Adaptive synchronisation of fractional-order chaotic systems*

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(Received 30 June 2009; revised manuscript received 19 August 2009)

A new stability theory of nonlinear dynamic systems is proposed, and a novel adaptive synchronisation method is presented for fractional-order chaotic and hyperchaotic systems based on the theory described in this paper. In comparison with previous methods, not only is the present control scheme simple but also it employs only one control strength, converges very fast, and it is also suitable for a large class of fractional-order chaotic and hyperchaotic systems. Moreover, this scheme is analytical and simple to implement in practice. Numerical and circuit simulations are used to validate and demonstrate the effectiveness of the method.

Keywords: new stability theory, fractional-order chaotic system, adaptive synchronisation

PACC: 0545

1. Introduction

Even though fractional calculus is a mathematical topic with more than 300 years of history, its application to physics and engineering has attracted a great deal of attention only in recent years. It has been found that many systems in interdisciplinary fields can be described by fractional differential equations, such as viscoelastic systems, dielectric polarisation, electrode–electrolyte polarisation, some finance systems and electromagnetic waves.[1,2] Moreover, applications of fractional calculus have been reported in many areas such as signal processing,[3] image processing,[4] automatic control[5] and robotics.[6,7] These examples and many other similar samples perfectly clarify the importance of consideration and analysis of dynamical systems with fractional-order models.

Nowadays, it has been found that some fractional-order differential systems can demonstrate chaotic behaviours, such as the fractional-order Chua circuit,[8] the fractional-order Duffing system,[9] the fractional-order jerk model,[10] the fractional-order Chen system,[11] the fractional-order Lü system,[12] the fractional-order Rossler system,[13] the fractional-order Arneodo system,[14] the fractional-order conjugate Chen system[15] and the fractional-order generalized Lorenz system.[16]

Recently, studying fractional-order differential systems has become an active research field. Synchronisation of chaotic fractional-order differential systems starts to attract increasing attention due to its potential applications in secure communication and control processing. There are many methods of synchronisation, such as the PC method, the one-way method and the bidirectional coupled method.[17−22] Using the partially coherent (PC) method, Li[20] investigated chaos synchronisation of two identical Chua systems with some fractional orders, and the sufficient conditions for achieving synchronisation are derived via the Laplace transformation theory. But some assume in the discussion that they are impossible to check if they hold or not, and the analysis of the synchronisation scheme by the Laplace transformation is not sufficient and immensurable.[23] In this paper, a new stability theory of nonlinear dynamic systems is presented, and an adaptive synchronisation method is proposed for fractional-order chaotic and hyperchaotic systems based on the new stability theory. The scheme is not only simple but also suitable for almost all chaotic and hyperchaotic systems. Furthermore, this controller includes only one control strength k_1 and the convergence speed is very fast. In addition, in this scheme the feedback strength is automatically adapted to an updated law which is significantly different from the usual linear feedback. In comparison with previous

*Project supported by the Natural Science Foundation of Hebei Province of China (Grant No. A2008000136).
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methods, in principle this technique does not need any numerical calculation and is simple to implement in practice. Numerical and circuit simulations are used to demonstrate the effectiveness and validity of this method.

2. Preliminary

This section presents a new stability theory of nonlinear dynamics systems based on the Lyapunov stability theory and the stability theory of the fractional-order system.

Theorem 1 Consider an arbitrary autonomous system

$$\frac{\mathrm{d}^\alpha x}{\mathrm{d}t^\alpha} = A(x)x, \quad (0 < \alpha \leq 1), \tag{1}$$

where $x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}}$ is the state variable, $A(x)$ is the coefficient matrix. If

$$H(x) = x^{\mathrm{T}} \frac{\mathrm{d}^\alpha x}{\mathrm{d}t^\alpha} \leq 0,$$

then the system (1) is asymptotically stable.

Proof (i) When $\alpha = 1$, we choose function $V(x)$ as follows:

$$V(x) = \frac{1}{2} x^{\mathrm{T}} x.$$

Then the derivative of V is given by

$$\dot{V} = x^{\mathrm{T}} \dot{x} = H(x) \leq 0.$$

According to the Lyapunov asymptotical stability theory, the system (1) is asymptotically stable.

