Consistency of the directionality of partially coherent beams in turbulence expressed in terms of the angular spread and the far-field average intensity*

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Under the quadratic approximation of the Rytov's phase structure function, this paper derives the general closed-form expressions for the mean-squared width and the angular spread of partially coherent beams in turbulence. It finds that under a certain condition different types of partially coherent beams may have the same directionality as a fully coherent Gaussian beam in free space and also in atmospheric turbulence if the angular spread is chosen as the characteristic parameter of beam directionality. On the other hand, it shows that generally, the directionality of partially coherent beams expressed in terms of the angular spread is not consistent with that in terms of the normalized far-field average intensity distribution in free space, but the consistency can be achieved due to turbulence.

Keywords: turbulence, partially coherent beam, angular spread, far-field average intensity

PACC: 4225B, 4225K, 9265R

1. Introduction

It was shown theoretically[1,2] and verified experimentally[3,4] that in free space partially coherent beams such as Gaussian Schell-model (GSM) beams may have the same far-field radiant intensity distribution as a fully coherent laser beam. It means that full spatial coherence is not a necessary condition for highly directional light beams. On the other hand, it is very important to study the propagation of laser beams through atmospheric turbulence for many practical applications such as the remote sensing and atmospheric optical communication, etc.[5] Much work has been carried out concerning the spreading of laser beams propagating through atmospheric turbulence, and it was shown both theoretically and experimentally that partially coherent beams are less affected by turbulence than fully coherent ones.[6−12] Recently, Wolf and his collaborators have extended the result in Refs. [1] and [2] from free space to atmospheric turbulence, and showed that under a certain condition there exist the equivalent GSM beams which may have the same directionality (i.e., the same angular spread) as a fully coherent Gaussian laser beam.[13] Furthermore, very recently we demonstrated that, besides the GSM beams, there also exist other partially coherent beams such as partially coherent Hermite-cosh-Gaussian beams, partially coherent Hermite–Gaussian (H–G) beams and partially coherent annular flat-topped beams which may have the same angular spread as a fully coherent Gaussian beam both in free space and in turbulence.[14−16] Thus, an interesting question arises: does the result hold true for general partially coherent beams?

On the other hand, as usual, it is recognized that the directionality of beams can be expressed in terms of the far-field divergence angle (i.e., the angular spread), or equivalently, in terms of the far-field radiant intensity distribution.[1,2,13,17] However, it has recently been shown that, unlike GSM beams, the directionality of partially coherent cosh-Gaussian beams expressed in terms of the far-field divergence angle is not consistent with that in terms of the far-field radiant intensity distribution in free space.[18] Therefore, another interesting question may arise: does the directionality of general partially coherent beams expressed in terms of the far-field divergence angle be consistent with that in terms of the far-field radiant intensity distribution in turbulence?

In this paper, the angular spread is chosen as the characteristic parameter of beam directionality, and the directionality of general partially coherent beams

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propagating through atmospheric turbulence is studied. In addition, the consistency of the directionality of general partially coherent beams expressed in terms of the angular spread and of the normalized far-field average intensity distribution is also examined, and some interesting results are obtained.

2. Directionality expressed in terms of the angular spread

The cross-spectral density of a partially coherent beam at the source plane $z = 0$ can be expressed as[19]

$$W_0(x_1', x_2') = \sqrt{I_0(x_1')}\sqrt{I_0(x_2')}\mu_0(x_2' - x_1'), \quad (1)$$

where $I_0(x')$ is the average intensity and $\mu_0(x_2' - x_1')$ is the spectral degree of coherence of the partially coherent beam at the plane $z = 0$.

