# A novel one equilibrium hyper-chaotic system generated upon Lü attractor*

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By introducing an additional state feedback into a three-dimensional autonomous chaotic attractor Lü system, this paper presents a novel four-dimensional continuous autonomous hyper-chaotic system which has only one equilibrium. There are only 8 terms in all four equations of the new hyper-chaotic system, which may be less than any other four-dimensional continuous autonomous hyper-chaotic systems generated by three-dimensional (3D) continuous autonomous chaotic systems. The hyper-chaotic system undergoes Hopf bifurcation when parameter $c$ varies, and becomes the 3D modified Lü system when parameter $k$ varies. Although the hyper-chaotic system does not undergo Hopf bifurcation when parameter $k$ varies, many dynamic behaviours such as periodic attractor, quasi periodic attractor, chaotic attractor and hyper-chaotic attractor can be observed. A circuit is also designed when parameter $k$ varies and the results of the circuit experiment are in good agreement with those of simulation.

**Keywords:** hyper-chaotic, bifurcation, circuit implementation, Lyapunov exponents

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## 1. Introduction

Since the first chaotic attractor[1] is found by Lorenz in 1963 and the hyper-chaotic attractor[2] is reported by Rossler, the study on the chaotic attractor or hyper-chaotic attractor attracts more and more interest.

For example, many new chaotic attractors are found which include Chen system,[3] Lü system,[4−6] and so on.[7−14] In addition, some fractional chaotic systems[15−18] are also studied.

On the basis of the chaotic systems, some hyper-chaotic systems are generated. For example, Li *et al.* found the hyper-chaos[19,20] based on Lorenz system. Gao *et al.* presented another hyper-chaos[21] based on Lorenz system. Cang *et al.* studied a four-dimensional (4D) non-autonomous hyper-chaos[22] based on the system presented by Gao. In addition, some other hyper-chaotic systems[23−27] are also presented based on some known chaotic systems.

It is well known that chaos or hyper-chaos have been also studied in some application fields, such as control and synchronization,[28−30] nonlinear circuit,[31,32] etc. Particularly, in recent years, the generation or the application of hyper-chaos has become a hot topic of study, because of its higher unpredictability than chaotic system. So the generation and circuit implementation of hyper-chaos looks very important.

In this paper, a new 4D continuous autonomous hyper-chaotic system with only one equilibrium point, can be obtained by introducing a new state variable to the second equation of Lü system in Ref. [5].

The new system may be the simplest in all the hyper-chaotic attractors generated by the unified chaotic system. It has only 8 terms in all the four equations, which may be less than any other hyper-chaotic attractors generated by 3D autonomous systems.

Although the system has only one equilibrium point, its many attractors can be observed, such as periodic attractor, quasi-periodic attractor, chaotic attractor and hyper-chaotic attractor. A circuit is also designed when analysing and simulating the new hyper-chaotic system, the results of circuit experiment are in good agreement with those of simulation.

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## 2. The fourth-order hyper-chaotic system

Recently, Lü *et al.* reported a unified chaotic system in Ref. [4], which is

$$\begin{aligned}\dot{x} &= (25a+10)(y-x),\\ \dot{y} &= (28-35a)x - xz + (29a-1)y,\\ \dot{z} &= xy - \frac{a+8}{3}z.\end{aligned} \tag{1}$$

It can be reformulated in the following canonical form in Ref. [7]:

$$\begin{aligned}\dot{x} &= a_{11}x + a_{12}y,\\ \dot{y} &= a_{21}x + a_{22}y - xz,\\ \dot{z} &= a_{33}z + xy.\end{aligned} \tag{2}$$

When it satisfies the conditions $a_{12}a_{21} > 0$, $a_{12}a_{21} < 0$ and $a_{12}a_{21} = 0$, the system is referred to Lorenz system, Chen system and Lü system, respectively. The three systems are similar, but their topological structures are unequal. Lü system is described as

$$\begin{aligned}\dot{x} &= a(y-x),\\ \dot{y} &= -xz + cy,\\ \dot{z} &= xy - bz.\end{aligned} \tag{3}$$

where $a = 36,\ b = 3,\ c = 20$, it has a typical chaotic attractor and the Lyapunov dimension is 2.0669. The corresponding chaotic attractors are shown in Fig. 1.

