$

and

$$|\Phi(0)\rangle = \cos\left(\frac{\theta}{2}\right)|ee\rangle + \sin\left(\frac{\theta}{2}\right)|gg\rangle,$$

respectively. Finally, the conclusions are presented in Section 5.

2. Hamiltonian of the MM competition model

The model discussed in the present paper is based on a two-mode Jaynes–Cummings (J–C) model, in which there are two two-level atoms labeled A and B (with states $|e\rangle$ and $|g\rangle$ separated by the frequency ω_0) inside an ideal cavity supporting two modes with frequencies ω_1 and ω_2. We assume that one of the modes, a, interacts with each atom through a three-photon process and the other mode, b, through a single-photon process. For simplicity, we further assume that the two cavity modes are in perfect resonance with the atomic transition, i.e., ω_0=3ω_1=ω_2. In the interaction picture, the interaction Hamiltonian for the system in rotating-wave approximation can be written as ($\hbar$=1)[33]

$$\begin{aligned} H_{\rm I} = {} & k_1(\sigma_{\rm A}^+ a^3 + \sigma_{\rm A}^- a^{\dagger 3}) + g_1(\sigma_{\rm A}^+ b + \sigma_{\rm A}^- b^\dagger) \\ & + k_2(\sigma_{\rm B}^+ a^3 + \sigma_{\rm B}^- a^{\dagger 3}) + g_2(\sigma_{\rm B}^+ b + \sigma_{\rm B}^- b^\dagger), \end{aligned} \quad (1)$$

where $\sigma_i^\pm$ (i=A, B) are the raising and lowering operators of the i-th two-level atom, a ($a^\dagger$) and b ($b^\dagger$) are the annihilation (creation) operators for the field modes a and b, respectively. k_1 (g_1) is the coupling constant between atom A and mode a (b), and analogously for k_2 (g_2). In the following, we will investigate the effect of MM competition on the atom–atom entanglement dynamics for two sets of pure initial states.

3. The atom–atom entanglement for the initial atomic state $|\Psi(0)\rangle = \cos(\theta/2)|eg\rangle + \sin(\theta/2)|ge\rangle$

It is convenient to quantify the degree of atom–atom entanglement by concurrence defined as[41]

$$C = \max\left(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right), \quad (2)$$

where $\lambda_1, \ldots, \lambda_4$ are the eigenvalues of the matrix $\tilde{\rho} = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$; σ_y is the Pauli spin matrix and the asterisk denotes complex conjugation; ρ is the density matrix of an arbitrary two-qubit state. The two qubits are maximally entangled when C=1, while they are separated when $C = 0$.

We take the two atoms initially in an atomic pure state, which is a superposition of Bell states usually denoted by $|\Psi^\pm\rangle \sim |eg\rangle \pm |ge\rangle$ and the field initially in vacuum state $|0\rangle_a \otimes |0\rangle_b = |00\rangle$. Then the initial state for the system can be written as

$$|\Psi(0)\rangle = \left\{\cos\left(\frac{\theta}{2}\right)|eg\rangle + \sin\left(\frac{\theta}{2}\right)|ge\rangle\right\} \otimes |00\rangle, \quad (3)$$

where $0 \le \theta \le \pi/2$. The total state of the system at any time t can be expressed as

$$\begin{aligned} |\Psi(t)\rangle = {} & x_1(t)|eg00\rangle + x_2(t)|ge00\rangle \\ & + x_3(t)|gg30\rangle + x_4(t)|gg01\rangle, \end{aligned} \quad (4)$$

where the complex amplitudes are derived from the Schrödinger equation in the interaction picture.