(ii) When $0 < \alpha < 1$, suppose that λ is one of eigenvalues of matrix A, and there should be a nonzero vector $\xi = (\xi_1, \xi_2 \ldots, \xi_n)^{\mathrm{T}}$ being an eigenvector corresponding to the eigenvalue λ, i.e.

$$A\xi = \lambda\xi. \tag{2}$$

One can gain

$$\xi^H A\,\xi = \lambda\,\xi^H \xi, \tag{3}$$

$$\xi^H A^{\mathrm{T}} \xi = \bar{\lambda}\,\xi^H \xi. \tag{4}$$

According to Eqs. (3) and (4), we have

$$\xi^H (A + A^{\mathrm{T}})\xi = (\lambda + \bar{\lambda})\xi^H \xi,$$

or

$$\lambda + \bar{\lambda} = \frac{\xi^H (A + A^{\mathrm{T}})\xi}{\xi^H \xi}. \tag{5}$$

Since

$$H(x) = x^{\mathrm{T}} \frac{\mathrm{d}^\alpha x}{\mathrm{d}t^\alpha} = x^{\mathrm{T}} A x = \frac{1}{2} x^{\mathrm{T}} (A + A^{\mathrm{T}}) x \leq 0,$$

$A + A^{\mathrm{T}}$ is a negative semi-definite matrix, and then

$$\lambda + \bar{\lambda} = \frac{\xi^H (A + A^{\mathrm{T}})\xi}{\xi^H \xi} \leq 0. \tag{6}$$

Formulation $|\arg(\lambda)| \geq \pi/2 > \alpha\pi/2$ is obvious. According to the stability theory of fractional-order system,[24] the system (1) is asymptotically stable.

3. Adaptive synchronisation theory of fractional-order chaotic systems

Consider the following class of fractional-order chaotic systems as the drive system

$$\frac{\mathrm{d}^\alpha x}{\mathrm{d}t^\alpha} = Ax + f(x), \tag{7}$$

where $0 < \alpha < 1$, $x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in R^n$ is the state vector, $A \in R^{n\times n}$ is a constant matrix, and $f(x)$ is a continuous differential nonlinear function.

The response system is as follows:

$$\frac{\mathrm{d}^\alpha y}{\mathrm{d}t^\alpha} = Ay + f(y) + u, \tag{8}$$

where $y = (y_1, y_2, \ldots, y_n)^{\mathrm{T}} \in R^n$ is the state vector, and $u \in R^n$ is a controller vector.

We now assume that the following hypothesis is pertaining to the drive system (7) and the response system (8).

Hypothesis 1 For a bounded matrix $M_{x,y}$ with elements depending on x and y

$$f(y) - f(x) = M_{x,y}(y - x).$$

Remark 1 At this point, it is important to note that many known fractional-order chaotic systems can be described by Hypothesis 1.

With the Hypothesis 1, the error system from Eqs. (7) and (8) is as follows:

$$\frac{\mathrm{d}^\alpha e}{\mathrm{d}t^\alpha} = (A + M_{x,y})e + u, \tag{9}$$

where $e = y - x$.

Theorem 2 Suppose that Hypothesis 1 holds, and then the error system (9) is asymptotically stable under the adaptive controllers

$$u = -k_1 e, \tag{10}$$

and updating law

$$\frac{\mathrm{d}^\alpha k_1}{\mathrm{d}t^\alpha} = \theta e^{\mathrm{T}} e = \sum_{i=1}^{n} \theta ||e_i||_2^2, \tag{11}$$

where $k_1 e = (k_1 e_1, k_1 e_2, \ldots, k_1 e_n)^{\mathrm{T}}$, $e_i = y_i - x_i$, θ is a positive constant.