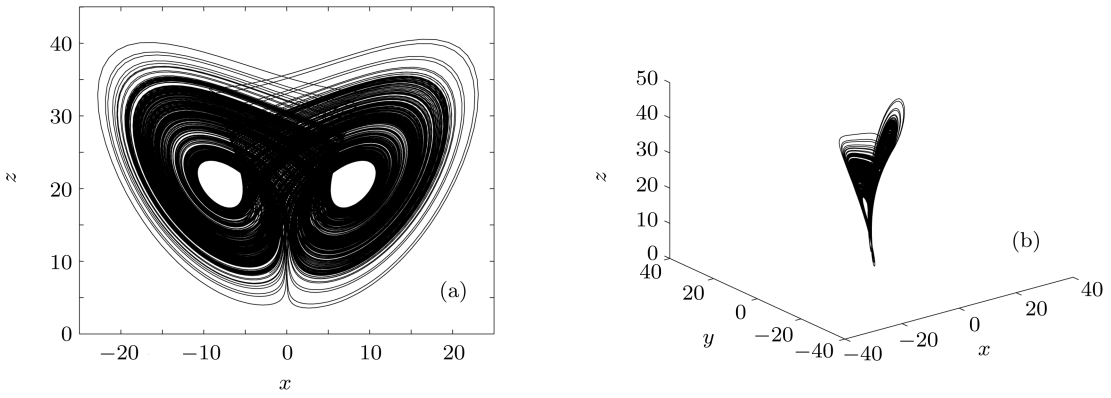



**Fig. 1.** Phase portraits of chaotic system (3) with $a = 36,\ b = 3,\ c = 20$: (a) attractor in $x$–$z$ plane; (b) attractor in $x$–$y$–$z$ plane.

Lü system is a bridge between Lorenz system and Chen system, and it realizes a transition from a system to another system. It is also the simplest chaotic attractor among the unified chaotic system.

Hyper-chaotic systems, generally, are classified as chaotic systems with more than one positive Lyapunov exponent, this shows that the chaotic dynamics of the systems are expanded in more than one direction and may give rise to more complex attractors.

Hyper-chaotic system also should satisfy some basic properties:

(i) Hyper-chaos exists only in higher-dimensional systems, i.e. not less than 4D autonomous system for the continuous time cases.

(ii) It was suggested that the number of terms in the coupled equations giving rise to instability should be at least two, in which one should be a nonlinear function.

Now, a fourth-order system can be obtained by introducing an additional state $u$ and coupling it to the second equation of Lü attractor, the new system is described by:

$$\begin{aligned}\dot{x} &= a(y-x),\\ \dot{y} &= -xz + cy - u,\\ \dot{z} &= xy - bz,\\ \dot{u} &= ky,\end{aligned} \tag{4}$$

where $a \in R^+,\ b \in R^+,\ c \in R$ and $k \in R$ are constant parameters of the system, $c$ and $k$ are two parameters determining the chaotic or hyper-chaotic behaviours and bifurcation of the system (4). The two system parameters will also be discussed in this paper.

Thus, the controller $u$ has made the chaotic system (3) into a 4D hyper-chaotic system (4), which may have two positive Lyapunov exponents, one zero, one negative Lyapunov exponent. When $a = 36$, $b = 3$, $c = 20$, $k = 10$, the hyper-chaotic attractors of the system (4) are shown in Fig. 2.

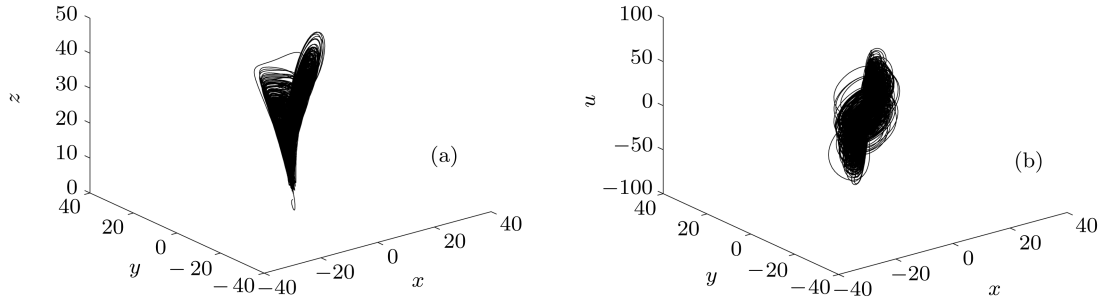



**Fig. 2.** Phase portraits of system (4) with $a = 36,\ b = 3,\ c = 20,\ k = 10$: (a) hyper-chaotic attractor in $x$–$y$–$z$ plane; (b) hyper-chaotic attractor in $x$–$y$–$u$ plane.

### 2.1. Dissipation

For the new system, it meets the equation

$$\Delta V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{u}}{\partial u} = -a + c - b,$$

that is, the system is dissipative with an exponential contraction rate

$$\frac{\mathrm{d}v}{\mathrm{d}t} = (c - a - b)$$

for any $k$ and $a + b > c$, this means that a volume element $V_0$ is contracted by the flow into a volume element $V_0\,\mathrm{e}^{-(c-a-b)}$ at time $t$.

### 2.2. Symmetry

The new system is symmetric about the $z$-axis, because it is invariant under the coordinate transformation $(x, y, z, u) \to (-x, -y, z, -u)$.

## 3. Equilibrium and Hopf bifurcation of the system (4)

### 3.1. Equilibrium

According to the property of the balance state, we can conclude the equation of the balance state, that is

$$\begin{aligned} &a(y - x) = 0,\\ &-xz + cy - u = 0,\\ &xy - bz = 0,\\ &ky = 0. \end{aligned} \tag{5}$$

It is obvious that $s_0 = (0, 0, 0, 0)$ is the only one equilibrium point of the new system (4).