Based on the extended Huygens–Fresnel principle, the average intensity of the partially coherent beam propagating through atmospheric turbulence at the plane z is given by the expression[5]

$$\begin{aligned}&\langle I(x,z)\rangle\\ &= \frac{k}{2\pi z}\int \mathrm{d}x_1' \int \mathrm{d}x_2' W_0(x_1', x_2')\\ &\quad \times \exp\left\{\left(\frac{\mathrm{i}k}{2z}\right)\left[(x'^2_1 - x'^2_2) - 2(xx_1' - xx_2')\right]\right\}\\ &\quad \times C_\psi(x_1', x_2', x, x, z), \end{aligned} \quad (2)$$

where the angle brackets denote averaging over the field ensemble, k is the wave number related to the wave length λ by $k = 2\pi/\lambda$, and[20]

$$\begin{aligned}C_\psi(x_1', x_2', x, x, z) &= \langle \exp[\psi^*(x_1', x, z) + \psi(x_2', x, z)]\rangle_m\\ &\cong \exp\left[-\frac{(x_1' - x_2')^2}{\rho_0^2}\right], \end{aligned} \quad (3)$$

where $\langle\ \rangle_m$ denotes averaging over the ensemble of the random medium, $\psi(x', x, z)$ is the phase function which depends on the properties of the medium. The $\rho_0 = (0.545C_n^2k^2z)^{-3/5}$ is the coherence length of a spherical wave propagating through atmospheric turbulence; C_n^2 is the structure constant of refractive index, which describes how strong the turbulence is. It is noted that a quadratic approximation of Rytov's phase structure function is used instead of the 5/3 power law in Eq. (3) to obtain an analytical expression for the average intensity in Section 3. This quadratic approximation has been shown to be reliable[20,21] and has been used widely.[8,9,14,15,20–22] We do test the validity of the quadratic approximation once more in numerical calculations (not presented here to save space), and find that the difference between the numerical result obtained with and without this quadratic approximation is quite small, and they give the same trends.

It is to be noted that C_ψ depends only on the difference between x_1' and x_2', so we may write

$$\begin{aligned}C_\psi(x_1', x_2', x, x, z) &= C_\psi(x_1' - x_2', 0, z)\\ &= C_\psi(u, 0, z), \end{aligned} \quad (4)$$

where $u = x_1' - x_2'$.

The mean-squared beam width is defined as[6]

$$w^2(z) = \frac{\int x^2 \langle I(x,z)\rangle \,\mathrm{d}x}{\int \langle I(x,z)\rangle \,\mathrm{d}x}. \quad (5)$$

By using the formulae for free space propagation[23] and the definition of the effective source[6] we can write Eq. (5) as

$$w^2(z) = \sigma_I^2 + \left[\sigma_J^2 - \frac{1}{k^2}\nabla_u^2 C_\psi(u,0,z)\,|_{u=0}\right] z^2, \quad (6)$$

where

$$\sigma_I^2 = \frac{\int x^2 I_0(x)\mathrm{d}x}{\int I_0(x)\mathrm{d}x}, \quad (7)$$

$$\sigma_J^2 = \frac{\int s_x^2 J(\boldsymbol{s})\mathrm{d}s_x}{\int J(\boldsymbol{s})\mathrm{d}s_x}, \quad (8)$$

$$J(\boldsymbol{s}) = 2\pi k\tilde{W}_0(-ks_x, ks_x), \quad (9)$$

where $J(\boldsymbol{s})$ is the power radiated by the source per unit solid angle around the $\boldsymbol{s}$ direction[19] and $\tilde{W}_0(-ks_x,\ ks_x)$ is the spatial Fourier transform of the cross-spectral density at the source plane, i.e.,

$$\begin{aligned}\tilde{W}_0(f_1, f_2) = \frac{1}{(2\pi)^2}\iint & W_0(x_1', x_2')\\ &\times \exp\left[-\mathrm{i}\left(f_1x_1' + f_2x_2'\right)\right]\mathrm{d}x_1'\mathrm{d}x_2'. \end{aligned} \quad (10)$$

Using Eq. (3), we may evaluate the second expression in the square brackets of Eq. (6) , which is given by

$$\begin{aligned}&-\frac{1}{k^2}\nabla_u^2 C_\psi(u,0,z)\,|_{u=0}\\ &= \frac{2}{k^2\rho_0^2} = 2(0.545C_n^2)^{6/5}k^{2/5}z^{6/5}. \end{aligned} \quad (11)$$

