Numerical study of electromagnetic scattering from one-dimensional nonlinear fractal sea surface*

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In recent years, linear fractal sea surface models have been developed for the sea surface in order to establish an electromagnetic backscattering model. Unfortunately, the sea surface is always nonlinear, particularly at high sea states. We present a nonlinear fractal sea surface model and derive an electromagnetic backscattering model. Using this model, we numerically calculate the normalized radar cross section (NRCS) of a nonlinear sea surface. Comparing the averaged NRCS between linear and nonlinear fractal models, we show that the NRCS of a linear fractal sea surface underestimates the NRCS of the real sea surface, especially for sea states with high fractal dimensions, and for dominant ocean surface gravity waves that are either very short or extremely long.

Keywords: fractals, nonlinearities, sea surface electromagnetic scattering

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1. Introduction

Fractal geometry[1,2] is an ideal tool for characterizing the natural sea surface because it can simulate semi-stochastic and semi-periodic features.[3,4] Different fractal sea surface models have been used to simulate the surfaces of the ocean[5−9] and land.[10−12] Recently, many studies have considered electromagnetic (EM) scattering from a fractal surface. Common forms of fractal surface models include the band-limited fractional Brownian motion (fBm) fractal surface[6−12] and the band-limited Weierstrass fractal function model.[7−9] The Kirchhoff approximation method,[5,7−9,13] the so-called forward–backward approach,[12] and the extended boundary condition method[14] are used to calculate the normalized radar cross sections (NRCS) for scattering from a fractal surface. Results from electromagnetic scattering from a fractal sea surface can be used in the remote sensing of ocean variables, for example, retrievals of wave spectra and marine winds, ship detection, oil slick recognition, pollution analysis and related problems. However, it is vital to establish a reliable model to simulate the real sea surface, then one can accurately predict the EM backscattering intensity, which can be expressed by the NRCS.

The studies mentioned above are based on a linear fractal Gaussian sea surface model. Unfortunately, the real sea surface always consists of nonlinear ocean waves, and nonlinear interactions between long waves and short waves commonly occur.[15] There are large discrepancies between the EM echo signals scattering from a linear sea surface and those scattering from a nonlinear sea surface,[16] particularly in numerical calculations of the EM echo from a rough sea surface. To reduce these EM echo signal discrepancies, it is necessary to develop a nonlinear fractal sea surface model. In this paper, we present a two-scale nonlinear fractal sea surface model for a one-dimensional (1D) deep sea. We calculate the associated NRCS of the sea surface with this model and then compare the NRCS characteristics of the backscatter resulting from a linear fractal sea surface.

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2. Electromagnetic scattering on 1D nonlinear fractal sea surface

To numerically study EM scattering from a 1D nonlinear fractal sea surface, one can start from a linear fractal Gaussian sea surface model as presented by Berizzi.[7] Applying the nonlinear interaction relationships between long and short waves as presented by Longuet–Higgins and Stewart,[15] a nonlinear fractal sea surface model can be set up to simulate the nonlinear sea surface. Here, the Kirchhoff approximation method is used to develop the EM model of the 1D nonlinear fractal sea surface.

2.1. The 1D nonlinear fractal sea surface model

The 1D linear sea surface can be expressed[8,17] as

$$f_{\rm linear}(x,t)=\sum_{n=1}^{N}a_n\sin\psi_n\ , \tag{1}$$

where subscript n ($1\leq n\leq N$) denotes the $n^{\rm th}$ sine wave component of the surface with amplitude $a_n=\sigma C b^{(s-2)(n-1)}$, phase $\psi_n=K_1b^{n-1}(x+Vt)-\Omega_n t+\Phi_n$, wave number $k_n=K_1b^{(n-1)}$, frequency $\Omega_n=\sqrt{gk_n}$, and surface current velocity $V=v_{\rm o}+v_f$[17–19] where $v_{\rm o}$ is Stokes velocity of large scale wave, v_f is the shift in current velocity caused by wind

$$v_{\rm o}=\omega_1K_1\left(\frac{H_1}{2}\right)^2\frac{\cosh(2K_1d)}{2\sinh^2(K_1d)}-\omega_1\left(\frac{H_1}{2}\right)^2\frac{\coth(2K_1d)}{2d}, \tag{2}$$

$$v_f=0.03U_{19.5}, \tag{3}$$

where wave number, frequency and wave height of the fundamental wave are $K_1\approx 0.77g/U_{19.5}^2$, $\omega_1\approx 0.877g/U_{19.5}$ and $H_1\approx 0.0212U_{19.5}^2$, respectively. Here, $U_{19.5}$ is the wind speed at 19.5 m above the sea surface, d is the depth of water, $C=\sqrt{2[1-b^{2(s-2)}]/(1-b^{2N(s-2)})}$, b is a constant, s is the fractal dimension ($1<s<2$), σ is the variance of the linear sea surface, and g is the gravitational acceleration. Initial phases Φ_n are uniformly distributed in the interval $[-\pi,\pi]$. The mean value of $f_{\rm linear}(x,t)$ is zero.