In the absence of MM competition, corresponding to $k_2 = 0$ and $g_1 = 0$, the model we considered is reduced to a Tavis–Cummings model in which each atom interacts only with one of the modes (atom A interacts with mode a through a three-photon transition, atom B interacts with mode b through a single-photon transition). In this case, the coefficients are given by

$$\begin{aligned} x_1(t) &= \cos\left(\frac{\theta}{2}\right)\cos(\sqrt{6}k_1 t), \\ x_2(t) &= \sin\left(\frac{\theta}{2}\right)\cos(g_2 t), \\ x_3(t) &= -\,\mathrm{i}\cos\left(\frac{\theta}{2}\right)\sin(\sqrt{6}k_1 t), \\ x_4(t) &= -\,\mathrm{i}\sin\left(\frac{\theta}{2}\right)\sin(g_2 t). \end{aligned} \quad (5)$$

Then we take the MM competition into consideration. When $k_1 = k_2 = k$ and $g_1 = g_2 = g$, the corresponding coefficients are now given by

$$
\begin{aligned}
x_1(t) &= \frac{1}{2}\left\{\cos\left(\frac{\theta}{2}\right)+\sin\left(\frac{\theta}{2}\right)\right\}\cos(\Omega g t)-\frac{1}{2}\sin\left(\frac{\theta}{2}\right)+\frac{1}{2}\cos\left(\frac{\theta}{2}\right),\\
x_2(t) &= \frac{1}{2}\left\{\cos\left(\frac{\theta}{2}\right)+\sin\left(\frac{\theta}{2}\right)\right\}\cos(\Omega g t)+\frac{1}{2}\sin\left(\frac{\theta}{2}\right)-\frac{1}{2}\cos\left(\frac{\theta}{2}\right),\\
x_3(t) &= \frac{-\mathrm{i}\sqrt{6}k}{\Omega g}\sin(\Omega g t),\\
x_4(t) &= \frac{-\mathrm{i}}{\Omega}\sin(\Omega g t)
\end{aligned}
\tag{6}
$$

with $\Omega=\sqrt{2+12k^2/g^2}$. When $k_1=g_1=\alpha$ and $k_2=g_2=\beta$, the corresponding coefficients become

$$
\begin{aligned}
x_1(t) &= \frac{7}{\omega^2}\left\{\frac{\alpha^2}{\beta^2}\cos\left(\frac{\theta}{2}\right)\cos(\omega\beta t)+\frac{\alpha}{\beta}\sin\left(\frac{\theta}{2}\right)\cos(\omega\beta t)-\frac{\alpha}{\beta}\sin\left(\frac{\theta}{2}\right)+\cos\left(\frac{\theta}{2}\right)\right\},\\
x_2(t) &= \frac{7}{\omega^2}\left\{\frac{\alpha^2}{\beta^2}\sin\left(\frac{\theta}{2}\right)-\frac{\alpha}{\beta}\cos\left(\frac{\theta}{2}\right)+\frac{\alpha}{\beta}\cos\left(\frac{\theta}{2}\right)\cos(\omega\beta t)+\sin\left(\frac{\theta}{2}\right)\cos(\omega\beta t)\right\},\\
x_3(t) &= \sqrt{6}x_4(t),\\
x_4(t) &= \frac{-\mathrm{i}}{\omega}\sin(\omega\beta t)\left\{\frac{\alpha}{\beta}\cos\left(\frac{\theta}{2}\right)+\sin\left(\frac{\theta}{2}\right)\right\}
\end{aligned}
\tag{7}
$$

with $\omega=\sqrt{7(1+\alpha^2/\beta^2)}$.

In order to obtain the atom–atom density operator $\rho^{\mathrm{AB}}(t)$, we trace over the field variables from Eq. (4). For the initial atomic state $|\Psi(0)\rangle$, the atom–atom density matrix $\rho^{\mathrm{AB}}(t)$ written in the basis of $|ee\rangle$, $|eg\rangle$, $|ge\rangle$, $|gg\rangle$ is given by

$$
\rho^{\mathrm{AB}}(t)=\begin{pmatrix}
0 & 0 & 0 & 0\\
0 & |x_1(t)|^2 & x_1(t)x_2^*(t) & 0\\
0 & x_1^*(t)x_2(t) & |x_2(t)|^2 & 0\\
0 & 0 & 0 & |x_3(t)|^2+|x_4(t)|^2
\end{pmatrix}. \tag{8}
$$

The concurrence of the density matrix (8) can be written as

$$
C^{\mathrm{AB}}(t)=2|x_1(t)x_2(t)|. \tag{9}
$$

In the following, we will discuss the effect of MM competition on the atom–atom entanglement when $\theta=0$ and $\pi/8$.