Proof Since A is a given constant matrix, and $M_{x,y}$ is abound matrix with elements depending on x and y, there exist two nonnegative constants β and γ such that $||A||_2 \leq \beta$, and $||M_{x,y}||_2 \leq \gamma$. It follows that

$$\left\| \frac{A + A^{\mathrm{T}}}{2} \right\|_2 \leq \beta, \left\| \frac{M_{x,y} + M_{x,y}^{\mathrm{T}}}{2} \right\|_2 \leq \gamma.$$

Substituting Eq. (10) into Eq. (9), we can get

$$\begin{aligned} \frac{\mathrm{d}^\alpha e}{\mathrm{d}t^\alpha} &= (A + M_{x,y})e - k_1 e \\ &= (A + M_{x,y})e - k^* e - (k_1 - k^*)e \\ &= (A + M_{x,y})e - k^* e - e e_k, \end{aligned} \tag{12}$$

where $e_k = k_1 - k^*, k^*$ is a positive constant.

From Eqs. (11) and (12), we have

$$\begin{aligned} H(e, e_k) &= \left(e^{\mathrm{T}} \frac{1}{\sqrt{\theta}} e_k \right) \begin{pmatrix} \dfrac{\mathrm{d}^\alpha e}{\mathrm{d}t^\alpha} \\ \dfrac{1}{\sqrt{\theta}} \dfrac{\mathrm{d}^\alpha e_k}{\mathrm{d}t^\alpha} \end{pmatrix} \\ &= \left(e^{\mathrm{T}} \ e_k \right) \begin{pmatrix} \dfrac{\mathrm{d}^\alpha e}{\mathrm{d}t^\alpha} \\ \dfrac{1}{\theta} \dfrac{\mathrm{d}^\alpha k_1}{\mathrm{d}t^\alpha} \end{pmatrix} \\ &= \left(e^{\mathrm{T}} \ e_k \right) \begin{pmatrix} A + M_{x,y} - k^* I & -e \\ e^{\mathrm{T}} & 0 \end{pmatrix} \begin{pmatrix} e \\ e_k \end{pmatrix} \\ &= e^{\mathrm{T}}(A + M_{x,y} - k^* I)e + e_k e^{\mathrm{T}} e - e^{\mathrm{T}} e e_k \\ &= e^{\mathrm{T}}(A + M_{x,y} - k^* I)e \\ &= e^{\mathrm{T}}(\frac{A + A^{\mathrm{T}}}{2} + \frac{M_{x,y} + {M_{x,y}}^{\mathrm{T}}}{2} - k^* I)e \\ &\leq \sum_{i=1}^{n} (\beta + \gamma - k^*) \left\| e_i \right\|_2^2 . \end{aligned}$$

Since β and γ are nonnegative constants, one can select a suitable constant k^* to make $\beta+\gamma-k^*$ a negative number, and then $H(e, e_k) \leq \sum\limits_{i=1}^{n} (\beta + \gamma - k^*) \left\| e_i \right\|_2^2 \leq 0$. According to Theorem 1, the error system (9) is asymptotically stable under the controllers (10) and updating law (11). That is $e \to 0, e_k \to 0$ as $t \to +\infty$. This completes the proof.

Remark 2 The control strength k_1 is automatically adapted to a suitable strength k^* depending on the initial values, which is significantly different from the usual linear feedback.

Remark 3 The controller $k_1 e$ can include only one feedback term e_i for some chaotic systems or two feedback terms e_i for some hyperchaotic systems.

4. Examples

This section presents three examples to show the effectiveness of the above synchronisation method.

Example 1 The famous fractional-order Rössler chaotic system.[13]

The system is written as follows:

$$\begin{aligned} \frac{\mathrm{d}^\alpha x_1}{\mathrm{d}t^\alpha} &= -x_2 - x_3, \\ \frac{\mathrm{d}^\alpha x_2}{\mathrm{d}t^\alpha} &= x_1 + 0.4 x_3, \\ \frac{\mathrm{d}^\alpha x_3}{\mathrm{d}t^\alpha} &= 0.2 + x_1 x_3 - 10 x_3. \end{aligned} \tag{13}$$

According to Theorem 2, we can select the controller $u = -k_1 e = -k_1 (e_1, e_2, e_3)^{\mathrm{T}}$ and set

$$\frac{\mathrm{d}^\alpha k_1}{\mathrm{d}t^\alpha} = 0.3 e^{\mathrm{T}} e = 0.3(e_1^2 + e_2^2 + e_3^2)$$