Substituting Eq. (11) into Eq. (6) we get

$$w^2(z) = \sigma_I^2 + \sigma_J^2 z^2 + 2(0.545C_n^2)^{6/5}k^{2/5}z^{16/5}. \quad (12)$$

Equation (12) is the mean-squared width of general partially coherent beams propagating through atmospheric turbulence if the quadratic approximation

of the Rytov's phase structure function is adopted, which is one of the main analytical results obtained in this paper. Equation (12) indicates that the spread of partially coherent beams due to atmospheric turbulence is contained within the third term, the first two terms represent the effect of the diffractive spreading of partially coherent beams in free space. It is mentioned that there are some differences between Eq. (12) in this paper and Eq. (30) in Ref. [6]. The reason is that a quadratic approximation of the Rytov's phase structure function is used in this paper, which results in $z^{16/5}$ occurring in the third term of Eq. (12) in this paper, while z^3 appearing in Ref. [6].

From Eq. (12), the angular spread of general partially coherent beams in turbulence turns to be

$$\theta_{\rm sp}(z) \equiv \lim_{z\to\infty} \frac{w(z)}{z} = \sqrt{\sigma_J^2 + 2(0.545C_n^2)^{6/5}k^{2/5}z^{6/5}}. \tag{13}$$

Equation (13) shows that σ_J is a measure of the far-zone angular spread of beam in free space, while the second term denotes the effect of turbulence on the angular spread. It can be seen that in free space, different types of partially coherent beams with the same value of σ_J will generate the same directionality expressed in terms of the angular spread. Furthermore, the result is valid in atmospheric turbulence if the wavelength λ is kept fixed, which is assumed to be fulfilled in this paper.

On the other hand, the mean-squared width and the angular spread of fully coherent Gaussian beams specified by subscripts 'Gs' in turbulence are expressed as[15]

$$w^2(z)\big|_{\rm Gs} = \frac{w_{0\rm Gs}^2}{4} + \frac{z^2}{k^2 w_{0\rm Gs}^2} + 2(0.545C_n^2)^{6/5}k^{2/5}z^{16/5}, \tag{14}$$

and

$$\theta_{\rm sp}(z)\,|_{\rm Gs} = \sqrt{\frac{1}{k^2 w_{0\rm Gs}^2} + 2(0.545C_n^2)^{6/5}k^{2/5}z^{6/5}}. \tag{15}$$

From a comparison of Eqs. (13) and (15) we conclude that a general partially coherent beam will have the same directionality (i.e., the same angular spread) as a fully coherent Gaussian beam both in free space and in atmospheric turbulence if the condition

$$\sigma_J = \frac{1}{k w_{0\rm Gs}} \tag{16}$$

is satisfied.

Based on the general formulae Eq. (12) and (13), the analytical expressions for the mean-squared width and the angular spread of two types of partially coherent beams are derived as follows, and a numerical example is given to illustrate the results obtained above.

(i) Partially coherent flat-topped beam

The cross-spectral density function of partially coherent flat-topped beams at the source plane $z = 0$ is written as[24,25]

$$W_0(x_1', x_2') = \sum_{m=1}^{M}\sum_{m'=1}^{M} \alpha_m \alpha_{m'}' \exp\left[-\left(m p_m \frac{x_1'^2}{w_0^2} + m' p_{m'} \frac{x_2'^2}{w_0^2}\right)\right] \exp\left[-\frac{(x_1' - x_2')^2}{2\sigma_0^2}\right], \tag{17}$$

where M is beam order of partially coherent flat-topped beams, and[24]

$$\alpha_t = (-1)^{t+1}\frac{M!}{t!(M-t)!}, \quad p_t = \sum_{t=1}^{M}\frac{\alpha_t}{t} \quad (t = m, m'). \tag{18}$$