Now we consider the case of θ=0, that is to say, the two atoms are initially in a separated state $|eg\rangle$, which is sensitive to the effect of MM competition. In the absence of MM competition, it is evident from Eq. (5) that the concurrence $C^{\mathrm{AB}}(t)$=0, i.e., the two atoms are disentangled through the time evolution. In the presence of MM competition, the time evolution of the atom–atom entanglement is plotted in Fig. 1. Figure 1(a) displays the concurrence $C^{\mathrm{AB}}(t)$ as a function of time and the parameter k/g (the relative coupling strength between two competing processes) in the case of $k_1=k_2=k$ and $g_1=g_2=g$. We find that the two atoms are entangled periodically, and the periods are π/Ω. Obviously, the larger the parameter k/g, the smaller the period. Furthermore, the maximum of atom–atom entanglement is always 0.5, which is easily found from the expression $C^{\mathrm{AB}}(t)=\frac{1}{2}|\cos^2(\Omega g t)-1|$. However, when we suppose that $k_1=g_1=\alpha$ and $k_2=g_2=\beta$, the concurrence is expressed by

$$
C^{\mathrm{AB}}(t)=\frac{98\alpha}{\omega^4\beta}\left|\left[\frac{\alpha^2}{\beta^2}\cos(\omega\beta t)+1\right][\cos(\omega\beta t)-1]\right|. \tag{10}
$$

In this case, the effect of parameter α/β (the relative coupling strength between two atoms) on the time evolution of the atom–atom concurrence is quite different, as shown in Fig. 1(b). The more exciting case is that a larger and even maximal entanglement between the atoms can be created at time $\tau_n=(2n+1)\pi/\omega$ (n=0,1,2...), as shown in Fig. 1(b). Furthermore, the concurrence can arrive at 1 when $\alpha/\beta=0.41,\ 2.41$, which can be found from the expression (10). This implies that the initially separated atoms can be maximally entangled due to the MM competition.

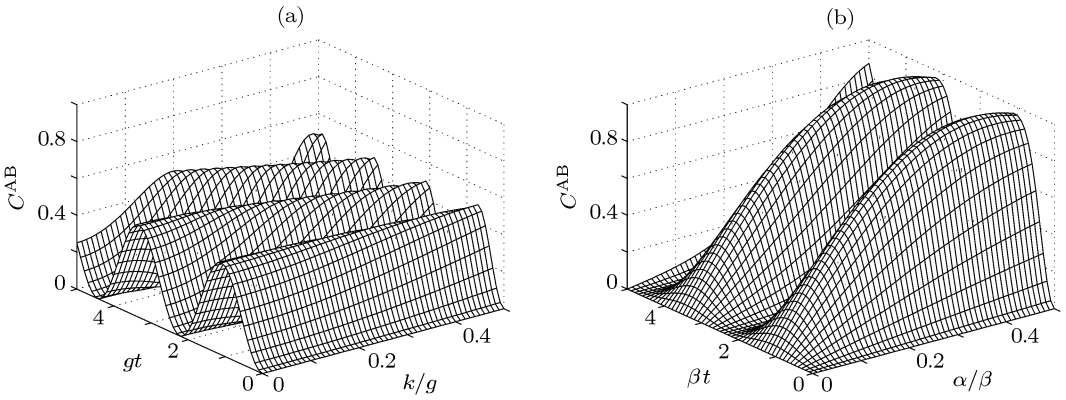


Fig.1. Variation of concurrence C^{AB} with time and parameters k/g (a), α/β (b), for the initial atomic state being $|\Psi(0)\rangle = |eg\rangle$.