In this formulation, the nonlinear interaction relationship between long and short waves[15] is

$$\begin{aligned}&f_{ls}(x,t)\\&=\left(a_l\sin\psi_l-\frac{1}{2}a_l^2k_l\sin 2\psi_l\right)\\&\quad+\left(a_s\sin\psi_s-\frac{1}{2}a_s^2k_s\sin 2\psi_s\right)\\&\quad-a_la_s\left(k_s\cos\psi_l\cos\psi_s-k_l\sin\psi_l\sin\psi_s\right),\end{aligned} \tag{4}$$

where subscripts l and s denote long and short waves, respectively. Applying the two-scale model to the linear sea surface model, the linear sea surface function given in Eq. (1) can be divided into two parts. One is the long wave part, and the other is the short wave part. The separation criterion for these two parts is provided by the critical value $n_{\rm c}=N_0$. Sine wave components whose subscripts satisfy $n\leq N_0$ belong to the long wave part; otherwise they can be regarded as short waves. Here, N_0 can be determined by

$$b^{(N_0+1)}-b^{(N_0)}\geq 10. \tag{5}$$

Equation (4) implies that $k_{N_0+1}\gg k_{N_0}$, so that the relations for nonlinear interactions between long and short waves[15] can be used. Therefore, the nonlinear fractal sea surface model is

$$\begin{aligned}&f_{\rm nonlinear}(x,t)\\&=f_{\rm linear-part}(x,t)+f_{\rm nonlinear-part}(x,t)\\&=\sum_{n=1}^{N}\left(a_n\sin\psi_n-\frac{1}{2}a_n^2k_n\sin 2\psi_n\right)-\frac{1}{2(N-N_0)}\sum_{n=N_0+1}^{N}\sum_{m=1}^{N_0}a_na_m\left[\begin{array}{l}(k_n+k_m)\cos(\psi_n+\psi_m)\\+(k_n-k_m)\cos(\psi_n-\psi_m)\end{array}\right].\end{aligned} \tag{6}$$

Details are presented by Xie *et al.*[20]

Considering the wind effects on the nonlinear fractal sea surface, we relate fractal dimensions to the wind speeds, using component amplitudes of sine waves. The $n^{\rm th}$ amplitude of wave component of the sine waves is

$$a_n=2\sqrt{W(k_n)\Delta k}, \tag{7}$$

where the 1D Pierson–Mospkowitz (PM) spectrum[17,21] of the sea surface is

$$W(k_n) = \frac{\alpha}{2\,|k_n|^4} \exp\left(-\frac{\beta g^2}{k_n^2 U_{19.5}^4}\right), \tag{8}$$

where $\alpha = 0.0081$ and $\beta = 0.74$. Using Eqs. (7) and (8) and expression $a_n = \sigma C b^{(s-2)(n-1)}$, the relationship between wind speed and fractal dimension s can be retrieved.

Figure 1 shows the relationship between wind speeds at 19.5 m above the sea surface, in terms of the number of components of sea surface and fractal dimension, when $\sigma = 1$.

From Fig. 1, one can find that (a) wind speeds at 19.5 m above the sea surface are decreasing when the fractal dimensions are increasing and N is held constant, (b) wind speeds also are decreasing when the numbers of components N of sea surface are increasing, when the fractal dimensions are held constant. These characteristics are obvious in the case of larger fractal dimensions rather than in the case of lower fractal dimension.

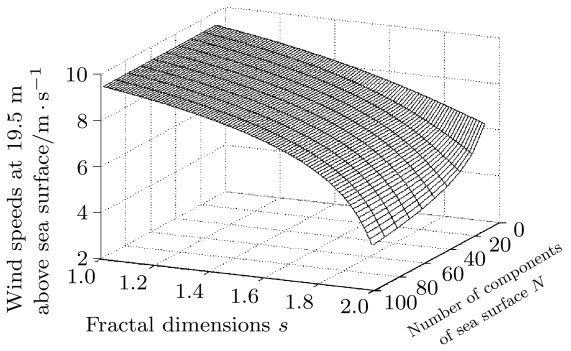


Fig. 1. Relationship between wind speeds at 19.5 m above the sea surface, number of sea surface components N and fractal dimensions when $\sigma = 1$.

