Impulsive synchronization and control of directed transport in chaotic ratchets*

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The impulsive synchronization problem of two identical chaotic ratchets is investigated in this paper. We demonstrate that the impulsive method to control directed transport is applicable when there are multiple co-existing attractors in phase space transporting particles in different directions. Numerical simulations are carried out to illustrate the effectiveness of the proposed method.

Keywords: chaos synchronization, impulsive control, Brownian motion, chaotic transport

PACC: 0545, 0540, 0560

1. Introduction

During the last decade, the synchronization of chaotic systems has been explored intensively by many researchers in various fields including physics, mathematics and engineering. The possible application of chaotic synchronization for spectrum communication,[1] secure communication[2] and chaotic cryptography[3] has led to a variety of chaotic synchronization techniques, such as bi-directional coupling,[4] nonlinear control,[5] adaptive control,[6] backstepping control,[7] time-delay feedback,[8] active control,[9,10] impulse control,[11−16] and so forth.

Recently, impulsive control has been widely used to stabilize and synchronize the chaotic systems.[12,13,16−19] The impulsive control may give a more efficient method for dealing with systems that cannot endure continuous disturbance. Using this technique the response system receives the samples of state variables (or functions of state variables) called synchronization impulses, from the drive system only in discrete times and the amount of transmitted synchronizing signals is, therefore, reduced considerably. Moreover, this is suitable in practice because of the reduced control cost.

In a different context, there has been an increasing interest in the study of transport phenomena of nonlinear systems that can extract usable work from unbiased non-equilibrium fluctuations in the past decade.[20] These so-called Brownian ratchets can be modelled by a Brownian particle undergoing random walk on a periodic asymmetric potential and being acted upon by an external time-dependent zero-average force. Research activities in this area are motivated by a series of scientific and technological reasons in devising and understanding noise-induced separation and pumping techniques (see Refs. [20] and [21] and the references therein).

Although the majority of literature in this research area considers the presence of noise, very recently there have been attempts to model the transport properties of classical deterministic ratchets as well.[22−28] In these works, the dynamics of classical deterministic inertia ratchets exhibiting chaotic motion has been intensively studied. It is well known that the chaotic dynamics of classical inertia ratchets modifies significantly the transport properties. For example, current reversal and multiple current reversals have been attributed to changes in the bifurcation structure. In fact, the study of the current reversal phenomena has given rise to a research activity on its own. In addition, the implication of chaotic dynamics in deterministic ratchets has been recently addressed in the quantum domain, together with the possible connection with quantum chaos.[29−33]

Synchronization of two coupled chaotic deterministic inertia ratchets has been recently investigated[34−41] and it is believed that the syn-

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chronization of coupled ratchets could provide some information concerning the transport properties of inertia ratchets. For instance, the authors of Refs. [35] have investigated numerically the anticipated synchronization of chaotic ratchets via time-delay feedback; the authors of Refs. [36]–[38] have presented phase synchronization in unidirectionally and bidirectionally coupled chaotic ratchets; very recently, Vincent *et al.* have shown complete synchronization of two identical/non-identical chaotic ratchets via active control[39,40] and backstepping control.[41] However, all above-mentioned works belong to continuous synchronization scheme, in which chaotic systems are coupled to each other continuously such that synchronization errors can be controlled within a given range or converge to zero. Due to the good performance of impulsive synchronization, it would be very instructive and significant to consider the synchronization problem of two chaotic ratchets via impulsive control method.

In this paper, we investigate the impulsive synchronization problem of two chaotic ratchets transporting particles in two directions. The paper is organized as follows. In the next section, some basic definitions and supporting results are presented. Impulsive synchronization of chaotic ratchets is investigated in Section 3. Results of numerical simulation are given in Section 4, which indicate that our suggested method is suitable in practice for controlling chaotic ratchet. Finally, conclusions are drawn in Section 5.

2. Stability theory of impulsive differential equations

We first address some basic results in this section which is useful in this paper. These results can also be found in Refs. [11], [17] and [18].

