Measurements of femtosecond temporal speckle field of a random medium*

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The femtosecond temporal speckle field of a random medium is studied theoretically and experimentally. Femtosecond temporal speckle arises from the interference of multiple randomly scattered electric fields. The femtosecond temporal speckle field is measured with a cross-correlation frequency-resolved optical gating method. The spatial average of the speckle field yields a smooth transmitted profile. The speckle field is a circular complex Gaussian variable because the scattered light beams from different trajectories have no correlation with each other. The field and the intensity profiles of individual speckle spots fluctuate randomly in time. The ensemble average of the temporal intensity profiles converges, thereby yielding the photon travel time probability distribution function.

Keywords: temporal speckle, femtosecond pulse, statistical mechanism

PACC: 4225B, 1180L, 4230D, 4280W

1. Introduction

Wave fluctuation and pulse propagation in a random medium is an important problem in communication, remote sensing and medical optical imaging.[1−13] A coherent light propagates through a random medium, and it will result in two components in the transmitted wave: one is the unscattered wave (coherent or ballistic), the other is the multiple scattered wave (incoherent or diffusive).[14] Multiple scattered photons acquire a complicated distribution of phases due to the various different path lengths travelling from source to detector. The transmitted light is a combination of frequency-dependent phase shifts and amplitude modulation. The light waves vary randomly in amplitude and phase and must be described in terms of statistical averages and probability densities.

The spatial characteristic of speckle produced by a continuous wave laser has received much attention but the temporal property was usually ignored due to the characteristic of continuous wave laser. With a coherent light pulse for which the pulse length is short compared with the typical size of the object, large fluctuations will appear in the space and the time domains.[15−17] Spatial fluctuation relies on measurements of intensity fluctuations,[18] as opposed to measurements of optical field directly. In the present paper, we study broadband femtosecond pulse propagation and scattering in a random medium. When femtosecond pulses propagate through a random medium, the output field is time-fluctuating waves, and random variation will occur in each of the space, time and frequency domains, because both the amplitude and the phase are changed by numerous linear (or nonlinear) optical factors such as dispersion, absorption and scattering. Temporal speckle on the femtosecond scale is predicted in theory and verified by experiment. Femtosecond temporal speckle is characterized by using the cross-correlation frequency resolved optical gating (XFROG). Frequency-resolved optical gating (FROG) can fully characterize ultrashort optical pulses because it yields information on both amplitude and phase of a pulse field. Among the various apparatuses of the FROG family, second harmonic generation FROG (SHG-FROG) is one of the sensitive methods of characterizing the weak ultrashort pulses.[19] For characterization of an even more weaker pulse, XFROG is more suitable because this technique is based on the sum-frequency signal between a weak unknown pulse and a strong reference pulse. With information contained in the cross-correlated spectrum, complete characterization of the temporal speckle field is available.

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2. Theory

Consider a femtosecond pulse that is normally incident on the front surface of a sample consisting of random scatters. The thickness of the sample is L. $E(\xi, \zeta, z, \omega)$ is known at the plane $z = 0$, which is the front surface of the sample. For a Gaussian-shaped femtosecond pulse at the input plane $z = 0$, E $(\xi, \zeta, 0, \omega)$ can be written as[20]

$$\begin{aligned} E(\xi,\zeta,0,\omega) &= S(\omega)A(\xi,\zeta,0) \\ &= \exp[-(\xi^2+\zeta^2)/2a^2]S(\omega), \end{aligned} \quad (1)$$

where $S(\omega)$ is the complex spectral amplitude of the pulse, and normalized such that $\int S\ (\omega)\mathrm{d}\omega = 1$, and $A\ (\xi, \zeta, 0)$ is the spatial part of the pulse at the input plane $z = 0$. By using the Green-function approach and making paraxial approximation, we can obtain the result for each spectral component of the beam[20]

$$\begin{aligned} & E(\xi,\zeta,z,\omega) \\ &= S(\omega)A(\xi,\zeta,z) \\ &= \frac{1}{1+\mathrm{i}d}\exp\left[-\frac{(\xi^2+\zeta^2)}{2a^2(1+\mathrm{i}d)}\right]\exp(\mathrm{i}kz)S(\omega), \end{aligned} \quad (2)$$

where d is the normalized propagation distance. At any position (ξ, ζ, z) and time t the propagated field is obtained by taking the inverse Fourier transform of Eq. (2) and given by

$$\begin{aligned} & E(\xi,\zeta,z,t) \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)A(\xi,\zeta,z)\exp(-\mathrm{i}\omega t)\mathrm{d}\omega. \end{aligned} \quad (3)$$