(selecting $\theta = 0.3$). Then the corresponding response system is

$$\begin{aligned} \frac{\mathrm{d}^\alpha y_1}{\mathrm{d}t^\alpha} &= -y_2 - y_3 - k_1 e_1, \\ \frac{\mathrm{d}^\alpha y_2}{\mathrm{d}t^\alpha} &= y_1 + 0.4 y_3 - k_1 e_2, \\ \frac{\mathrm{d}^\alpha y_3}{\mathrm{d}t^\alpha} &= 0.2 + y_1 y_3 - 10 y_3 - k_1 e_3. \end{aligned} \tag{14}$$

In the simulations, we choose $\alpha = 0.9$, the initial conditions $x(0) = (2, 1, 1)^{\mathrm{T}}$, $y(0) = (-3.5, -1, 7)^{\mathrm{T}}$; the numerical results are illustrated in Figs. 1 and 2. Figure 1 shows the errors converging to zero as time increases, which implies that the synchronisation can be realized. Figure 2 displays the evolution of k_1. Obviously, with time passing, k_1 tends to a constant.

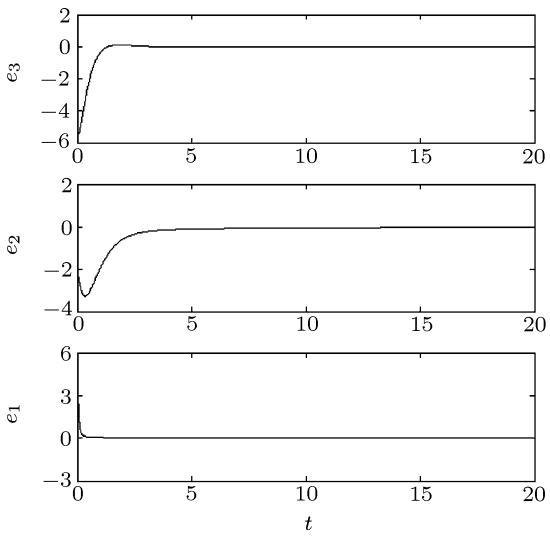


Fig. 1. Time evolution of synchronization errors for systems (13) and (14) with time t.

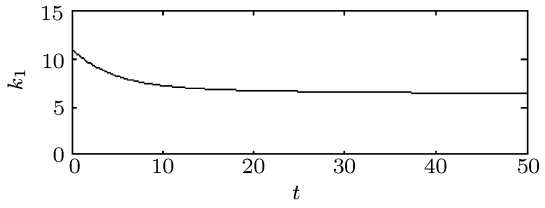


Fig. 2. The plot of control strength k_1 tending to a negative constant as t tends to ∞.

Example 2 The famous fractional-order Chen chaotic system.[11]

The fractional-order Chen chaotic system is described by

$$\begin{aligned}\frac{\mathrm{d}^\alpha x_1}{\mathrm{d}t^\alpha} &= 35(x_2 - x_1),\\ \frac{\mathrm{d}^\alpha x_2}{\mathrm{d}t^\alpha} &= -7x_1 + 28x_2 - x_1x_3,\\ \frac{\mathrm{d}^\alpha x_3}{\mathrm{d}t^\alpha} &= x_1x_2 - 3x_3,\end{aligned} \tag{15}$$

which has a chaotic attractor at $0.83 < \alpha \leq 1$. The corresponding response system is

$$\begin{aligned}\frac{\mathrm{d}^\alpha y_1}{\mathrm{d}t^\alpha} &= 35(y_2 - y_1) + u_1,\\ \frac{\mathrm{d}^\alpha y_2}{\mathrm{d}t^\alpha} &= -7y_1 + 28y_2 - y_1y_3 + u_2,\\ \frac{\mathrm{d}^\alpha y_3}{\mathrm{d}t^\alpha} &= y_1y_2 - 3y_3 + u_3.\end{aligned} \tag{16}$$

Let $e_i = y_i - x_i$ $(i = 1, 2, 3)$. Subtracting Eq. (16) from Eq. (15) yields an error dynamical system between Eqs. (15) and (16)

$$\begin{aligned}\frac{\mathrm{d}^\alpha e_1}{\mathrm{d}t^\alpha} &= 35(e_2 - e_1) + u_1,\\ \frac{\mathrm{d}^\alpha e_2}{\mathrm{d}t^\alpha} &= (-7 - y_3)e_1 + 28e_2 - x_1e_3 + u_2,\\ \frac{\mathrm{d}^\alpha e_3}{\mathrm{d}t^\alpha} &= y_2e_1 + x_1e_2 - 3e_3 + u_3.\end{aligned} \tag{17}$$