Substituting Eq. (17) into Eqs. (7) and (8), recalling the integral formulae

$$\int \exp[-\beta x^2 + \gamma x]{\rm d}x = \frac{\sqrt{\pi}}{\sqrt{\beta}} \exp\left[\frac{\gamma^2}{4\beta}\right], \tag{19}$$

and

$$\int \exp[-\beta x^2] x^2 {\rm d}x = \frac{\sqrt{\pi}}{\beta^{3/2}}, \tag{20}$$

after straightforward integral calculations, we obtain

$$\sigma_{I1}^2 = \frac{\sum_{m=1}^{M}\sum_{m'=1}^{M} \dfrac{\alpha_m \alpha_{m'} w_0^2}{2\left(mp_m + m'p_{m'}\right)^{3/2}}}{\sum_{m=1}^{M}\sum_{m'=1}^{M} \dfrac{\alpha_m \alpha_{m'}}{\sqrt{mp_m + m'p_{m'}}}}, \tag{21}$$

$$\sigma_{J1}^2 = \frac{2\sum_{m=1}^{M}\sum_{m'=1}^{M} \dfrac{\alpha_m \alpha_{m'}}{\sqrt{mp_m + m'p_{m'}}} \left[\dfrac{1}{2\sigma_0^2} + \dfrac{mp_m m'p_{m'}}{w_0^2\left(mp_m + m'p_{m'}\right)}\right]}{k^2 \sum_{m=1}^{M}\sum_{m'=1}^{M} \dfrac{\alpha_m \alpha_{m'}}{\sqrt{mp_m + m'p_{m'}}}}. \tag{22}$$

From Eqs. (21) and (22), the mean-squared width and the angular spread of partially coherent flat-topped beams turn to be

$$w_1^2(z) = \sigma_{I1}^2 + \sigma_{J1}^2 z^2 + 2(0.545C_n^2)^{6/5} k^{2/5} z^{16/5}, \tag{23}$$

and

$$\theta_{\mathrm{sp1}}(z) = \sqrt{\sigma_{J1}^2 + 2(0.545C_n^2)^{6/5} k^{2/5} z^{6/5}}. \tag{24}$$

For $M = 1$, equations (23) and (24) reduce to the expressions for the mean-squared width and the angular spread of GSM beams, which are given by

$$w_{\mathrm{GSM}}(z) = \frac{w_0^2}{4} + \left(\frac{1}{w_0^2} + \frac{1}{\sigma_0^2}\right)\frac{z^2}{k^2} + 2(0.545C_n^2)^{6/5} k^{2/5} z^{16/5}, \tag{25}$$

$$\theta_{\mathrm{spGSM}}(z) = \sqrt{\left(\frac{1}{w_0^2} + \frac{1}{\sigma_0^2}\right)\frac{1}{k^2} + 2(0.545C_n^2)^{6/5} k^{2/5} z^{6/5}}. \tag{26}$$

(ii) Partially coherent annular Gaussian beam

The partially coherent annular Gaussian beam is modeled as the difference between two Gaussian beams with different values of the waist width w_0 and εw_0, where $0 < \varepsilon < 1$. The cross-spectral density function of partially coherent annular Gaussian beams at the source plane reads as[26]

$$W_0(x_1', x_2') = \left\{\exp\left[-\frac{x_1'^2}{w_0^2}\right] - \exp\left[-\frac{x_1'^2}{\varepsilon^2 w_0^2}\right]\right\}\left\{\exp\left[-\frac{x_2'^2}{w_0^2}\right] - \exp\left[-\frac{x_2'^2}{\varepsilon^2 w_0^2}\right]\right\}\exp\left[-\frac{(x_1' - x'_2)^2}{2\sigma_0^2}\right]. \tag{27}$$

Similarly, we can obtain the mean-squared width and the angular spread of partially coherent annular Gaussian beams, which are given by

$$w_2^2(z) = \sigma_{I2}^2 + \sigma_{J2}^2 z^2 + 2(0.545C_n^2)^{6/5} k^{2/5} z^{16/5}, \tag{28}$$

and

$$\theta_{\mathrm{sp2}}(z) = \sqrt{\sigma_{J2}^2 + 2(0.545C_n^2)^{6/5} k^{2/5} z^{6/5}}, \tag{29}$$

where

$$\sigma_{I2}^2 = \frac{w_0^2}{4}\left(\frac{1 + \varepsilon^3 - \dfrac{2^{5/2}\varepsilon^3}{(1+\varepsilon^2)^{3/2}}}{1 + \varepsilon - \dfrac{2^{3/2}\varepsilon}{\sqrt{1+\varepsilon^2}}}\right), \tag{30}$$