### 3.2. Hopf bifurcation

For the system (4) at $s_0 = (0, 0, 0, 0)$ , one obtains Jacobian matrix

$$J = \begin{bmatrix} -a & a & 0 & 0\\ 0 & c & 0 & -1\\ 0 & 0 & -b & 0\\ 0 & k & 0 & 0 \end{bmatrix}, \tag{6}$$

and its characteristic equation is

$$f(\lambda) = (\lambda + b)(\lambda + a) f_1(\lambda) = 0, \tag{7}$$

where

$$f_1(\lambda) = (\lambda^2 - c\lambda + k), \tag{8}$$

and its characteristic values are

$$\begin{aligned} \lambda_1 &= -a,\\ \lambda_2 &= -b,\\ \lambda_3 &= \frac{c + \sqrt{c^2 - 4k}}{2},\\ \lambda_4 &= \frac{c - \sqrt{c^2 - 4k}}{2}. \end{aligned} \tag{9}$$

Obviously, $-a$, $-b$ are two negative real roots, thus, according to Routh–Hurwitz criterion, the real parts of the other roots in $f(\lambda) = 0$ are negative if and only if $c < 0$, $k > 0$. Furthermore, equilibrium point $s_0 = (0, 0, 0, 0)$ of the system (4) is asymptotically stable when $a \in R^+$, $b \in R^+$, $c < 0, k > 0$, and the system (4) has no chaotic or hyper-chaotic attractor.

Now, by fixing $a \in R^+$, $b \in R^+$, parameter $c$ and $k$ are discussed as bifurcation parameter.

### 3.2.1. The bifurcation of the system (4) when $c$ varies

The system (4) may have Hopf bifurcation when the parameter $c$ varies.

Now, suppose the system (4) has a pure imaginary root $\lambda = \mathrm{i}w$, $w \in R^+$, substituting it into Eq. (8) yields

$$-w^2 - \mathrm{i}cw + k = 0. \tag{10}$$

It then follows that

$$\begin{aligned} -cw &= 0, \\ -w^2 + k &= 0. \end{aligned} \tag{11}$$

Then, when $k > 0$, solving Eq. (11) gives

$$c = 0, \quad w_0 = \sqrt{k}.$$

Thus, substituting $c = 0$ into Eq. (7), one obtains

$$\begin{aligned} \lambda_1 &= -a, \\ \lambda_2 &= -b, \\ \lambda_3 &= j\sqrt{k}, \\ \lambda_4 &= -j\sqrt{k}, \end{aligned} \tag{12}$$

where $a \in R^+$, $b \in R^+$, and $k > 0$. So according to Refs. [33] and [34], the first condition for Hopf bifurcation is satisfied.

From Eq. (7), one obtains

$$\mathrm{Re}\left(\lambda'(c)\right)\Big|_{\lambda = j\sqrt{k}} = \frac{1}{2} \neq 0. \tag{13}$$

Thus, according to Refs. [33] and [34], the second condition for Hopf bifurcation is satisfied.

So the system (4) undergoes a Poincare–Andronov–Hopf bifurcation at equilibrium point $s_0 = (0, 0, 0, 0)$, furthermore, if $c > 0$ the system (4) is a periodic attractor, if $c < 0$ the system (4) is an asymptotically stable attractor.

Now select $c = -1$ and $c = 1$, we can observe that the system (4) is asymptotically stable attractor and periodic attractor, respectively. The corresponding attractors are shown in Figs. 3(a) and 3(b).

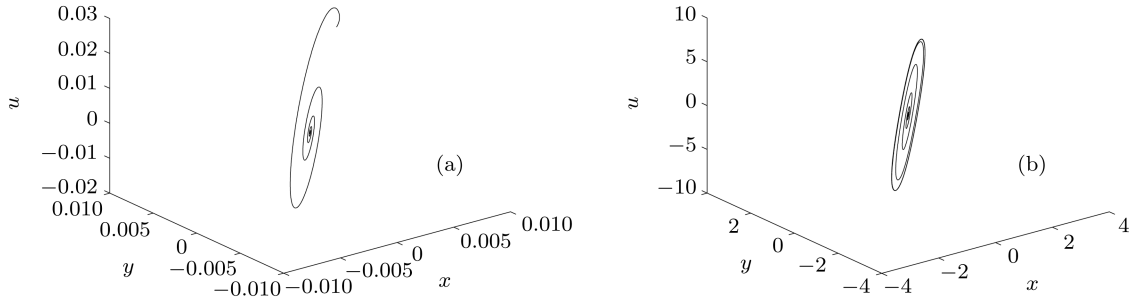



**Fig. 3.** Phase portraits of hyper-chaotic system (4) with $a = 36$, $b = 3$, $k = 10$ and initial points $(0.01, 0.01, 0.01, 0.01)$: (a) $c = -1$; (b) $c = 1$.