Theorem 3 The error system (17) is asymptotically stable under the adaptive controllers

$$u_1 = u_3 = 0, \quad u_2 = -k_1e_2, \tag{18}$$

and updating law

$$\frac{\mathrm{d}^\alpha k_1}{\mathrm{d}t^\alpha} = \theta e_2^2, \tag{19}$$

where θ is a positive constant.

Proof Assume $e_k = k_1 - k^*$, where k^* is a positive constant. Substituting Eq. (18) into Eq. (17), we can obtain

$$\begin{aligned}\frac{\mathrm{d}^\alpha e_1}{\mathrm{d}t^\alpha} &= 35(e_2 - e_1),\\ \frac{\mathrm{d}^\alpha e_2}{\mathrm{d}t^\alpha} &= (-7 - y_3)e_1 + 28e_2 - x_1e_3 - k^*e_2 - e_2e_k,\\ \frac{\mathrm{d}^\alpha e_3}{\mathrm{d}t^\alpha} &= y_2e_1 + x_1e_2 - 3e_3.\end{aligned} \tag{20}$$

Then by Eqs. (19) and (20), we have

$$H(e, e_k) = \left(\sqrt{\sigma}e_1, e_2, e_3, \frac{1}{\sqrt{\theta}}e_k\right)\begin{pmatrix}\sqrt{\sigma}\dfrac{\mathrm{d}^\alpha e_1}{\mathrm{d}t^\alpha}\\ \dfrac{\mathrm{d}^\alpha e_2}{\mathrm{d}t^\alpha}\\ \dfrac{\mathrm{d}^\alpha e_3}{\mathrm{d}t^\alpha}\\ \dfrac{1}{\sqrt{\theta}}\dfrac{\mathrm{d}^\alpha e_k}{\mathrm{d}t^\alpha}\end{pmatrix} = (e_1, e_2, e_3, e_k)\begin{pmatrix}\sigma\dfrac{\mathrm{d}^\alpha e_1}{\mathrm{d}t^\alpha}\\ \dfrac{\mathrm{d}^\alpha e_2}{\mathrm{d}t^\alpha}\\ \dfrac{\mathrm{d}^\alpha e_3}{\mathrm{d}t^\alpha}\\ \dfrac{1}{\theta}\dfrac{\mathrm{d}^\alpha e_k}{\mathrm{d}t^\alpha}\end{pmatrix}$$

$$= (e_1, e_2, e_3, e_k)\begin{pmatrix}-35\sigma & 35\sigma & 0 & 0\\ -7 - y_3 & 28 - k^* & -x_1 & -e_2\\ y_2 & x_1 & -3 & 0\\ 0 & e_2 & 0 & 0\end{pmatrix}\begin{pmatrix}e_1\\ e_2\\ e_3\\ e_k\end{pmatrix}$$

$$= (e_1, e_2, e_3)\begin{pmatrix}-35\sigma & 35\sigma & 0\\ -7 - y_3 & 28 - k^* & -x_1\\ y_2 & x_1 & -3\end{pmatrix}\begin{pmatrix}e_1\\ e_2\\ e_3\end{pmatrix}$$

$$
\begin{aligned}
&= (e_1, e_2, e_3) \begin{pmatrix} -35\sigma & (35\sigma - 7 - y_3)/2 & y_2/2 \\ (35\sigma - 7 - y_3)/2 & 28 - k^* & 0 \\ y_2/2 & 0 & -3 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} \\
&\leq -(|e_1|, |e_2|, |e_3|) \begin{pmatrix} 35\sigma & -(35\sigma - 7 + m)/2 & -m/2 \\ -(35\sigma - 7 + my_3)/2 & k^* - 28 & 0 \\ -m/2 & 0 & 3 \end{pmatrix} \begin{pmatrix} |e_1| \\ |e_2| \\ |e_3| \end{pmatrix} \\
&= -(|e_1|, |e_2|, |e_3|) P \begin{pmatrix} |e_1| \\ |e_2| \\ |e_3| \end{pmatrix},
\end{aligned}
$$