$$\sigma_{J2}^2 = \frac{\dfrac{1}{w_0^2}\left(1 + \dfrac{1}{\varepsilon} - \dfrac{2^{5/2}\varepsilon}{(1+\varepsilon^2)^{3/2}}\right) + \dfrac{1}{\sigma_0^2}\left(1 + \varepsilon - \dfrac{2^{3/2}\varepsilon}{(1+\varepsilon^2)^{1/2}}\right)}{k^2\left[1 + \varepsilon - \dfrac{2\sqrt{2}\varepsilon}{\sqrt{1+\varepsilon^2}}\right]}. \tag{31}$$

It is mentioned that equation (28) is essentially in agreement with Eq. (16) in Ref. [16], which is derived in different way.

From Eqs. (15), (24) and (29) it can be seen that, whether in free space or in atmospheric turbulence a partially coherent flat-topped beam and a partially coherent annular Gaussian beam will have the same angular spread as a fully coherent Gaussian beam if the condition

$$\sigma_{J1} = \sigma_{J2} = \frac{1}{kw_{0\mathrm{Gs}}} \tag{32}$$

is fulfilled.

It is noted that, in the limiting case $\sigma_0 \to \infty$ the mean-squared width and the angular spread of the two partially coherent beams mentioned above reduce to those of the two corresponding fully coherent beams.

Figure 1 gives the mean-squared width $w(z)$ of partially and fully coherent beams which satisfy the condition (32) and the corresponding fully coherent Gaussian beam propagating both in free space and in atmospheric turbulence, where $\lambda = 1.06$ μm is kept fixed, and the other beam parameters are listed in Table 1. As can be expected, the fully and partially coherent flat-topped beams (i.e., "b" and "c"), the GSM beam (i.e., "d"), and the fully and partially coherent annular Gaussian beams (i.e., "e" and "f") exhibit the same directionality (i.e., angular spread) as the corresponding fully coherent Gaussian beam (i.e., "a") not only in free space but also in atmospheric turbulence.

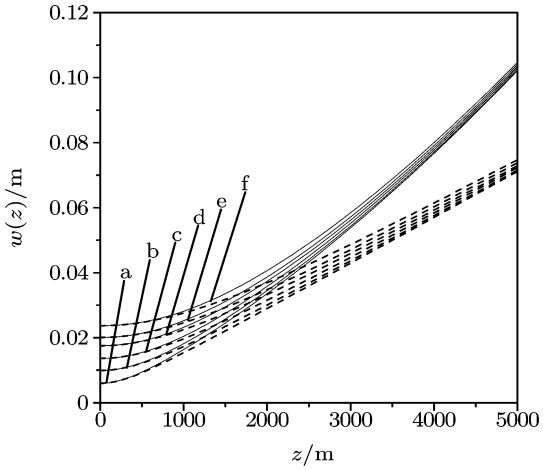


Fig. 1. Mean-squared width $w(z)$ versus propagation distance z. The calculation parameter is $\lambda = 1.06$ μm, and the other parameters are listed in Table 1, where condition (32) is satisfied. line a: fully coherent Gaussian beam; lines b and c: fully and partially coherent flat-topped beams; line d: GSM beam; lines e and f: fully and partially coherent annular Gaussian beams. dash line for $C_n^2 = 0$; solid line for $C_n^2 = 10^{-14}$ m$^{-2/3}$.

Table 1. Beam parameters corresponding to Fig. 1.

	M	w_0/mm	σ_0/mm	ε
a (fully coherent Gaussian beam)		11.90	∞	
b (fully coherent flat-topped beam)	$M = 12$	15.81	∞	
c (partially coherent flat-topped beam)	M=10	26.12	14.70	
d (GSM beam)		35.00	12.71	
e (fully coherent annular Gaussian beam)		21.30	∞	0.7
f (partially coherent annular Gaussian beam)		22.29	258.10	0.9

3. Consistency of the directionality expressed in terms of the angular spread and of the normalized far-field average intensity distribution

The partially coherent annular Gaussian beam is taken as a typical example of general partially coherent beams, the consistency of the directionality expressed in terms of the angular spread and of the normalized far-field average intensity distribution is examined in this section.