Consider the general nonlinear system described by

$$\dot{x} = f(t, x), \tag{1}$$

where $x \in R^n$ is the state variable, $\dot{x} \equiv \mathrm{d}x/\mathrm{d}t$ and $f : R_+ \times R^n \longrightarrow R^n$ is continuous. Suppose that a discrete set $\{t_k\}$ of time instants satisfies

$$0 \leq t_0 < t_1 < \cdots < t_k < t_{k+1} < \cdots,$$
$$t_k \longrightarrow \infty \ \text{ as } \ k \longrightarrow \infty.$$

Let

$$I_k(x) = \delta x|_{t=t_k} \equiv x(t_k^+) - x(t_k^-) \tag{2}$$

be the "jump" in the state variable at time instant t_k, where $x(t_k^+) = \lim_{t \longrightarrow t_k^+} x(t)$, $x(t_k^-) = \lim_{t \longrightarrow t_k^-} x(t)$. In general, for simplicity, it is assumed that $x(t_k^-) = x(t_k)$. Then, the impulsive control system with the initial condition in time t_0 can be described by

$$\begin{aligned} &\dot{x} = f(t, x), \quad t \neq t_k, \\ &\delta x = I_k(x), \quad t = t_k, \quad k = 1, 2, 3, \ldots, \\ &x(t_0) = x_0. \end{aligned} \tag{3}$$

Equation (3) is called an impulsive differential equation. To study the stability of the impulsive differential equation, we may assume that $f(t, 0) = 0$ and $I_k(0) = 0$ for all k. Additionally, we call function $V : R_+ \times R^n \longrightarrow R_+$ an element of class V_0 if: (i) V is continuous in $(t_{k-1}, t_k] \times R^n$, and for each $x \in R^n$, $k = 1, 2, 3, \ldots$, $\lim_{(t,y) \longrightarrow (t_k^+, x)} V(t, y) = V(t_k^+, x)$ exists; (ii) V is locally Lipschitzian in x.

For $(t, x) \in (t_{k-1}, t_k] \times R^n$, the right and upper Dini's derivative of $V \in V_0$ is defined as

$$D^+V(t, x) \equiv \limsup_{h \longrightarrow 0^+} \frac{1}{h}\{V[t+h, x+hf(t, x)] - V(t, x)\}. \tag{4}$$

Several approaches including the comparison system approach, Lyapunov-type approach as well as two measure approaches have been established to study the stability of Eq. (3). Here we consider the comparison approach to derive the asymptotical stability conditions of system (3).

Let $V \in V_0$ and assume that

$$\begin{aligned} &D^+V(t, x) \leq g(t, V(t, x)), \quad t \neq t_k, \\ &V(t, x + I_k(x)) \leq \Psi_k(V(t, x))), \quad t = t_k, \end{aligned} \tag{5}$$

where $g : R_+ \times R_+ \longrightarrow R$ is continuous and $g(t, 0) = 0$, $\Psi_k : R_+ \longrightarrow R_+$ is nondecreasing. Then, the system

$$\begin{aligned} &\dot{w} = g(t, w), \quad t \neq t_k, \\ &w(t_k^+) = \Psi_k(w(t_k)), \quad t = t_k, \\ &w(t_0^+) = w_0 \geq 0 \end{aligned} \tag{6}$$

is called the comparison system of Eq. (3).

We give the following notation $S_\rho = \{x \in R^n : \|x\| < \rho\}$ where $\|\cdot\|$ denotes the Euclidean norm on R^n. A function a is said to belong to class κ if $a \in C[R_+, R_+], a(0) = 0$ and $a(x)$ is strictly increasing in x.

The following propositions[11,17] are presented to support our analysis in later section.

Proposition 1 Assume that the following three conditions are satisfied:

(i) $V : R_+ \times S_\rho \longrightarrow R_+, \rho > 0$, $V \in V_0$, $D^+V(t, x) \leq g(t, V(t, x)), t \neq t_k$.

(ii) There exists a $\rho_0 > 0$ such that $x \in S_{\rho_0}$ implies that $x+I_k(x) \in S_\rho$ for all k and $V(t, x+I_k(x)) \leq \Psi_k(V(t,x)), t = t_k, x \in S_{\rho_0}$.

(iii) $b(\|x\|) \leq V(t,x) \leq a(\|x\|)$ on $R_+ \times S_\rho$, where $a(\cdot), b(\cdot) \in \kappa$.