where σ is a positive constant, $m = \max(|y_2|, |y_3|)$. If we choose

$$\sigma > \frac{m^2}{420}$$

and

$$k^* > 28 + \frac{3(35\sigma - 7 + m)^2}{420\sigma - m^2},$$

the matrix P is positive, then

$$
\begin{aligned}
H(e, e_k) &\leq -(|e_1|, |e_2|, |e_3|) P (|e_1|, |e_2|, |e_3|)^{\mathrm{T}} \\
&\leq 0\,.
\end{aligned}
$$

According to Theorem 1, the response system (16) synchronises the drive system (15) by the controller (18) and updating law (19). This completes the proof.

According to the circuit of $1/s^{\alpha}$,[15,16] using the circuitry theory and the circuitry design method in Refs. [15] and [16], we design a circuit to realize the synchronisation of two identical fractional-order Chen chaotic systems with $\alpha = 0.9$ in Fig. 3, where unit (I) is circuitry of drive system, unit (II) is circuitry of response system and unit (III) is circuitry of controller $u_2 = -k_1 e_2$. AD633 is a multiplier with an output coefficient of 0.1, and LM741 is an operational amplifier; $R_1 = 63.10$ MΩ, $R_2 = 1.598$ MΩ, $R_3 = 15.81$ kΩ, $C_1 = 0.4410$ μF, $C_2 = 0.4870$ μF, $C_3 = 0.2937$ μF, $R_4 = R_5 = R_8 = R_9 = R_{13} = R_{16} = R_{19} = R_{20} = 10$ kΩ, $R_6 = 35$ kΩ, $R_7 = R_{14} = R_{18} = 100$ kΩ, $R_{10} = 14.3$ kΩ, $R_{11} = R_{15} = 1$ kΩ, $R_{12} = 3.57$ kΩ, $R_{17} = 33.3$ kΩ; $R_{21} = R_{22} = R_{23} = R_{24} = 10$ kΩ, $R_{25} = 100$ kΩ, $R_{26} = 10$ kΩ (i.e. selecting $\theta = 1$). When we set delay-time $t = 10$ s, the circuit simulation results are depicted in Fig. 4. From which, one can find that the state variables y_1, y_2 and y_3 quickly converge to the state variables x_1, x_2 and x_3 respectively, as switch S is turned on. It is clear that the two fractional systems (16) and (15) can achieve synchronisation quickly by the adaptive controller.

As Multisim 2001 software is the model which is based on the actual circuit components, its simulation results should be basically consistent with those of the actual circuit experiment. So the results of circuit simulation are effective.

Example 3 For further illustrating the effectiveness of the proposed scheme, we consider a new fractional-order hyperchaotic system, which is described by the following equations[25]

$$
\begin{aligned}
\frac{\mathrm{d}^{\alpha} x_1}{\mathrm{d}t^{\alpha}} &= 10(x_2 - x_1) + x_4, \\
\frac{\mathrm{d}^{\alpha} x_2}{\mathrm{d}t^{\alpha}} &= 40x_1 + x_1 x_3 - x_4, \\
\frac{\mathrm{d}^{\alpha} x_3}{\mathrm{d}t^{\alpha}} &= -4x_1^2 - 2.5x_3, \\
\frac{\mathrm{d}^{\alpha} x_4}{\mathrm{d}t^{\alpha}} &= 2.5x_1,
\end{aligned} \tag{21}
$$

and it is hyperchaotic with $\alpha = 0.95$. We can select the controller $u = -k_1 e = -k_1(0, e_2, 0, e_4)^{\mathrm{T}}$ and set

$$\frac{\mathrm{d}^{\alpha} k_1}{\mathrm{d}t^{\alpha}} = \theta e^{\mathrm{T}} e = \theta(e_2^2 + e_4^2),$$

then the corresponding slave system is as follows:

$$
\begin{aligned}
\frac{\mathrm{d}^{\alpha} y_1}{\mathrm{d}t^{\alpha}} &= 10(y_2 - y_1) + y_4, \\
\frac{\mathrm{d}^{\alpha} y_2}{\mathrm{d}t^{\alpha}} &= 40y_1 + y_1 y_3 - y_4 - k_1 e_2, \\
\frac{\mathrm{d}^{\alpha} y_3}{\mathrm{d}t^{\alpha}} &= -4y_1^2 - 2.5y_3, \\
\frac{\mathrm{d}^{\alpha} y_4}{\mathrm{d}t^{\alpha}} &= 2.5y_1 - k_1 e_4.
\end{aligned} \tag{22}
$$