### 3.2.2. The bifurcation of the system (4) when $k$ varies

If $k = 0$, then the corresponding 4D controlled Lü system (4) becomes the 3D modified Lü system

$$\begin{aligned} \dot{x} &= a(y - x), \\ \dot{y} &= -xz + cy - u_0, \\ \dot{z} &= xy - bz, \end{aligned} \tag{14}$$

on the subspace

$$\Lambda_0 = \{(x, y, z, u) \,|\, (x, y, z) \in R^3, u = u_0\}. \tag{15}$$

Thus, by varying $u_0$, it finds that the system may be chaotic or periodic for any initial point $(x_0,\ y_0,\ z_0)$ when selecting $a = 36$, $b = 3$, $c = 20$, $u_0 \in [-30, 30]$. The Lyapunov exponents of the system (14) are shown in Fig. 4(a), and the corresponding bifurcation diagram of the system (14) is shown in Fig. 4(b). According to the Lyapunov exponents and bifurcation diagram of the system (14) in Fig. 4, one obtains the exits of chaos depending on initial $u_0$ for the system (14). Now select $u_0 = 10, 19$, the corresponding chaotic attractor and periodic attractor are shown in Fig. 5.

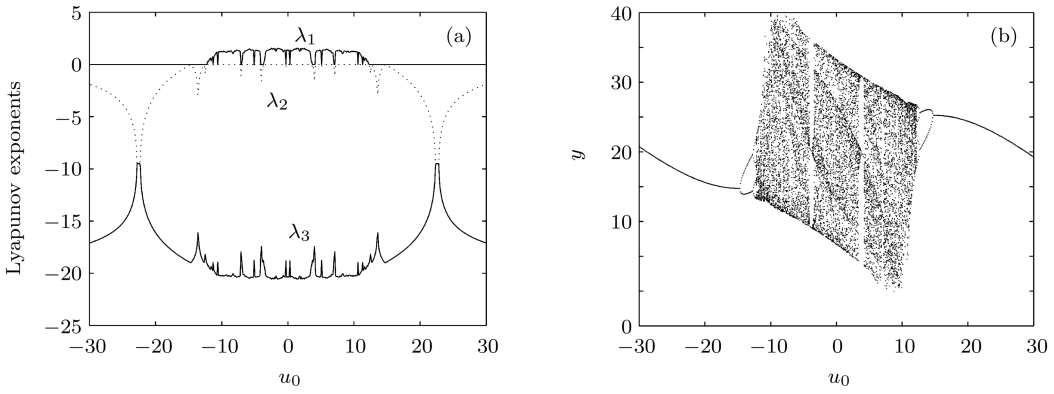



**Fig. 4.** The Lyapunov exponents and bifurcation diagram for any initial point $(x_0,\ y_0,\ z_0)$ when $u_0$ varies: (a) the Lyapunov exponents; (b) bifurcation diagram.

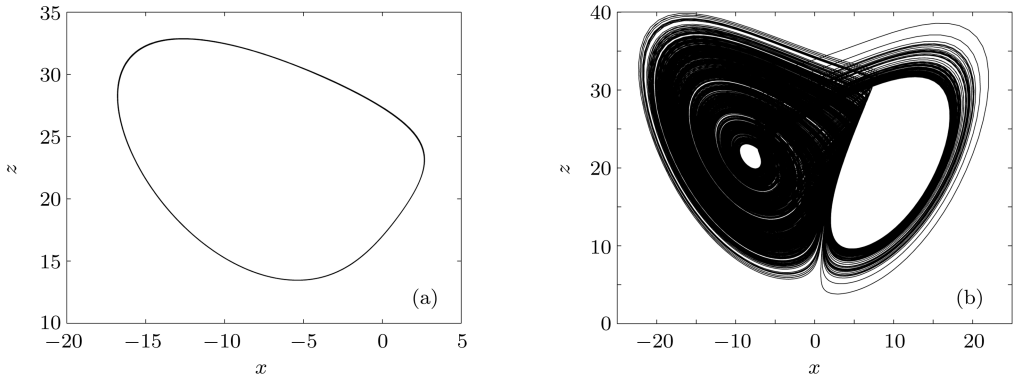



**Fig. 5.** Phase portraits of the system (14) with $a = 36,\ b = 3,\ c = 20$, with any initial point $(x_0,\ y_0,\ z_0)$: (a) $u_0 = 19$; (b) $u_0 = 10$.

In addition, according to Eq. (9), it is impossible that $\mathrm{Re}(\lambda_i) = 0$, $\mathrm{Im}(\lambda_i) \neq 0$, $i = 1, 2, 3, 4$, when $k$ varies, so the system (4) has no Hopf bifurcation when $k$ varies.

Although the system (4) has no Hopf bifurcation when $k$ varies, its many attractors including periodic attractor, quasi-periodic attractor, chaotic attractor and hyper-chaotic attractor can be observed. The complex dynamic behaviours of the system (4) will be discussed in the next section when $k$ varies.

# 4. Numerical simulations and circuit implementation of the hyper-chaotic system

For system (4), when $k$ varies, by fixing $a = 36,\ b = 3,\ c = 20, 0 \leq k \leq 80$, the Lyapunov exponents of the system are shown in Figs. 6(a) and 6(b), and the corresponding bifurcation diagram of the system is shown in Fig. 6(c).

A hyper-chaotic attractor is classified as a chaotic attractor with more than one positive Lyapunov exponent, which indicates that the dynamics of the system is expanded in more than one direction.