Then the stability properties of the trivial solution of the comparison system (6) imply the corresponding stability properties of the trivial solution of Eq. (3).

Proposition 2 Let $g(t,\omega) = \dot{\lambda}(t)\omega$, $\lambda \in C^1[R_+, R_+], \Psi_k(\omega) = d_k\omega, d_k \geq 0$ for all k, then the origin of system (3) is asymptotically stable if the following conditions hold:

(I) $\sup_k\{d_k \exp[\lambda(t_{k+1}) - \lambda(t_k)]\} = \varepsilon_0 < \infty$;

(II) There exists a $\gamma > 1$ such that $\lambda(t_{2k+3}) + \ln(\gamma d_{2k+2}d_{2k+1}) \leq \lambda(t_{2k+1})$ holds for all $d_{2k+2}d_{2k+1} \neq 0, k = 1, 2, \ldots$;

(III) $\dot{\lambda}(t) \geq 0$;

(IV) There exists $a(\cdot)$ and $b(\cdot)$ in class κ such that $b(\|x\|) \leq V(t,x) \leq a(\|x\|)$.

3. Impulsive synchronization of chaotic inertial ratchets

To start out, let us consider now the same equation of motion studied in Ref. [23], that is, a one-dimensional problem of a particle driven by a periodic time-dependent external force, under the influence of an asymmetric periodic potential of the ratchet type. The time average of the external deterministic force is zero. The dimensionless equation of motion for a particle of unit mass moving in the ratchet potential $V(x)$ is given by

$$\ddot{x} + b\dot{x} + \frac{\mathrm{d}V(x)}{\mathrm{d}x} = a\cos(\omega t), \tag{7}$$

where the time t has been normalized in unit of ω_0^{-1}, the small resonant frequency of the system, b is the dimensionless friction coefficient, a is the amplitude of the external force and ω is the ratio of the frequencies of the external driving force and the linear motion around the minima of the potential, $V(x)$ is the external asymmetric periodic potential. The dimensionless ratchet potential is given by

$$V(x) = C - \frac{1}{4\pi^2\epsilon}\left[\sin 2\pi(x-x_0) + \frac{1}{4}\sin 4\pi(x-x_0)\right] \tag{8}$$

and is shown in Fig. 1, where the potential is shifted by an amount x_0 in order that the minima of the potential are located at the origin, and ϵ is defined by $\epsilon = \sin(2\pi|x_0|) + \sin(4\pi|x_0|)$. Picking $x_0 = -0.19$ therefore $\epsilon \simeq 1.6$ and $C \simeq 0.0173$. In addition, it is easy to see that

$$\frac{\mathrm{d}V(x)}{\mathrm{d}x} = \frac{1}{4\pi\epsilon}[2\cos 2\pi(x-x_0) + \cos 4\pi(x-x_0)].$$

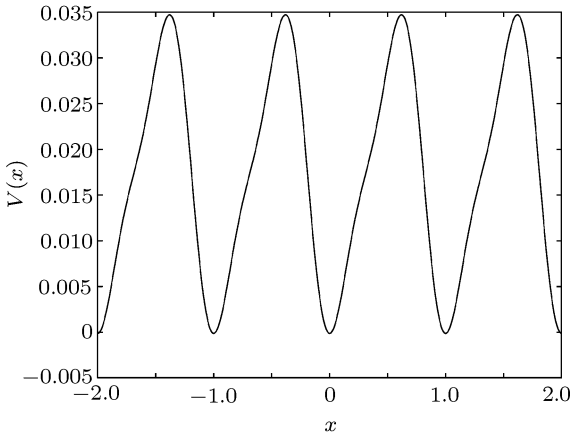


Fig. 1. The ratchet potentials $V(x)$.