Substituting Eqs. (3) and (27) into Eq. (2), after very tedious but straightforward integral calculations we obtain the average intensity of partially coherent annular Gaussian beams, i.e.,

$$\langle I(x,z)\rangle = I_1(x,z)+I_2(x,z)-I_3(x,z)-I_4(x,z), \tag{33}$$

where

$$I_1(x,z) = \frac{k}{2z\beta_1\beta_2}\exp\left[-\frac{k^2}{2z^2\beta_1^2\beta_2^2}\cdot\frac{x^2}{w_0^2}\right], \tag{34}$$

$$I_3(x,z)=\frac{k}{2z\beta_1\beta_5}\exp\left[-\frac{k^2}{4z^2\beta_1^2\beta_5^2}\cdot\frac{(1+\varepsilon^2)x^2}{\varepsilon^2w_0^2}\right], \quad (35)$$

$$\beta_1^2=\frac{1}{w_0^2}+\frac{1}{2\sigma_0^2}+\frac{1}{\rho_0^2}-\mathrm{i}\frac{k}{2z}, \quad (36)$$

$$\beta_2^2=\frac{1}{w_0^2}+\frac{1}{2\sigma_0^2}+\frac{1}{\rho_0^2}+\mathrm{i}\frac{k}{2z}-\frac{\eta^2}{\beta_1^2}, \quad (37)$$

$$\beta_3^2=\frac{1}{\varepsilon^2w_0^2}+\frac{1}{2\sigma_0^2}+\frac{1}{\rho_0^2}-\mathrm{i}\frac{k}{2z}, \quad (38)$$

$$\beta_4^2=\frac{1}{\varepsilon^2w_0^2}+\frac{1}{2\sigma_0^2}+\frac{1}{\rho_0^2}+\mathrm{i}\frac{k}{2z}-\frac{\eta^2}{\beta_3^2}, \quad (39)$$

$$\beta_5^2=\frac{1}{\varepsilon^2w_0^2}+\frac{1}{2\sigma_0^2}+\frac{1}{\rho_0^2}+\mathrm{i}\frac{k}{2z}-\frac{\eta^2}{\beta_1^2}, \quad (40)$$

$$\beta_6^2=\frac{1}{w_0^2}+\frac{1}{2\sigma_0^2}+\frac{1}{\rho_0^2}+\mathrm{i}\frac{k}{2z}-\frac{\eta^2}{\beta_3^2}, \quad (41)$$

$$\eta=\frac{1}{2\sigma_0^2}+\frac{1}{\rho_0^2}. \quad (42)$$

$I_2\ (x,z)$ can be obtained if β_1, β_2 and w_0 are replaced by β_3, β_4 and εw_0 in Eq. (34); $I_4\ (x,z)$ can be obtained if β_1 and β_5 are replaced by β_3 and β_6 in Eq. (35).

Figure 2 shows the mean-squared width $w(z)$ of partially coherent annular Gaussian beams with different values of ε and the corresponding fully coherent Gaussian beam propagating both in free space and in atmospheric turbulence, where the condition (32) is satisfied. The calculation parameters are listed in Table 2, and $\lambda=1.06$ μm is taken. Figure 2 shows that the curves of the mean-squared width $w(z)$ are close to each other with the increase of the propagation distance z, which means that the three partially coherent annular Gaussian beams with different values of ε have the same angular spread as the corresponding fully coherent Gaussian beam both in free space and in turbulence.