According to the Lyapunov exponents and bifurcation diagram in Fig. 6, we can approximately make the following coclusions: when $k \in \{(0, 5), (5, 20.3), (20.8, 23.8), (26.3, 28.8)\}$, the system is hyper-chaotic, because of existing two positive Lyapunov exponents, one zero and one negative Lyapunov exponent; when $k \in \{(20.4, 20.7), (23.9, 26.2), (28.9, 29.2), (33.3, 34.1)\}$, the system is chaotic, because of existing one positive Lyapunov exponent, one zero and two negative Lyapunov exponents; when $k \in \{(29.3, 31.4), (34.2, 35.5), (39.1, 53.1), (68.1, 100)\}$, the system is periodic, because of existing one zero and three negative Lyapunov exponents; when $k \in \{(31.5, 33.2), (35.6, 39), (53.2, 68)\}$, the system is quasi-periodic, because of existing two zero and two negative Lyapunov exponents.

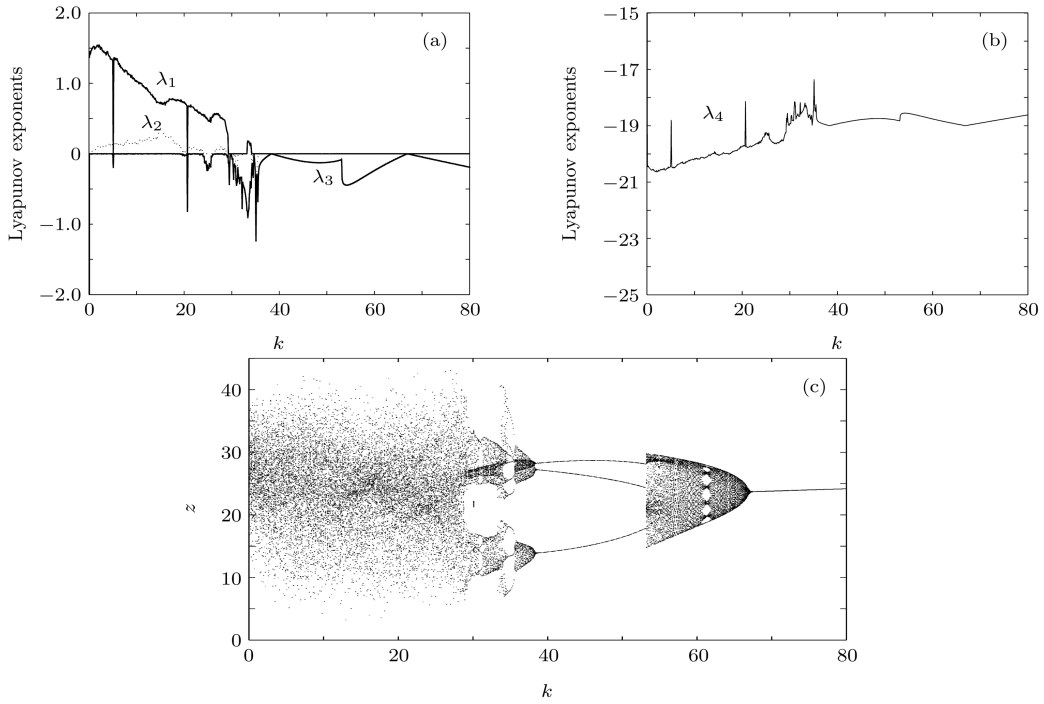
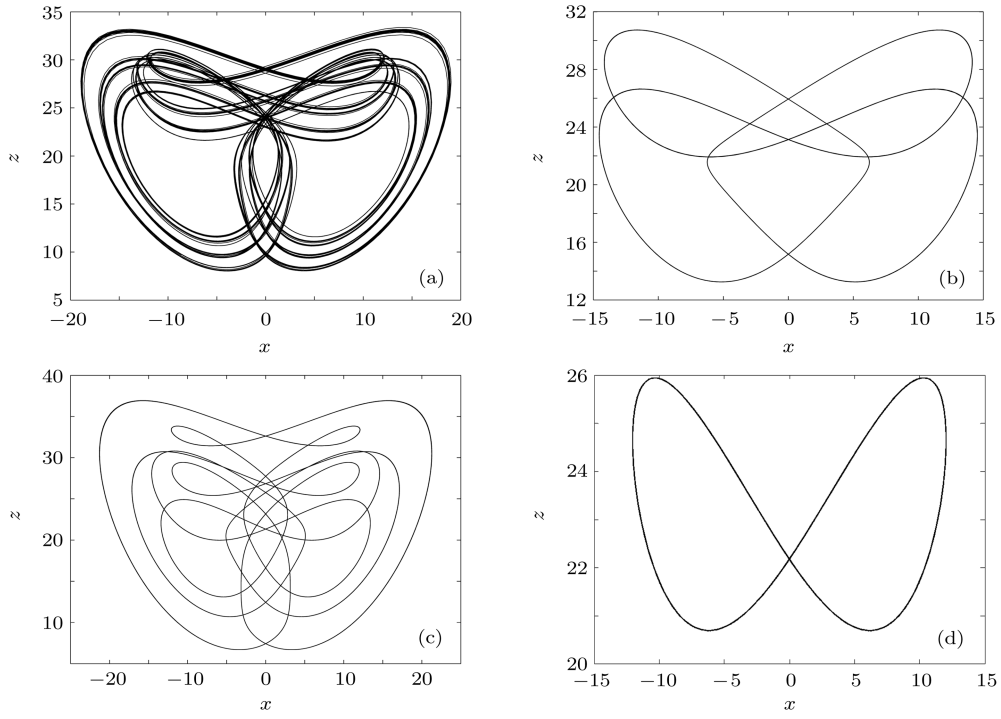



**Fig. 6.** The Lyapunov exponents and bifurcation diagram of the system (4) when $a = 36,\ b = 3,\ c = 20, 0 \leq k \leq 80$: (a) the biggest three Lyapunov exponents; (b) the fourth Lyapunov exponent; (c) bifurcation diagram.