Here we consider two chaotic ratchets in a drive-response framework, such that the drive ratchet with the subscript 1 is to control the response ratchet with subscript 2. Similar to Refs. [39]–[41], we can express the drive ratchet system as

$$\begin{aligned} \dot{x}_1 &= y_1, \\ \dot{y}_1 &= a\cos(\omega t) - by_1 - \frac{\mathrm{d}V(x_1)}{\mathrm{d}x_1}, \end{aligned} \tag{9}$$

and the response ratchet as

$$\begin{aligned} \dot{x}_2 &= y_2, \\ \dot{y}_2 &= a\cos(\omega t) - by_2 - \frac{\mathrm{d}V(x_2)}{\mathrm{d}x_2}. \end{aligned} \tag{10}$$

In order to synchronize the response system (10) with the drive system (9) via impulsive control method, we introduce the impulses at instant time t_k in response system (10):

$$\begin{aligned} \dot{x}_2 &= y_2, \\ \dot{y}_2 &= a\cos(\omega t) - by_2 - \frac{\mathrm{d}V(x_2)}{\mathrm{d}x_2}, \quad t \neq t_k, \\ \delta e &= Be, \quad t = t_k. \end{aligned} \tag{11}$$

Let $e = (e_1, e_2)^{\mathrm{T}} = (x_2 - x_1, y_2 - y_1)^{\mathrm{T}}$ be the synchronization error and

$$\Phi(x_1, x_2) = \left(0, \frac{\mathrm{d}V(x_1)}{\mathrm{d}x_1} - \frac{\mathrm{d}V(x_2)}{\mathrm{d}x_2}\right)^{\mathrm{T}}.$$

Then the error system of the impulsive synchronization is given by

$$\begin{aligned} &\dot{e} = Ae + \Phi(x_1, x_2), \quad t \neq t_k, \\ &\delta e = Be, \quad t = t_k, \quad k = 1, 2, \ldots, \end{aligned} \tag{12}$$

where

$$A = \begin{pmatrix} 0 & 1 \\ 0 & -b \end{pmatrix}.$$

By now, we have translated the impulsive synchronization problem of systems (9) and (11) into the stability problem of origin of the error system (12).

Theorem 1 Let the impulses be equidistant and separated by interval Δ. $Q \in R^{n\times n}$ is a symmetric and positive-definite matrix. Constant $r > 0$ is an upper bound of the maximum eigenvalue of $A^{\mathrm{T}}Q + QA$. Suppose that there exist constant scalars $d > 0, \gamma > 1$ such that

I) $\Omega_1 = (I + B)^{\mathrm{T}}Q(I + B) - dQ \leq 0$,

II) $\dfrac{2(r + r_1)}{\lambda_{\min}(Q)}\Delta + \ln(\gamma d^2) \leq 0$,

where I denotes the identity matrix,

$$r_1 = (2/\epsilon)\lambda_{\max}(Q)\,.$$

Then the origin of the error system (12) is asymptotically stable, i.e. the drive system (9) and response system (11) realize complete synchronization under the effects of impulses.

Proof Let us construct a Lyapunov function

$$V(t, e) = e^{\mathrm{T}}Qe, \tag{13}$$

where Q is a symmetric and positive-definite matrix. Let

$$b(\|e\|) = \lambda_{\min}(Q)\|e\|^2$$

and

$$a(\|e\|) = \lambda_{\max}(Q)\|e\|^2\,,$$

where $\lambda_{\min}$ and $\lambda_{\max}$ denote the smallest and largest eigenvalues of a square matrix, respectively. Then $a(\cdot)$ and $b(\cdot)$ belong to κ. Moreover, equation (13) implies that $b(\|e\|) \leq V(t, e) \leq a(\|e\|)$. For $t \neq t_k$, combining the fact that

$$\begin{aligned} \frac{\mathrm{d}V(x_1)}{\mathrm{d}x_1} - \frac{\mathrm{d}V(x_2)}{\mathrm{d}x_2} &= \frac{1}{4\pi\epsilon}[2\cos 2\pi(x_1 - x_0) + \cos 4\pi(x_1 - x_0)] - \frac{1}{4\pi\epsilon}[2\cos 2\pi(x_2 - x_0) + \cos 4\pi(x_2 - x_0)] \\ &= \frac{1}{4\pi\epsilon}[(2\cos 2\pi(x_1 - x_0) - 2\cos 2\pi(x_2 - x_0)) + (\cos 4\pi(x_1 - x_0) - \cos 4\pi(x_2 - x_0))] \\ &\leq \frac{1}{4\pi\epsilon}[4\pi|e_1| + 4\pi|e_1|] \leq \frac{2}{\epsilon}|e_1|, \end{aligned}$$