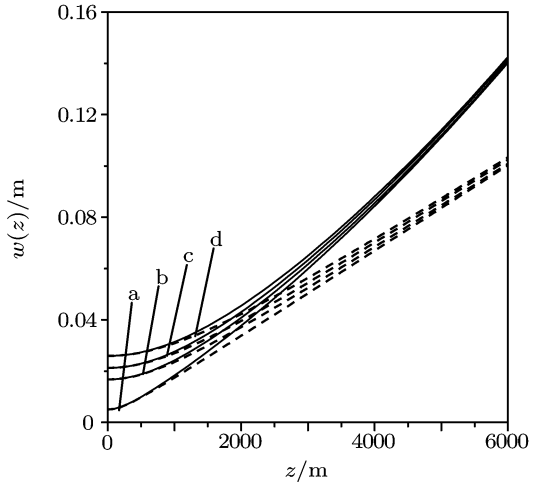


Fig. 2. Mean-squared width $w(z)$ versus propagation distance z. The calculation parameter is $\lambda=1.06$ μm, and the other parameters are listed in Table 2, where condition (32) is satisfied. a: Fully coherent Gaussian beam; b, c and d: partially coherent annular Gaussian beams. hash line for $C_n^2=0$; solid line for $C_n^2=10^{-14}$ m$^{-2/3}$.

Table 2. Beam parameters corresponding to Figs. 2 and 3.

	w_0/mm	σ_0/mm	ε
a (fully coherent Gaussian beam)	10.11	∞	
b (partially coherent annular Gaussian beam)	22.49	180.00	0.4
c (partially coherent annular Gaussian beam)	20.00	199.80	0.9
d (partially coherent annular Gaussian beam)	28.00	170.11	0.2

Figure 3 gives the normalized far-field average intensity distributions $\langle I(x,z)\rangle/\langle I(0,z)\rangle$ of partially coherent annular Gaussian beams with different values of ε and of the corresponding fully coherent Gaussian laser beam in accordance with Fig. 2, where z=20 km is taken, and other parameters are the same as those in Fig. 2. Figure 3(a) shows that $\langle I(x,z)\rangle/\langle I(0,z)\rangle$ of partially coherent annular Gaussian beams is different from that of the corresponding fully coherent Gaussian laser beam, and there are differences between $\langle I(x,z)\rangle/\langle I(0,z)\rangle$ of partially coherent annular Gaussian beams with different values of ε, though they have the same angular spread in free space (see Fig. 2). In free space, there are several side lobes in $\langle I(x,z)\rangle/\langle I(0,z)\rangle$ of partially coherent annular Gaussian beams, while $\langle I(x,z)\rangle/\langle I(0,z)\rangle$ of the fully coherent Gaussian laser beam is a Gaussian profile. However, in turbulence (see Fig. 3(b), $C_n^2=10^{-14}$ m$^{-2/3}$) $\langle I(x,z)\rangle/\langle I(0,z)\rangle$ of partially coherent annular Gaussian beams becomes Gaussian profile, and $\langle I(x,z)\rangle/\langle I(0,z)\rangle$ of partially coherent annular Gaussian beams is the same as that of the corresponding fully coherent Gaussian laser beam.

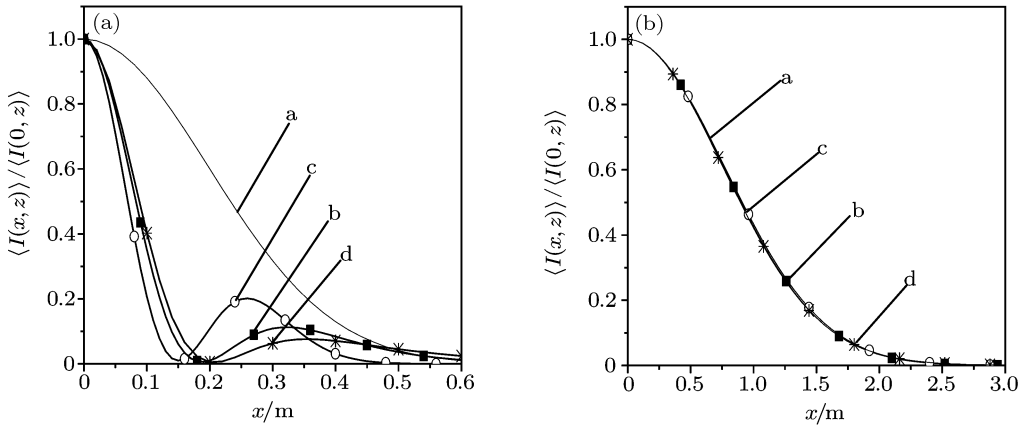


Fig. 3. Normalized far-field average intensity distributions $\langle I(x,z)\rangle/\langle I(0,z)\rangle$ in accordance with Fig. 2, where $z = 20$ km and other parameters are the same as those in Fig. 2. a: fully coherent Gaussian laser beam; b, c, and d: partially coherent annular Gaussian beams. (a) $C_n^2 = 0$; (b) $C_n^2 = 10^{-14}$ m$^{-2/3}$.