## 4.1. Numerical simulations

When $k$ varies, fixing $a = 36,\ b = 3,\ c = 20,\ 0 \leq k \leq 80$, we can find all kinds of attractors including periodic attractors, quasi-periodic attractors, chaotic attractor and hyper-chaotic attractor. Now, select $k$= 30, 50, 35 and 80, the phase portraits of some typical periodic attractor in $x$–$z$ plane are shown in Figs. 7(a)–7(d); select $k = 59.5$ and 68, the phase portraits of some typical quasi-periodic attractor in $x$–$z$ plane are shown in Figs. 7(e) and 7(f); select $k = 16$ and 25, the phase portraits of hyper-chaotic attractor in $x$–$z$ plane, and chaotic attractor in $x$–$z$ plane are shown in Figs. 7(g) and 7(h).



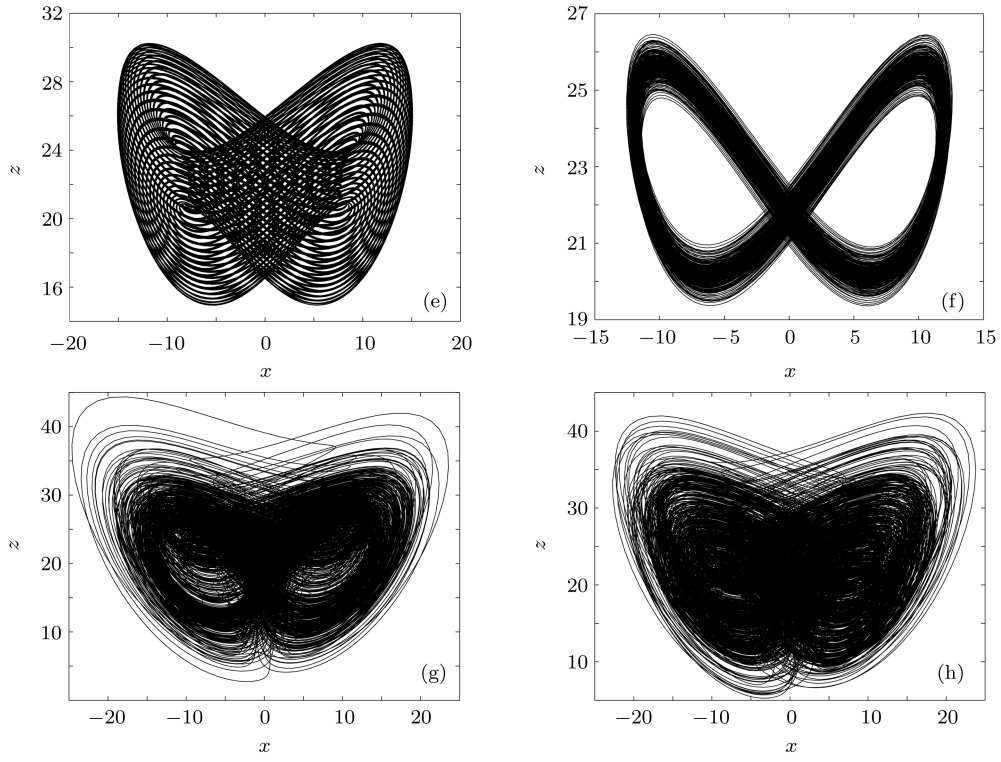



**Fig. 7.** Phase portraits of the new hyper-chaotic system (4) in $x$–$z$ plane: (a) $k = 30$; (b) $k = 50$; (c) $k = 35$; (d) $k = 80$; (e) $k = 59.5$; (f) $k = 68$; (g) $k = 16$; (h) $k = 25$.

## 4.2. Circuit implementation

For system (4), select $a = 36$, $b = 3$, $c = 20$, $k$ is variable, and let $x = 10X$, $y = 10Y$, $Z = 10Z$, $u = 10U$, thus, one obtains

$$\begin{aligned} 10\dot{X} &= 36(10Y - 10X), \\ 10\dot{Y} &= -100XZ + 200Y - 10U, \\ 10\dot{Z} &= 100XY - 30Z, \\ 10\dot{U} &= 10kY, \end{aligned} \tag{16}$$

In order to implement it by a circuit, equation (16) needs to be transformed into

$$\begin{aligned} -0.1\dot{X} &= 3.6(X - Y), \\ -0.1\dot{Y} &= XZ - 2Y + 0.1U, \\ -0.1\dot{Z} &= -XY + 0.3Z, \\ -0.1\dot{U} &= -0.1kY. \end{aligned} \tag{17}$$