By the procedure similar to Example 2, the chaos synchronisation between the systems (21) and (22) is achieved.

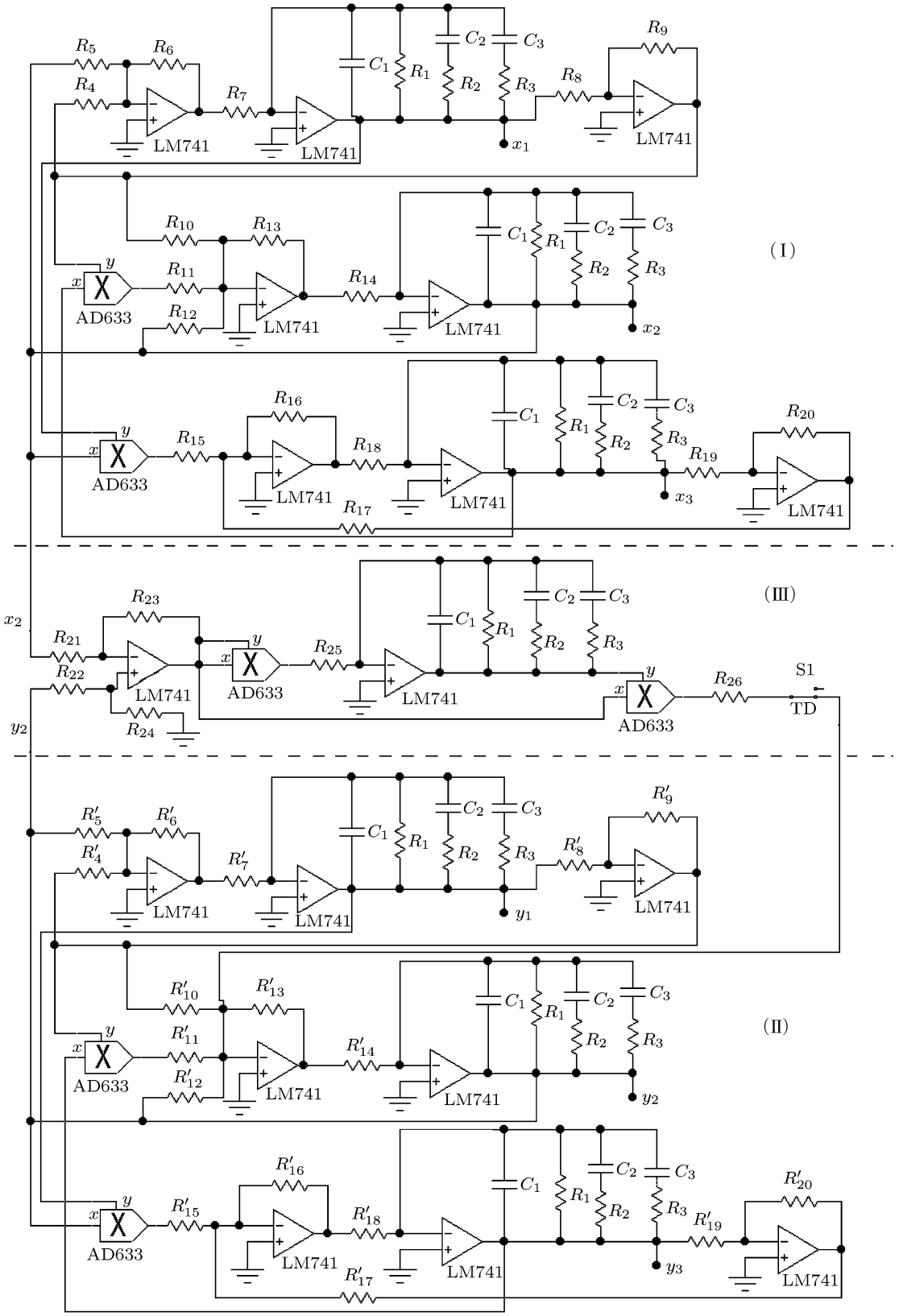


Fig. 3. Synchronisation circuitry between fractional-order Chen chaotic systems (16) and (17). (I) circuitry of drive system, (II) circuitry of response system, (III) circuitry of controller.