Figure 2 displays the dependence of atom–atom entanglement on the MM competition when $\theta = \pi/8$ (the initial atomic state $|\Psi(0)\rangle = \cos(\pi/16)|eg\rangle + \sin(\pi/16)|ge\rangle$). From this figure, we can see that, the time evolution of concurrence $C^{AB}(t)$ in the absence of MM competition (Fig. 2(a)) is different from that in the MM competing case (Fig. 2(b)). In the former, the maximal value of atom–atom entanglement is equal to the initial degree of entanglement between two atoms (0.38), while in the later, a larger atom–atom entanglement can be obtained periodically. Especially for $\alpha/\beta = 0.3$, the maximal entanglement can be achieved at time τ_n (in Fig. 2(b)). Therefore, the introduction of MM competition can enhance entanglement. In other words, the effect of MM competition can be to trigger a maximum entanglement, which indicates that, no matter what value of the initial entanglement between the atoms has, the maximal atom–atom entanglement can be obtained by the MM competition.

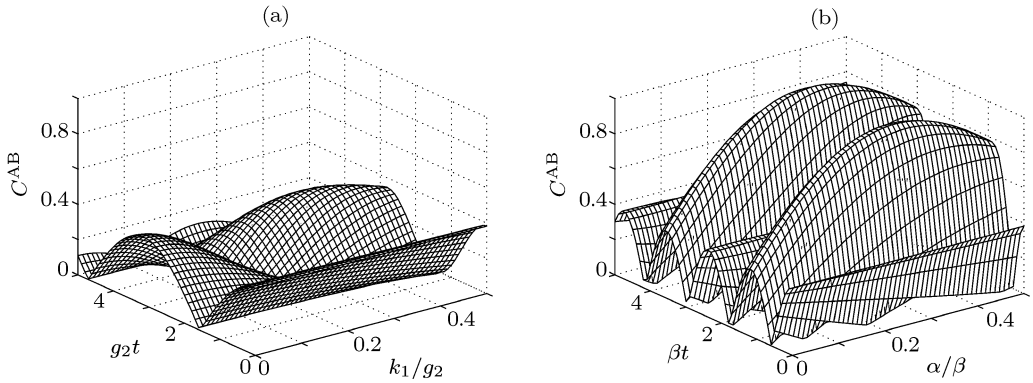


Fig. 2. Variation of concurrence C^{AB} with time and parameters k_1/g_2 (a), α/β (b) for the initial atomic state being $|\Psi(0)\rangle = \cos(\pi/16)|eg\rangle + \sin(\pi/16)|ge\rangle$.

4. The atom–atom entanglement for the initial atomic state $|\Phi(0)\rangle = \cos\left(\frac{\theta}{2}\right)|ee\rangle + \sin\left(\frac{\theta}{2}\right)|gg\rangle$

Alternatively, we assume that the initial state for the atoms is in a combination of the other two Bell states $|\Phi^{\pm}\rangle \sim |ee\rangle \pm |gg\rangle$. Accordingly, the initial state for the total system can be expressed as

$$|\Phi(0)\rangle = \left\{\cos\left(\frac{\theta}{2}\right)|ee\rangle + \sin\left(\frac{\theta}{2}\right)|gg\rangle\right\} \otimes |00\rangle. \tag{11}$$

In this case, the system state at time t can be written as

$$|\Phi_1(t)\rangle = y_1(t)|ee00\rangle + y_2(t)|gg00\rangle + y_3(t)|ge30\rangle + y_4(t)|eg01\rangle + y_5(t)|gg31\rangle \tag{12}$$