Thus, according to Eq. (17), a circuit shown in Fig. 8 can be designed, and the equations of the circuit can be described as

$$\begin{aligned} -R_5C_1\dot{X} &= -\frac{R_2}{R_1}Y + \frac{R_1 + R_2}{R_1}\frac{R_4}{R_3 + R_4}X, \\ -R_{12}C_2\dot{Y} &= \frac{R_{11}R_8}{R_{10}R_6}0.1XZ + \frac{R_{11}R_8}{R_{10}R_7}U - \frac{R_{11}}{R_9}Y, \\ -R_{17}C_3\dot{Z} &= -\frac{R_{14}}{R_{13}}0.1XY + \frac{R_{13} + R_{14}}{R_{13}}\frac{R_{16}}{R_{15} + R_{16}}Z, \\ -R_{20}C_4\dot{U} &= -\frac{R_{19}}{R_{18}}Y, \end{aligned} \tag{18}$$

which accords with Eq. (17). The circuit consists of four channels, the operational amplifiers LF347N perform the operation of addition, subtraction, and integration in every channel, the nonlinear terms in the equation are implemented with the analog multipliers AD633. The resistor $R_{19}$ is adjusted with the variation of parameter $k$, the other resistors and capacitors shown in Fig. 8 are $R_1$, $R_3$, $R_6$, $R_{13} = 1$ kΩ, $R_2$, $R_4 = 3.6$ kΩ, $R_5$, $R_7$, $R_{12}$, $R_{17}$, $R_{20} = 100$ kΩ, $R_8$, $R_{10}$, $R_{11}$, $R_{14}$, $R_{15}$, $R_{18} = 10$ kΩ, $R_9 = 5$ kΩ, $R_{16} = 0.271$ kΩ, and $C_1$, $C_2$, $C_3$, $C_4 = 1$ nF.

The dynamics of the new hyper-chaotic system (4) has been also confirmed by the circuit. Adjusting $R_{19}$, we can observe the different attractors including periodic attractors, quasi-periodic attractors, chaotic attractor and hyper-chaotic attractor by oscilloscope, as shown in Fig. 9.

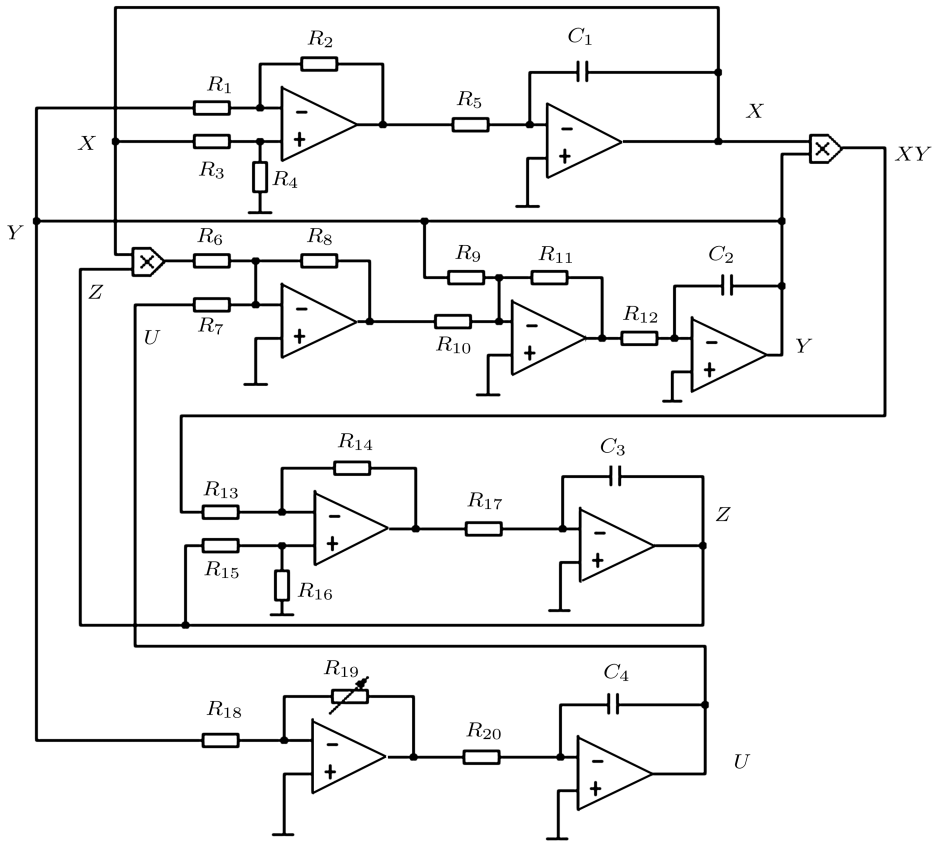
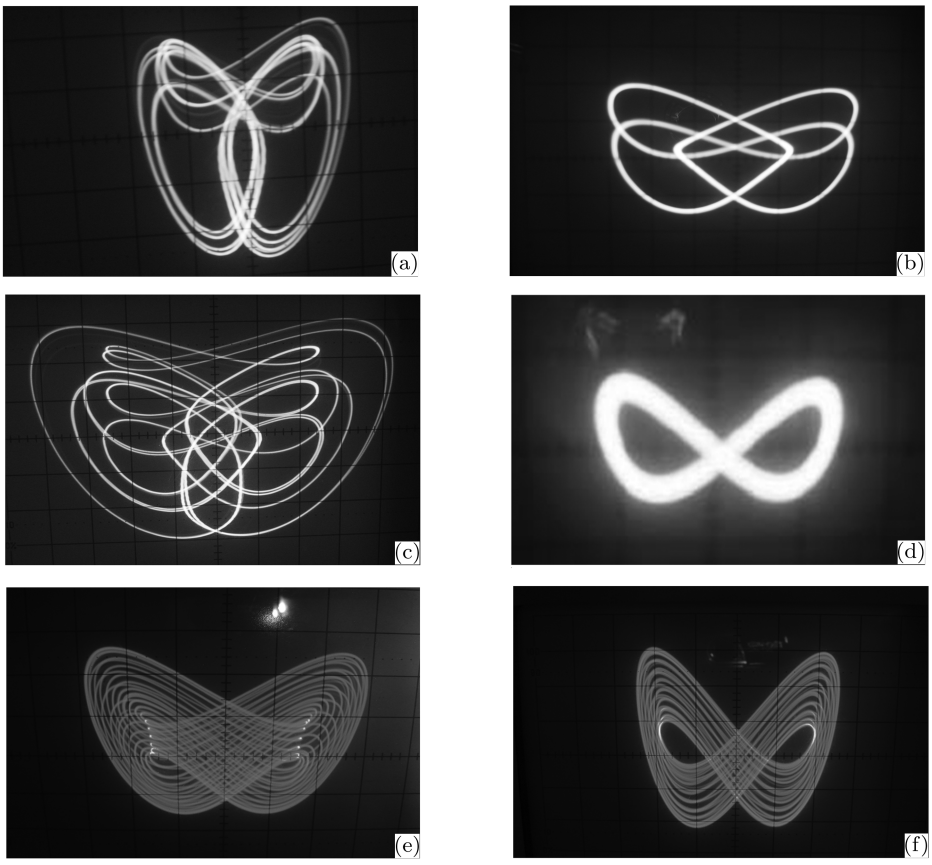