and

$$2|e_1|e_2 \leq e_1^2 + e_2^2,$$

the derivative of $V(t, e)$ along the solution of error system (12) is

$$\begin{aligned} D^+V(t, e) &= e^{\mathrm{T}}(A^{\mathrm{T}}Q + QA)e + e^{\mathrm{T}}Q\Phi(x_1, x_2) + \Phi(x_1, x_2)^{\mathrm{T}}Qe \\ &\leq re^{\mathrm{T}}e + \frac{4}{\epsilon}\lambda_{\max}(Q)|e_1|e_2 \leq \frac{r + r_1}{\lambda_{\min}(Q)}e^{\mathrm{T}}Qe = \frac{r + r_1}{\lambda_{\min}(Q)}V(t, e). \end{aligned} \tag{14}$$

For $t = t_k$, we have

$$\begin{aligned} V(t_k, e + Be) &= e^{\mathrm{T}}(I + B)^{\mathrm{T}}Q(I + B)e \\ &= e^{\mathrm{T}}[(I + B)^{\mathrm{T}}Q(I + B) - dQ]e + dV(t_k, e) \\ &= e^{\mathrm{T}}\Omega_1 e + dV(t_k, e) \\ &\leq dV(t_k, e). \end{aligned} \tag{15}$$

Let $\dot{\lambda}(t) = (r + r_1)/\lambda_{\min}(Q)$, $d_k = d, k = 1, 2, \ldots$. Then, it follows from Proposition 2 that the origin of the error system (12) is asymptotically stable. This completes the proof.

Let $Q = I$ in Theorem 1, noting that $\lambda_{\max}(A + A^{\mathrm{T}}) = \sqrt{b^2 + 1} - b > 0$, the following corollary holds.

Corollary 1 Assume that there exist constants $d > 0, \gamma > 1$ such that

1) $\Omega_1 = (I + B)^{\mathrm{T}}(I + B) - dI \leq 0$,

2) $2(\sqrt{b^2 + 1} - b + 2/\epsilon)\Delta + \ln(\gamma d^2) \leq 0$,

then the origin of the error system (12) is asymptotically stable.

4. Numerical results and discussions

The fourth-order Runge–Kutta method is used to solve the differential equations (9) and (10) with a time step size of 0.1 in all numerical simulations. Without loss of generality, we choose $Q = I$ and $B = kI$ where k is the control gain, fix the parameters of the driven system (9) and response system (11) as those in Ref. [23], i.e. $b = 0.1, \omega = 0.67, a = 0.156$. It follows easily from Corollary 1 that $-2 < k < 0$. In this case, one can obtain

$$d = (k+1)^2.$$

Then, an estimate of the boundaries of the stable region according to the condition 2) in Corollary 1 is given by

$$0 \leq \Delta \leq -\frac{\ln\gamma + \ln(k+1)^4}{2 \times 2.1550}, \quad -2 < k < 0. \qquad (16)$$

Figure 2 shows the stable region for impulsive synchronization with different γ. The whole region under the curve corresponding to $\gamma = 1$ is the predicted stable region. The stable region shrinks to a line $k = -1$ when $\gamma \longrightarrow +\infty$.

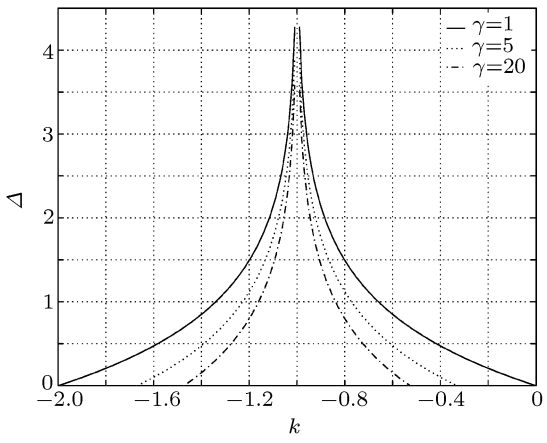


Fig. 2. Estimate of the boundaries of stable regions for synchronization with different γ used in expression (16).