The ensemble-averaged intensity is[21]

$$\begin{aligned} I(r,t) &= \left\langle |E(r,t)|^2 \right\rangle \\ &= \frac{1}{(2\pi)^2}\int\int_{-\infty}^{\infty} S[\omega+(\Delta\omega/2)]S^*[\omega-(\Delta\omega/2)]\exp\left(\frac{\mathrm{i}\omega z}{v}-\frac{z}{\bar{\ell}}\right)\exp[-\mathrm{i}(\Delta\omega)t]\mathrm{d}(\Delta\omega)\mathrm{d}\omega \\ &\quad +\frac{1}{16\pi^3\ell}\int\int_{-\infty}^{\infty}\int_0^L H(z-z_1)C(\omega,z_1)S[\omega+(\Delta\omega/2)]S^*[\omega-(\Delta\omega/2)] \\ &\quad \times\exp[-\mathrm{i}(\Delta\omega)t]\mathrm{d}(\Delta\omega)\mathrm{d}\omega\mathrm{d}z_1, \end{aligned} \quad (4)$$

where v is the phase velocity, $\bar{\ell}$ is the extinction length of the coherent intensity, and ℓ is the transport mean free path. Equation (4) is the sum of two components: the first integration term is the coherent ballistic contribution, while the second integration term describes the effects of scattering. Femtosecond temporal speckle is a direct result of random interference of scattered femtosecond pulse from different trajectories. A certain set of photons from shorter trajectories forms a certain spatial speckle pattern first on the observation plane, which stays for a moment on the order of the incident pulse duration. After this pattern has gone out, another set of photons from longer trajectories forms a new speckle pattern. In this way transmitted femtosecond pulse has temporal speckle in time on the order of the incident pulse duration. Because the femtosecond pulse duration is much shorter than the typical flight time of transmitted pulses in random medium, the trajectory distribution function does not change significantly in the pulse duration. The temporal correlation function between speckle fields observed at time t and those observed at the time $t+\tau$ can be expressed as[22]

$$\begin{aligned} & C(t,\tau) \\ &= \frac{1}{v^2}|T(t)|^2\left|\int_0^{\infty}\tilde{E}_{\mathrm{in}}^*(s/v)\tilde{E}_{\mathrm{in}}(s/v+\tau)\mathrm{d}s\right|^2, \end{aligned} \quad (5)$$

where s is the length of the trajectory, v is the velocity of light in the medium, $T(t)$ is the trajectory distribution function, and E_{in} is the electric field of the input pulse.

When the incident femtosecond pulse enters the random medium, it can be scattered many times before exiting. The measured field consists of the superposition of many scattered wavelets. For a particular time delay τ, the measured field can be described as the sum of a large number of independent fields with random phases and amplitudes. The sum of a large number of independent identically distributed random variables has Gaussian distribution. Thus, the statistics of both the real component $E_{\mathrm{r}} = \mathrm{Re}[E(t)]$ and the imaginary component $E_{\mathrm{i}} = \mathrm{Im}[E(t)]$ of the speckle

field can be expressed as[23]

$$P(a) = \frac{1}{\sqrt{2\pi\langle I\rangle}} \exp\left[-\frac{a^2}{2\langle I\rangle}\right], \tag{6}$$

where $a = \{E_\mathrm{r}, E_\mathrm{i}\}$, and $\langle I\rangle$ is the intensity of the scattered light averaged over all the measured data. Because of its quadratic relationship between electric field and the Gaussian statistics of electric field distribution, the intensity of temporal speckle field follows an exponential distribution as[23]

$$P(I) = \frac{1}{\langle I\rangle} \exp\left[-\frac{I}{\langle I\rangle}\right]. \tag{7}$$

3. Experiment

Our experimental setup is shown in Fig. 1. The femtosecond laser pulses directly come from a Ti:sapphire oscillator. The input pulse is split into two by a beam splitter. Reference pulses travel along the delay arm, and random medium is mounted on a translation stage and placed in the other arm. We use SHG-FROG technique to characterize the reference pulse, and it is also the input pulse to the random medium. The measured intensity and phase, the real and the imaginary components of electric field in time domain are shown in Fig. 2. The temporal full width at half maximum (FWHM) of the reference pulse is 56 fs with central wavelength 824 nm.