**Fig. 8.** Circuit for the new hyper-chaotic system (4).



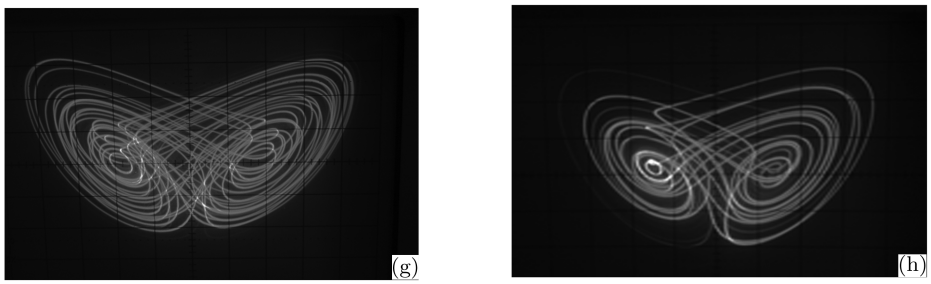



**Fig. 9.** Phase portraits of the new hyper-chaotic system (4) in $x$–$z$ plane, measured by oscilloscope: (a) $R_{19}$ = 30 kΩ; (b) $R_{19}$ = 50 kΩ; (c) $R_{19}$ = 35 kΩ; (d) $R_{19}$ = 80 kΩ; (e) $R_{19}$ = 59.5 kΩ; (f) $R_{19}$ = 68 kΩ; (g) $R_{19}$ = 16 kΩ; (h) $R_{19}$ = 25 kΩ.

By selecting $R_{19}$ = 30 kΩ, 50 kΩ, 35 $k\Omega$ and 80 $k\Omega$, the phase portraits of some typical periodic attractor in $x$–$z$ plane are shown in Figs. 9(a)–9(d). By selecting $R_{19}$ = 59.5 kΩ, 68 kΩ, the phase portraits of some typical quasi-periodic attractor in $x$–$z$ plane are shown in Figs. 9(e) and 9(f). By selecting $R_{19}$ = 16 kΩ, 25 kΩ, the phase portrait of hyper-chaotic attractor in $x$–$z$ plane, and the phase portrait of chaotic attractor in $x$–$z$ plane are shown in Figs. 9(g) and 9(h). Comparing the attractors in Fig. 9 with those in Fig. 7, we can conclude that the experimental realization are in good agreement with the numerical simulations.

## 5. Conclusion

In this paper, we present a new hyper-chaotic attractor with one equilibrium point. The bifurcation of the new hyper-chaotic system has been discussed when parameter $c$ or $k$ varies. The system is very simple, but its dynamics are very complex. Simulating the system, we can observe many attractors, such as the periodic attractor, quasi-periodic attractor, chaotic attractor and hyper-chaotic attractor. The system is also confirmed by the circuit. The results of the circuit experiment are in good agreement with those of simulation.

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