For the fixed parameters, we will choose the initial conditions of particular interest such that they correspond to the case where two attractors co-exist in phase space. The system (9) with initial values $(x(0), y(0)) = (-0.10, 0.25)$ corresponds to the chaotic attractor transporting particles in the positive direction while the system (9) with initial values $(x(0), y(0)) = (0.43, -0.12)$ is the periodic attractor, which transports particles in the negative direction as shown in Fig. 3 (one can refer to Ref. [23] for more details).

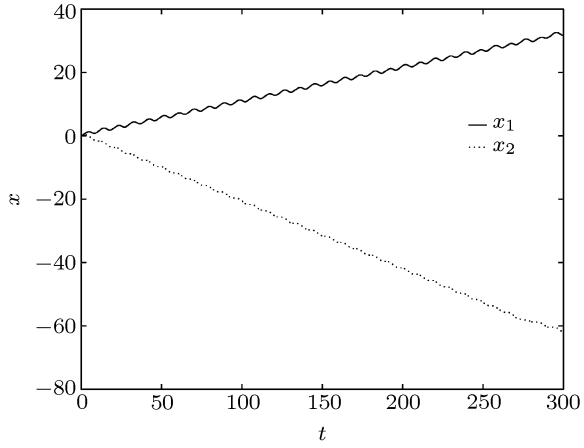
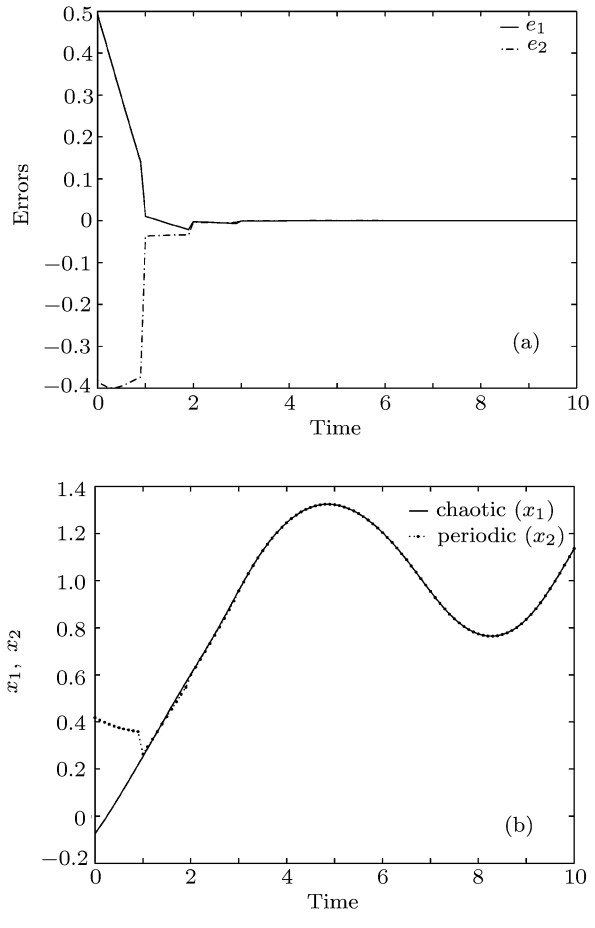


Fig. 3. Trajectories of two attractors: positive transport (x_1) generated by the chaotic attractor and negative transport (x_2) generated by the periodic attractor.

The simulation results of both impulsive synchronization and control of directed transport in chaotic ratchets are shown in Fig. 4 in the stable region for feedback gain $k = -0.9$ and impulsive interval $\Delta = 1$.



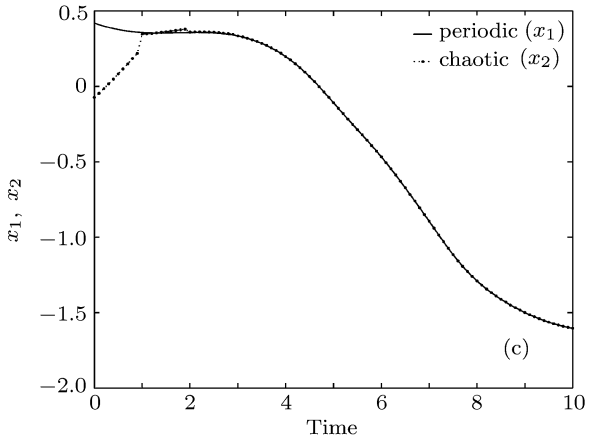


Fig. 4. Time response of (a) impulsive synchronization error, (b) and (c) state trajectories of x_1 and x_2 for $k = -0.9$ and $\Delta = 1$.