with

$$
\begin{aligned}
y_1(t) &= \frac{1}{4}\cos\left(\frac{\theta}{2}\right)\left\{\mathrm{e}^{Pt}+\mathrm{e}^{-Pt}+\mathrm{e}^{Qt}+\mathrm{e}^{-Qt}\right\},\\
y_2(t) &= \cos\left(\frac{\theta}{2}\right),\\
y_3(t) &= \frac{\mathrm{i}}{4PQ}\cos\left(\frac{\theta}{2}\right)\{Q(\sqrt{6}k_1+g_2)(\mathrm{e}^{-Pt}-\mathrm{e}^{Pt})+P(\sqrt{6}k_1-g_2)(\mathrm{e}^{-Qt}-\mathrm{e}^{Qt})\},\\
y_4(t) &= \frac{\mathrm{i}}{4PQ}\cos\left(\frac{\theta}{2}\right)\{Q(\sqrt{6}k_1+g_2)(\mathrm{e}^{-Pt}-\mathrm{e}^{Pt})+P(\sqrt{6}k_1-g_2)(\mathrm{e}^{Qt}-\mathrm{e}^{-Qt})\},\\
y_5(t) &= \frac{1}{4}\cos\left(\frac{\theta}{2}\right)\{\mathrm{e}^{Pt}+\mathrm{e}^{-Pt}-\mathrm{e}^{Qt}-\mathrm{e}^{-Qt}\},\\
P &= (-g_2^2-6k_1^2-2\sqrt{6}g_2k_1)^{1/2},\\
Q &= (-g_2^2-6k_1^2+2\sqrt{6}g_2k_1)^{1/2},
\end{aligned}
\tag{13}
$$

and

$$
\begin{aligned}
|\Phi_2(t)\rangle = {} & z_1(t)|ee00\rangle + z_2(t)|gg00\rangle + z_3(t)|ge30\rangle + z_4(t)|ge01\rangle + z_5(t)|eg30\rangle\\
& + z_6(t)|eg01\rangle + z_7(t)|gg60\rangle + z_8(t)|gg02\rangle + z_9(t)|gg31\rangle,
\end{aligned}
\tag{14}
$$

where $|\Phi_1(t)\rangle$ and $|\Phi_2(t)\rangle$ correspond to the non-competition case and the competing case, repectively. It is difficult to obtain the analytical solution for the coefficients in Eq. (14), so we use the fourth-order Runge–Kutta method to obtain their numerical solution. In the bases of $|ee\rangle, |eg\rangle, |ge\rangle, |gg\rangle$, the atom–atom density matrix $\rho^{\mathrm{AB}}(t)$ can be obtained by tracing over the field variables from Eqs. (12) and (14)

$$
\rho_1^{\mathrm{AB}}(t) = \begin{pmatrix} |y_1(t)|^2 & 0 & 0 & y_1(t)y_2^*(t) \\ 0 & |y_4(t)|^2 & 0 & 0 \\ 0 & 0 & |y_3(t)|^2 & 0 \\ y_2(t)y_1^*(t) & 0 & 0 & |y_2(t)|^2+|y_5(t)|^2 \end{pmatrix},
\tag{15}
$$

and

$$
\rho_2^{\mathrm{AB}}(t) = \begin{pmatrix} a(t) & 0 & 0 & w(t) \\ 0 & b(t) & z(t) & 0 \\ 0 & z^*(t) & c(t) & 0 \\ w^*(t) & 0 & 0 & d(t) \end{pmatrix}
\tag{16}
$$

with

$$
\begin{aligned}
a(t) &= |z_1(t)|^2, & b(t) &= |z_5(t)|^2+|z_6(t)|^2,\\
c(t) &= |z_3(t)|^2+|z_4(t)|^2, & d(t) &= |z_2(t)|^2+|z_7(t)|^2+|z_8(t)|^2+|z_9(t)|^2,\\
z(t) &= z_5(t)z_3^*(t)+z_6(t)z_4^*(t), & w(t) &= z_1(t)z_2^*(t).
\end{aligned}
\tag{17}
$$