4. Conclusions

Under the quadratic approximation of the Rytov's phase structure function, we have derived the general closed-form expressions for the mean-squared width and the angular spread of partially coherent beams. We show that different types of partially coherent beams with the same value of σ_J will generate the same directionality both in free space and in atmospheric turbulence. In general, under a certain condition different types of partially coherent beams may have the same directionality as a fully coherent Gaussian beam in free space and also in atmospheric turbulence.

On the other hand, from Refs. [1], [2], [13] and [17], it can be seen that for GSM beams, the directionality expressed in terms of the angular spread is consistent with that in terms of the normalized far-field average intensity distribution both in free space and in turbulence. However, this paper finds that generally, the directionality of partially coherent beams expressed in terms of the angular spread is not consistent with that in terms of the normalized far-field average intensity distribution in free space, while the consistency can be achieved due to turbulence.

References

[1] Collett E and Wolf E 1978 *Opt. Lett.* **2** 27
[2] Wolf E and Collett E 1978 *Opt. Commun.* **25** 293
[3] Santis P D, Gori F, Guattari G and Palma C 1979 *Opt. Commun.* **29** 256
[4] Farina J D, Narducci L M and Collett E 1980 *Opt. Commun.* **32** 203
[5] Andrews L C and Phillips R L 1998 *Laser Beam Propagation Through Random Media* (Bellingham, WA: SPIE)
[6] Gbur G and Wolf E 2002 *J. Opt. Soc. Am.* A **19** 1592
[7] Dogariu A and Amarande S 2003 *Opt. Lett.* **28** 10
[8] Eyyuboğlu H T and Baykal Y 2007 *Opt. Commun.* **278** 17
[9] Wang T and Pu J X 2007 *Acta Phys. Sin.* **56** 6754 (in Chinese)
[10] Ji X L, Huang T X and Lü B D 2006 *Acta Phys. Sin.* **55** 978 (in Chinese)
[11] Zhang E T, Ji X L and Lü B D 2009 *Chin. Phys.* B **18** 571
[12] Ji X L, Xiao X and Lü B D 2004 *Acta Phys. Sin.* **53** 3996 (in Chinese)
[13] Shirai T, Dogariu A and Wolf E 2003 *Opt. Lett.* **28** 610.
[14] Yang A L, Zhang E T, Ji X L and Lü B D 2008 *Opt. Express* **16** 8366
[15] Ji X L, Chen X W and Lü B D 2008 *J. Opt. Soc. Am.* A **25** 21
[16] Chen X W and Ji X L 2008 *Opt. Commun.* **281** 4765
[17] Foley J T and Zubairy M S 1978 *Opt. Commun.* **26** 297.
[18] Lü S Y and Lü B D 2008 *Opt. Commun.* **281** 3514
[19] Mandel L and Wolf E 1995 *Optical Coherence and Quantum Optics* (Cambridge: Cambridge University Press)
[20] Wang S C H and Plonus M A 1979 *J. Opt. Soc. Am.* **69** 1297
[21] Leader J C 1978 *J. Opt. Soc. Am.* **68** 175
[22] Cai Y J and He S 2006 *Opt. Express* **14** 1353
[23] Gbur G and Wolf E 2001 *Opt. Commun.* **199** 295
[24] Li Y 2002 *Opt. Commun.* **206** 225
[25] Qiu Y, Guo H and Chen Z 2005 *Opt. Commun.* **245** 21
[26] Cai Y J and Zhang L 2006 *J. Opt. Soc. Am.* B **23** 1398