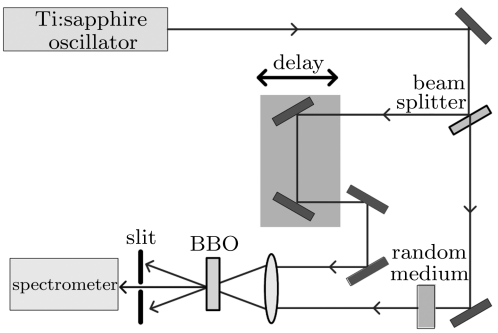


Fig. 1. Experimental setup for measuring femtosecond temporal speckle field.

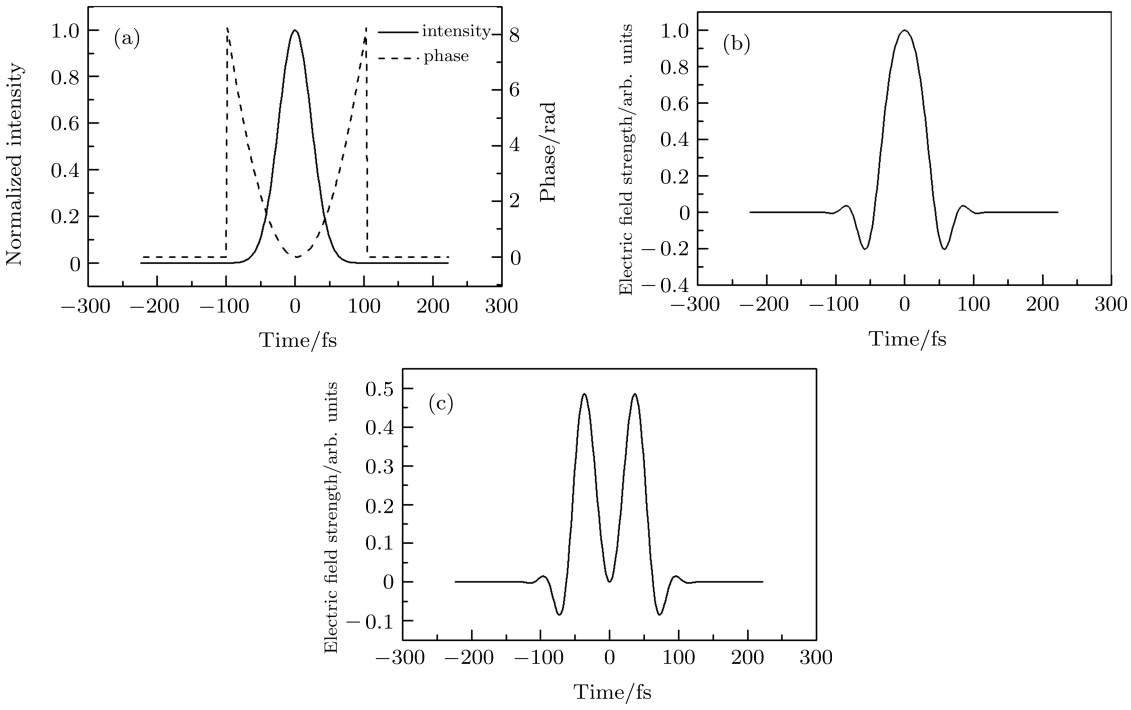


Fig. 2. Experimental results of reference pulse: (a) intensity and phase, (b) real component of electric field, (c) imaginary component of electric field.

The XFROG is used to measure the cross-correlation of weak speckle field and reference pulse, and the cross-correlation is achieved in a frequency-doubled crystal. We use a 100-μm thick β-barium borate (BBO) crystal to generate the cross-correlation signals. The beam crossing angle is about 5°, the delay time increment is 6.7 fs. In our experiment, we use white alkyd paint with small TiO_2 particles as the random medium. The average diameter of TiO_2 particles is about 60 nm. The sample thickness is 4 mm. The measured femtosecond

temporal speckle field from five measurements is shown in Fig. 3(a). The real and the imaginary components of speckle field are shown in Figs. 3(b) and 3(c), respectively. Femtosecond temporal speckle arises from the interference of multiple randomly scattered electric field. If we use coherent cw light as an incident light source, the time scale of the temporal speckle should become infinite because the coherence time of the cw light is much longer than the trajectory-loop length in the random medium.