The concurrences for these two matrixes can be written as

$$
C_1^{\mathrm{AB}}(t) = 2\max\{0, |y_1(t)y_2(t)| - |y_3(t)y_4(t)|\},
\tag{18}
$$

and

$$
C_2^{\mathrm{AB}}(t) = 2\max\{0, |z(t)| - \sqrt{a(t)d(t)}, |w(t)| - \sqrt{b(t)c(t)}\}.
\tag{19}
$$

In the following, we will discuss the effect of MM competition on the atom–atom entanglement in the same way. When the two atoms are initially in a separated state $|ee\rangle$ ($\theta = 0$), the expressions (18) and (19) are easily reduced to $C_1^{\mathrm{AB}}(t) = 0$ (non-competition case) and $C_2^{\mathrm{AB}}(t) = 2\max\{0, |z(t)| - \sqrt{a(t)d(t)}\}$ (competing case), respectively. In the absence of MM competition, it is clear that the atoms are always disentangled at all times. While, for the MM competing case, although the concurrence $C_2^{\mathrm{AB}}(t)$ is zero when $k_1 = k_2 = k$ and

$g_1 = g_2 = g$, as shown in Fig. 3(a), the atoms become entangled at particular discrete times when $k_1 = g_1 = \alpha$ and $k_2 = g_2 = \beta$ with ESD (Fig. 3(b)). Obviously, these results are quite different from that for the initial atomic state $|eg\rangle$ (Fig. 1). As a result, in this situation, the entanglement between two initially separated atoms can be induced by the MM competition only when $k_1 = g_1 = \alpha$ and $k_2 = g_2 = \beta$.

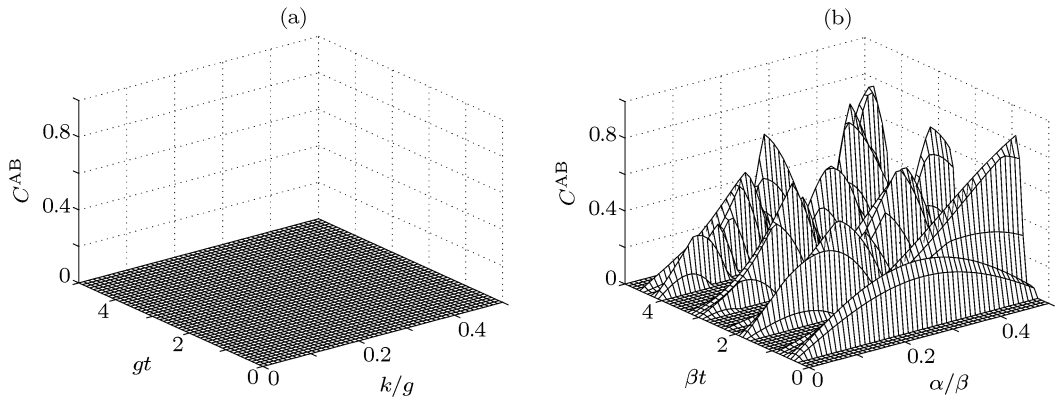


Fig. 3. Variation of concurrence C^{AB} with time and parameters k/g (a), α/β (b), for the initial atomic state $|\Phi(0)\rangle = |ee\rangle$.

Figure 4 shows the dependence of atom–atom entanglement on the MM competition when $\theta = \pi/8$ (the initial atomic state $|\Phi(0)\rangle = \cos(\pi/16)|ee\rangle + \sin(\pi/16)|gg\rangle$). Figure 4(a) plots the time evolution of the concurrence $C_1^{AB}(t)$, and Fig. 4(b) that of $C_2^{AB}(t)$. The former describes the non-competition case, and the latter the competition-involved case. We can see that, in the absence of MM competition, the atom–atom entanglement suffers ESD (see Fig. 4(a)), but, in the presence of MM competition, the entanglement between two atoms becomes larger and steady with the increase of the relative coupling strength k/g (see Fig. 4(b)). This indicates that the strong MM competition can prevent the two atoms entangled initially from disentangling. Hence, it is possible for the two atoms to avoid ESD by using the MM competition.