2.2. EM backscattering model from nonlinear fractal ocean surface

Following the approach of Berizzi and Mese,[8] one can obtain the scattering coefficient for a 1D nonlinear fractal sea surface,

$$\gamma = \sum_{p_1,p_2,\ldots p_N=-\infty}^{\infty} \gamma_1 \cdot \gamma_2 \cdot \gamma_3, \tag{9}$$

$$\begin{aligned}\gamma_1 \;=\; & \prod_{n=1}^{N} J_{p_n}(-2\sigma C k_{\mathrm{e}} b^{(s-2)(n-1)} \cos\theta_i) \cdot J_{p_n}(2\sigma^2 C^2 k_{\mathrm{e}} b^{(2s-3)(n-1)} \cos\theta_i) \\ & \times \prod_{n=N_0+1}^{N} \prod_{m=1}^{N_0} J_{q_m}\left(\frac{1}{N-N_0} k_{\mathrm{e}} K_1 \sigma^2 C^2 b^{(s-2)(m+n-2)} (b^{(n-1)} + b^{(m-1)}) \cos\theta_i\right) \\ & \times \prod_{n=N_0+1}^{N} \prod_{m=1}^{N_0} J_{q_m}\left(\frac{1}{N-N_0} k_{\mathrm{e}} K_1 \sigma^2 C^2 b^{(s-2)(m+n-2)} (b^{(n-1)} - b^{(m-1)}) \cos\theta_i\right), \end{aligned} \tag{10}$$

$$\gamma_2 \;=\; \sin c\left[\left(2k_{\mathrm{e}} \sin\theta_i + 3K_1 \sum_{n=1}^{N} p_n b^{(n-1)} + 2K_1 \sum_{n=N_0+1}^{N} \sum_{m=1}^{N_0} q_m b^{(n-1)}\right) X\right], \tag{11}$$

$$\gamma_3 \;=\; \exp\left\{j\left[3\underline{p}^{\mathrm{T}}\underline{\varphi}(t) + 2\sum_{n=N_0+1}^{N} \sum_{q_m=1}^{N_0} q_m \left(\varphi_n(t) + \frac{\pi}{2}\right)\right]\right\}, \tag{12}$$

where k_{e} is the EM wave number with incident angle θ_i. The illuminating area of radar is X and $\underline{p} = (p_1, p_2, \ldots, p_N)^{\mathrm{T}}$, $\underline{q} = (q_1, q_2, \ldots, q_{N_0})^{\mathrm{T}}$, $\underline{\varphi}(t) = (\varphi_1(t), \varphi_2(t), \ldots, \varphi_n(t), \ldots, \varphi_N(t))$, $\underline{\varphi'}(t) = (\varphi_1(t), \varphi_2(t), \ldots, \varphi_m(t), \ldots, \varphi_{N_0}(t))$, $\varphi_v(t) = \Phi_v(t) - \Omega_v t + K_1 b^{v-1} V t$, where $v = n, m$.

For the detailed derivations of Eqs. (9)–(12) and the above parameters, see Refs. [5] and [8].

3. Numerical results and discussion

In this section, we calculate sea surface elevation by using the linear and the nonlinear fractal sea surface models separately. We then compute the EM backscattered NRCS, denoted by σ_0, of the linear and the nonlinear sea surface models by using the relationship,

$$\sigma_0 = 10\log_{10}(\gamma). \tag{13}$$

Finally, the EM backscattered NRCS discrepancies between linear and nonlinear models caused by the variations in the longest ocean wavelength $2\pi/k_1$ and the value of the fractal dimensions are presented.

Numerical simulations of the linear and the nonlinear sea surfaces are shown in Fig. 2. Ocean waves are simulated with the initial conditions $t = 0.5$: main wavelength $\lambda_1 = 150$ m, $b = 2e/3$, fractal dimensions $s = 1.4$, $\sigma = 0.5$, number of sine wave components $N = 20$, and current velocity $V = 0.5$. We assume that the frequency is 5.405×10^9 Hz, which constitutes the frequency of the incidence EM wave, that the wind speed is 6.9 m/s and the incidence angle is 30°. Figure 2(a) shows the linear fractal sea surface elevation as a function of position in the x- direction; the corresponding nonlinear fractal sea surface elevations are shown in Fig. 2(b). The elevation differences between the linear and the nonlinear fractal surfaces are shown in Fig. 2(c). From Fig. 2(c), one can see that values of the elevation difference occurring in the locations of crests and troughs of the dominant wave are much larger than those occurring in other locations. This result coincides with the occurrence of nonlinear interactions between long and short waves.[15]