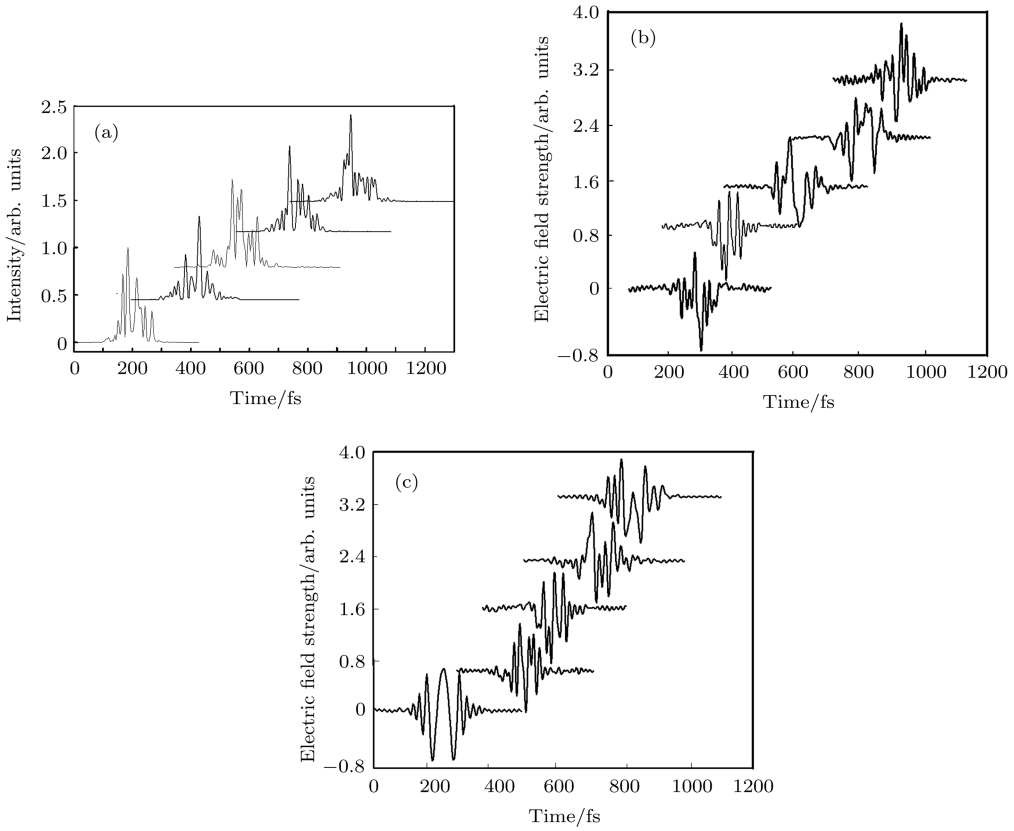


Fig. 3. (a) Intensity, (b) real component of transmitted field, (c) imaginary component of transmitted field.

In order to estimate the ensemble statistics of the speckle field we perform 10 measurements and then average the 10 results, moving the sample laterally at a step of 100 μm between two measurements, and record the XFROG traces. The probability distributions of real and imaginary components of speckle field are shown in Figs. 4(a) and 4(b), respectively. Each distribution is generated by computing and averaging the histograms of all the acquired data of the 10 measurements including those shown in Figs. 3(b) and 3(c). Both of them are normalized to unity at its peak. The dash-dot lines in Figs. 4(a) and 4(b) are for the best Gaussian fit. The statistics of speckle field are Gaussian distribution as described by Eq. (6). The outgoing speckle field is a circular complex Gaussian variable because the scattered light rays from different trajectories have no correlation with each other. The statistics of intensity distribution is shown in Fig. 5(a). The distribution is generated by computing and averaging the histograms of all the acquired data of the 10 measurements including those shown in Fig. 3(a). The solid line in Fig. 5(a) denotes the best exponential fit as described by Eq. (7). Circles in Fig. 5(b) represent the intensity correlation function of transmitted pulse, solid line refers to the best Gaussian fit, and dash-dot line denotes the intensity correlation function of input pulse. We average 10 independent measurements to estimate the ensemble-average intensity temporal response. In Fig. 6 we plot the ensemble-average intensity temporal response for the random sample, and the ensemble average of the temporal intensity profiles converges, thereby yielding the photon travel time probability distribution function.