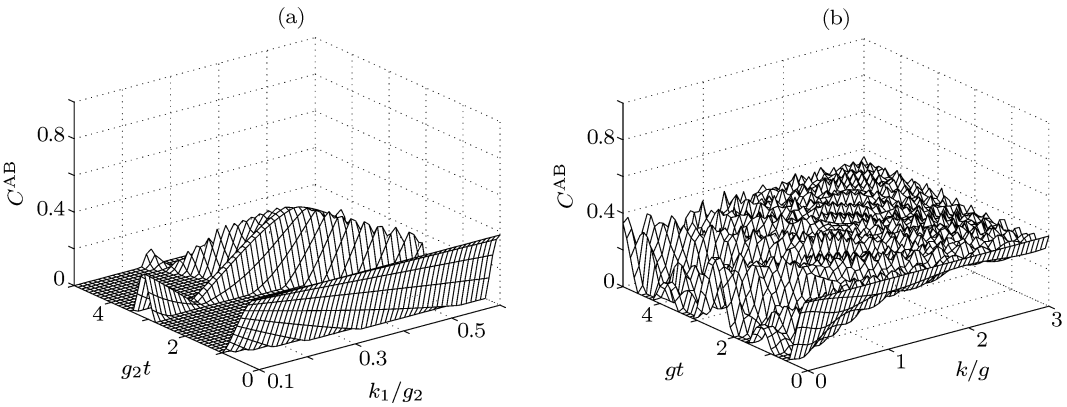


Fig. 4. Variation of concurrence C^{AB} with time and parameters k_1/g_2 (a), k/g (b), for the initial atomic state $|\Phi(0)\rangle = \cos(\pi/16)|ee\rangle + \sin(\pi/16)|gg\rangle$.

What is more interesting is the time evolution of atom–atom entanglement for $\theta = \pi/2$, i.e., the two atoms are initially in a maximally entangled state

$$|\Phi(0)\rangle = \sqrt{2}/2(|ee\rangle + |gg\rangle$$

with C=1. In this case, long-lived maximal entanglement between the two atoms can be created by the strong MM competition (see Fig. 5), which may provide us with a potential way to obtain the high amount atom–atom entanglement for some system parameters

chosen under the case of MM competition.

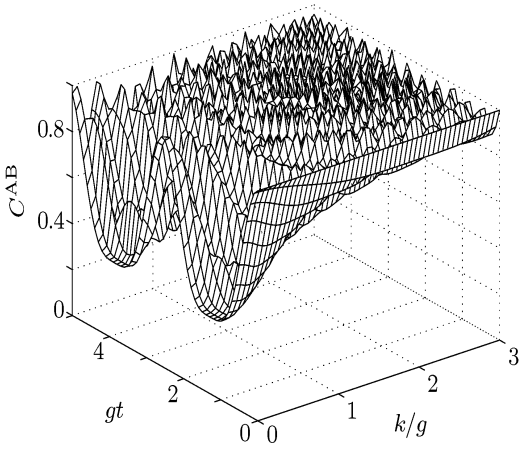


Fig. 5. Variation of concurrence C^{AB} as a function of the time and the parameter k/g, for the initial atomic state $|\Phi(0)\rangle = \sqrt{2}/2(|ee\rangle + |gg\rangle)$.

5. Conclusion

In this paper, we have investigated the effect of MM competition on the atom–atom entanglement in a system composed of two atoms, each interacting resonantly with a two-mode field through two different competing transitions.

For the initial atomic state $|\Psi(0)\rangle$, it is shown that the entanglement of the two initially separated atoms can be generated by the MM competition. Interestingly, whether or not the atoms are initially separated or entangled, the maximal entanglement between them can be obtained periodically for some system parameters chosen in the MM competing case.

For the initial atomic state $|\Phi(0)\rangle$, the MM competition can prevent the two atoms entangled initially from undergoing ESD. Moreover, it leads to a more stable and longer-lived entanglement between the two atoms than in the case of non-competition.

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