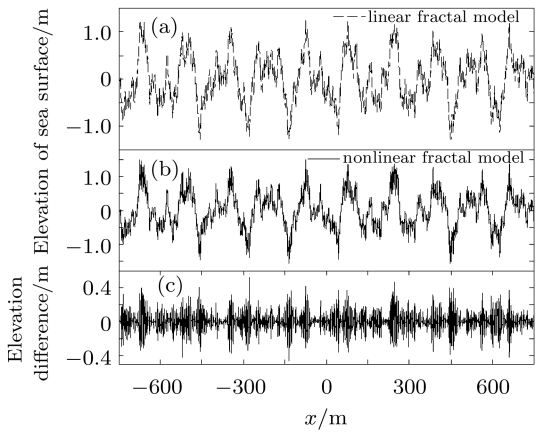


Fig. 2. Elevation of (a) linear fractal sea surface, (b) nonlinear fractal sea surface, and (c) corresponding elevation difference between nonlinear and linear fractal sea surfaces.

To study the EM backscatter discrepancies between the nonlinear and the linear fractal sea surfaces, one can begin with the numerically computing of the EM backscattered NRCS of the nonlinear fractal sea surface by using Eqs. (9)–(12). The NRCS of the linear fractal sea model can then be calculated by using similar equations, with the nonlinear terms neglected. The NRCS of the linear fractal model and that of the nonlinear model can be compared with each other by using the nonlinear PM nonlinear sea surface with following the method presented by Lo and Chiang.[22] Variations in NRCS with incident angle for the linear and the nonlinear sea surfaces are shown in Fig. 3. Incident angles vary from 0° to 70°.

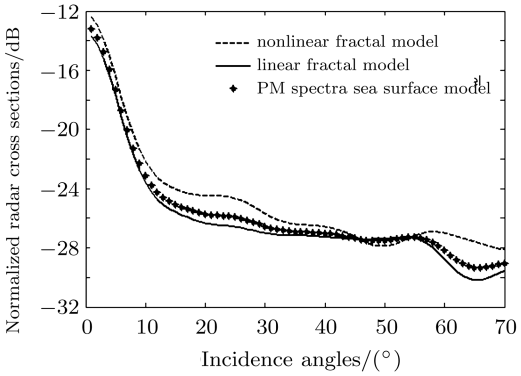


Fig. 3. EM backscattered NRCS of a nonlinear sea surface compared with that of a linear sea surface. Incident angle varies from 0° to 70°. The NRCS of nonlinear fractal sea surface is represented with dash line, the NRCS of linear fractal sea surface with solid line, and the NRCS of PM spectra nonlinear sea surface with the method of Lo and Chiang[22] with asterisk line.

Figure 3 shows that the difference in average NRCS backscatter from the sea surface between the nonlinear and the linear models is –1.0928 dB, averaged over [0°, 70°]. The largest discrepancies are mainly distributed around moderate incident angles ranging from 10° to 70°. This result can be interpreted as the Bragg scattering mechanism at moderate incident angles. Because of the nonlinear interactions between long and short waves, the frequency bands of the nonlinear fractal sea surface model are more densely populated.[17] Thus, the spectrum intensity at most frequency bands is strengthened, and the Bragg resonant intensity for a nonlinear fractal sea surface is therefore smaller than that of a linear fractal sea surface. Moreover, at moderate incident angles, most EM backscattered NRCS values are smaller for a nonlinear sea surface than for a linear sea surface.

Comparing the NRCS of the linear fractal model and that of the nonlinear fractal model with the NRCS of the PM spectra nonlinear sea surface, obtained with the method of Lo and Chiang[22] in Fig. 3, we suggest that the NRCS for a real (nonlinear) sea surface is underestimated by using a linear fractal model whereas it is underestimated by using a nonlinear fractal model.

However, the NRCS discrepancies between the nonlinear fractal model and the PM spectra sea surface model are smaller than those of linear fractal model over the entire incident angle domain. Thus, the nonlinear fractal model is more accurate than the linear fractal model.