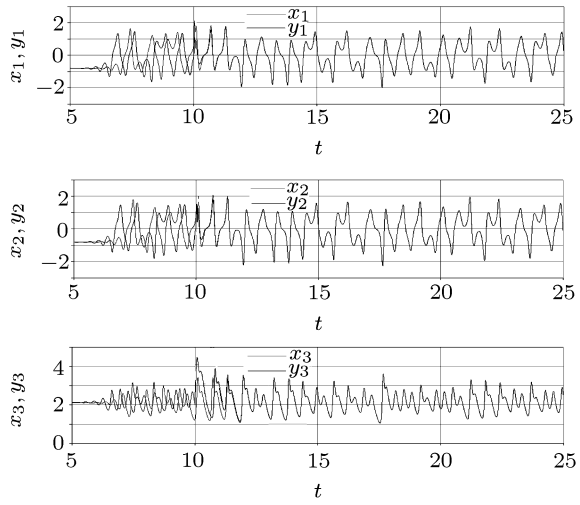


Fig. 4. Observed state variable evolution with time on electronic workbench Multisim2001.

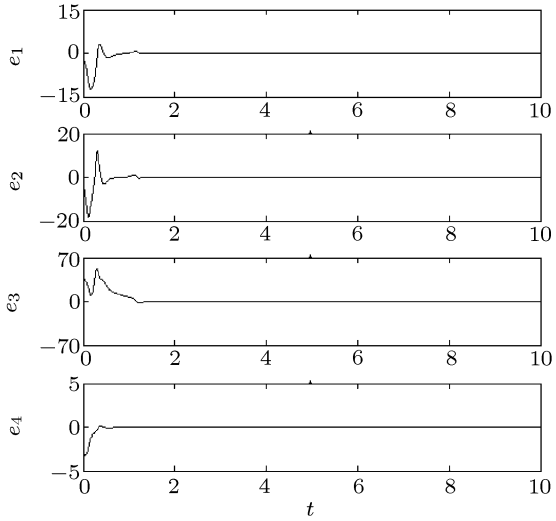


Fig. 5. Time evolution of synchronization errors for systems (21) and (22) with t.

In the simulations, we choose $\alpha = 0.95$, the initial conditions $x(0) = (1, 1, -20, 2)^{\mathrm{T}}$, $y(0) = (-1, -2, 18, -1)^{\mathrm{T}}$, $k_1(0) = 0$, $\theta = 1$. The numerical results are illustrated in Figs. 5 and 6. Figure 5 shows the errors converging to zero as time increases, which implies that the synchronisation can be realized. Figure 6 displays the evolution of k_1. Obviously, with time passing, k_1 tends to a constant.

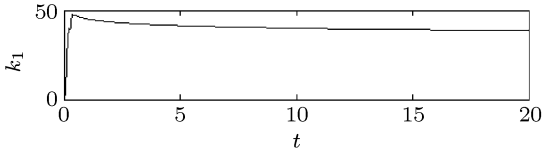


Fig. 6. The plot of control strength k_1 which tends to a negative constant as t approaches ∞.

5. Conclusions

A new stability theory of nonlinear dynamics systems is proposed, and a simple adaptive controller for fractional-order chaos synchronisation is obtained based on this new theory. In comparison with the previous methods, the present controller is not only simple but also capable of being generalized, especially as it can include only one feedback term e_i for some chaotic systems or two feedback terms e_i for some hyperchaotic systems. Therefore, it could be easily implemented in practice. The numerical simulation and circuit experiment demonstrate the effectiveness and validity of this controller. We also believe that such a simple adaptive controller will be very beneficial for the applications of fractional-order chaos synchronisation.

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