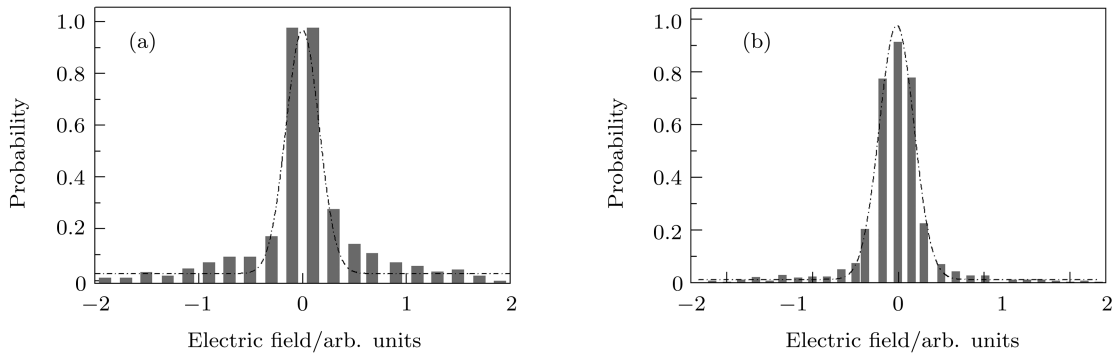


Fig. 4. Probability distributions of real (a) and imaginary (b) components of speckle field.

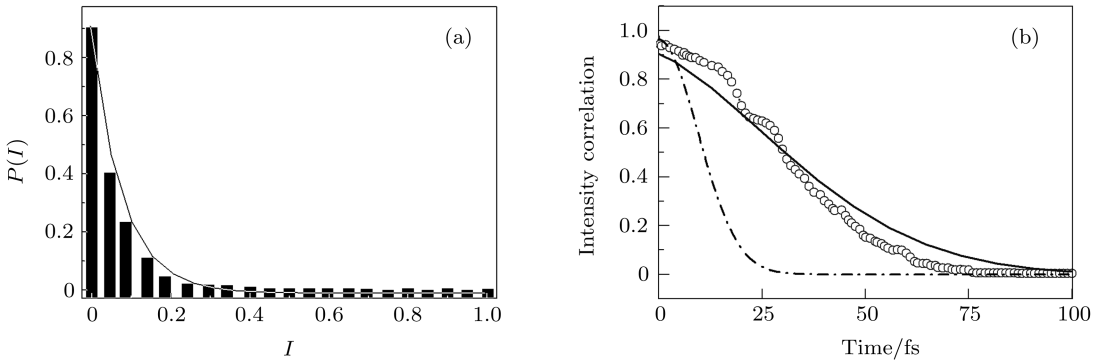


Fig. 5. (a) Probability distribution of intensity. (b) Circles represent the intensity correlation function of transmitted pulses. Solid line denotes the best Gaussian fit. Dash-dot line refers to the intensity autocorrelation function of input pulse.

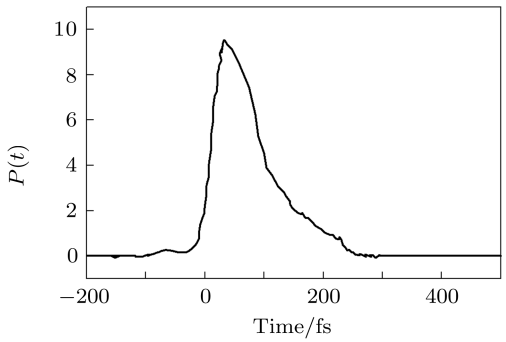


Fig. 6. Ensemble average intensity temporal response of random medium.

4. Conclusion

The femtosecond temporal speckle field of a random medium is explored theoretically and experimentally. Temporal speckle appears when a coherent femtosecond pulse propagates through a random medium. The transmitted pulse is distorted and broadened. The speckle field is measured by using cross-correlation frequency-resolved optical gating. We also measure statistical averages of the speckle field, from which we clearly show that the experimental results are in good agreement with theoretical predictions.

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