What is the relationship or trend between the NRCS discrepancy and sea state? To answer this question, we change the dominant wavelength $2\pi/k_1$ and the value of the fractal dimension s, but we keep the other parameters unchanged. We then calculate the average backscattered NRCS discrepancy between nonlinear and linear sea surfaces. Figure 4 shows the variation in NRCS discrepancy as a function of the dominant ocean wavelength. In order to clarify the effect of the dominant wavelength on the NRCS discrepancy, we use a running average to smooth the numerical results in Fig. 4, thereby eliminating the system variability. The smoothed data suggests that the magnitude of the NRCS discrepancy decreases more rapidly with short wavelength increasing from 10 m to 95 m, than with moderate wavelength increasing from 95 m to 250 m. On the other hand, when the dominant ocean wavelength exceeds 250 m, the magnitude of the NRCS discrepancy increases with the increase of wavelength. Overall, the minimum and the maximum values of the average NRCS discrepancy are –0.1153 dB and –1.544 dB, respectively.

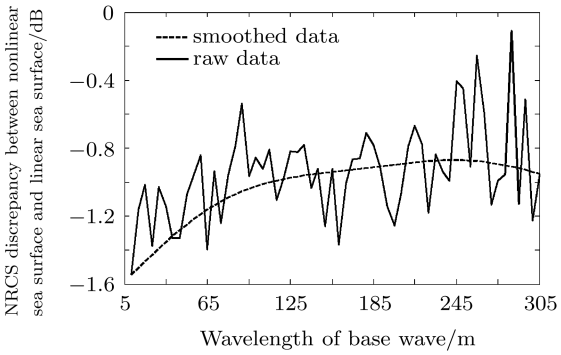


Fig. 4. NRCS discrepancy between nonlinear and linear sea surfaces as a function of dominant ocean wavelength ($5\ \text{m} \leq 2\pi/k_1 \leq 305\ \text{m}$).

We also compute the NRCS discrepancy between nonlinear and linear sea surfaces as a function of fractal dimension s. As shown in Fig. 5, when the fractal dimension is less than 1.7, the magnitude of the NRCS discrepancy increases very slowly, varying between 1.03 dB and 1.53 dB. However, when the fractal dimension increases from 1.63 to 1.96, the NRCS discrepancy magnitude increases rapidly to 8 dB, with the value of s increasing.

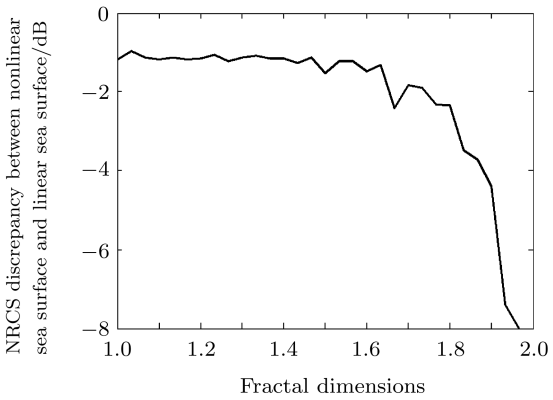


Fig. 5. NRCS discrepancy between nonlinear and linear sea surfaces as a function of fractal dimension s, where ($1 \leq s \leq 1.96$).

4. Conclusions

In recent years, linear fractal sea surface models have been used to calculate the sea surface EM backscattered NRCS. In these models, the ocean surface is assumed to be represented by many linearly superposed different harmonic waves. However, the real sea surface is always nonlinear. We present a nonlinear fractal sea surface model in a relevant paper.[17] In a nonlinear fractal sea surface, the spectra are distributed over additional frequency and wave number band domains compared with the case of a linear sea surface. Thus, the NRCS from a nonlinear sea surface is quite different from that of a linear sea surface.

In this paper, we derived the EM backscattered NRCS model for a nonlinear sea surface model, and compared the corresponding backscattered NRCS of a nonlinear sea surface with that of a linear fractal sea surface. The NRCS discrepancies between the nonlinear and linear models suggest that (i) from numerical results, a linear fractal sea surface model leads to underestimates in the NRCS, compared with the real sea surface, in all sea states; (ii) the NRCS discrepancy magnitude (between nonlinear and linear sea surfaces) is low for backscatter from sea states with dominant ocean wavelengths that are moderate, compared with those with notably short or long surface gravity waves; (iii) for fractal dimensions over 1.7, corresponding to comparatively rough sea surface conditions, the NRCS discrepancy magnitude becomes relatively large. A linear fractal model is therefore not suitable for the calculation of this NRCS.

We suggest that our nonlinear fractal EM backscattering model can potentially be more reliably used to numerically study EM backscattering from high sea states, or rough seas, compared with a linearly-based backscattering model.

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