From Fig. 4(a), it can be seen that the impulsive synchronization error will converge to zero finally and two chaotic ratchets are indeed achieving chaos synchronization. From Fig. 4(b) (and 4(c)), it is very easy to show that the directed motion in the negative (positive) direction has been controlled to follow the direction of positive (negative) transport via impulsive control.

It is worth while noting that the condition of impulsive synchronization is only a conservative sufficient condition. In other words, the true stable region in actual application is larger than that predicted in Fig. 2. To demonstrate this fact, we show in Fig. 5 the numerical results for $k = -0.9$ and $\Delta = 50$ (actually the upper bound of impulsive interval Δ derived from Eq. (16) is 2.67 when feedback gain k is -0.9. Here impulsive interval $\Delta = 50$ is much greater than the value of upper bound), the other conditions are the same as those in Figs. 3 and 4. In fact, It can be seen that the length of impulsive interval Δ can be arbitrarily selected from a large number of numerical experiments. These numerical simulations indicate that any width of impulsive interval Δ can be used to implement impulsive synchronization in coupled chaotic ratchets. The freedom to choose the discrete time instants with impulsive signals therefore leads to a large flexibility in applications, especially in biological realm and some technological applications.

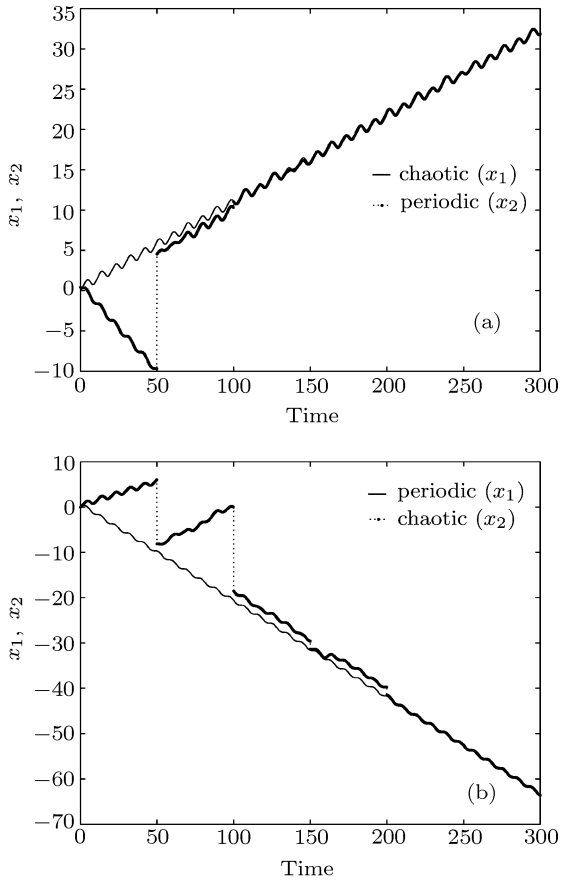


Fig. 5. Time response of the state trajectories of x_1 and x_2 for $k = -0.9$ and $\Delta = 50$.

5. Conclusions

In this paper, we have investigated the issue of reversing the direction of particle transport in nonequilibrium statistical physics based on impulsive synchronization. Thus, by identifying the directions of particle transport for co-existing attractors, one can choose a desired direction of particle transport in which a reverse to this direction is required via an impulsive control technique. Some theoretic conditions are obtained to guarantee that the synchronization between two chaotic ratchets can be realized via impulses, i.e. any desired direction of transport can be achieved. Computer simulations indicate that the suggested impulsive control method is effective and suitable in practice. We believe that the good performance of impulsive synchronization illustrated in this paper could be extended to ratchet devices.

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