Hierarchical-control-based output synchronization of coexisting attractor networks*

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This paper introduces the concept of hierarchical-control-based output synchronization of coexisting attractor networks. Within the new framework, each dynamic node is made passive at first utilizing intra-control around its own arena. Then each dynamic node is viewed as one agent, and on account of that, the solution of output synchronization of coexisting attractor networks is transformed into a multi-agent consensus problem, which is made possible by virtue of local interaction between individual neighbours; this distributed working way of coordination is coined as inter-control, which is only specified by the topological structure of the network. Provided that the network is connected and balanced, the output synchronization would come true naturally via synergy between intra and inter-control actions, where the rightness is proved theoretically via convex composite Lyapunov functions. For completeness, several illustrative examples are presented to further elucidate the novelty and efficacy of the proposed scheme.

Keywords: hierarchical control, passive control, composite Lyapunov function, the Newton–Leipnik equation attractor

PACC: 0545, 0547

1. Introduction

Various large-scale and complicated systems can be modeled by networks, such as the Internet, WWW, and many others. The most striking recent advances are the occurrence of the small world network model[1] and scale free network model,[2] which have been declared to be closer to most real-world networks as compared with their earlier counterpart, random-graph model.[3] Since that time, many scientists and scholars have contributed to this topic, and addressed some problems such as network stability as well as stabilization,[4−7] network spreading[8] and network growing.[9] In addition, network synchronization was also concentrated on, for its widespread applications in various fields of sciences.[10−20] Among the available results, synchronization cost, local synchronization with and without time varying topology for both continuous and discrete networks, phase synchronization and adaptive synchronization were investigated in detail. It is undoubted that those contributions are important to further researches. However, less attention has been paid to the physical characteristics of the network nodes hitherto, and this may be one of the possible reasons for the complexity of the results.

In this paper, node physical dynamics and topology of the networks are dealt with equivalently, synchronization is realized by the cooperative manipulation of the node physical characteristics and the network topology. The added control action consists of two different parts: one is only associated with the node dynamics itself, named intra-control, and the other is related with the outputs of the node neighbours, coined as inter-control. While the intra-control makes the node dynamics passive, the inter-control drives the whole network output synchronized. Unlike the existing results, our method does not take local linearization, so that the results obtained stand globally, whereas the more importance of our method is that the network synchronization problem is reformulated

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as multi-agent consensus, and owing to this, together with graph connectivity and balance of network topology, the coexisting attractor networks output synchronization comes naturally. In case of theoretical rigidity and completeness, the composite Lyapunov function, which comes from the convex composition of its individual node counterpart, is employed to prove the validity of the suggested strategy.

The rest of the paper is organized as follows: In Section 2, some preliminary results of passive control are reviewed to elucidate the intra-control. In Section 3, the inter-control is discussed, where much attention is paid to its distributive working mechanism. Illustrative examples will be provided in Section 4 to further clarify the efficacy of the hierarchical policy. Finally, concluding remarks are given in Section 5.

2. Preliminaries

Consider a nonlinear affine system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u, \\ y &= h(x), \end{aligned} \tag{1}$$

where the state $x \in \Re^n$, the input $u \in \Re^m$, and output $y \in \Re^m$; f together with the m column of g are smooth vector fields, h is a smooth mapping. Suppose that the vector field f has at least one equilibrium point, without loss of generality, we can assume that the equilibrium point is $x = 0$. If the equilibrium is not at $x = 0$, through coordinate shift we can transfer the equilibrium point to $x = 0$.

Definition 1 The system (1) can be said passive if there exists a real constant β such that for $\forall t \geq 0$, any one of the inequalities among expressions (2a)–(2c) can hold

$$\int_0^t u^{\mathrm{T}}(\tau)y(\tau)\mathrm{d}\tau \geq \beta, \tag{2a}$$

$$\int_0^t u^{\mathrm{T}}(\tau)y(\tau)\mathrm{d}\tau + \beta \geq \rho \int_0^t y^{\mathrm{T}}(\tau)y(\tau)\mathrm{d}\tau, \tag{2b}$$

$$V(x) - V(x_0) \leq \int_0^t y^{\mathrm{T}}(\tau)u(\tau)\mathrm{d}\tau,$$

$$V(x) > 0. \forall x \backslash \{0\}, \quad V(0) = 0. \tag{2c}$$

The passive system (1) with storage function $V(x)$ can be further said strictly passive if there exists a positive-definite function $S(x)$, such that for $\forall x \in X, \forall t \geq 0$, expression (2d) stands

$$V(x) - V(x_0) \leq \int_0^t y^{\mathrm{T}}(\tau)u(\tau)d\tau - \int_0^t S(x(\tau))\mathrm{d}\tau. \tag{2d}$$

The physical meaning of being passive is that the energy of the nonlinear system can be increased only through the external supply. The stability properties of passive system are well known.[21−23] Once the system has been rendered passive, the controller law such as $u = -\varphi(y)$ can asymptotically stabilize the equilibrium of origin $x = 0$ of Eq. (1), where $\varphi(y)$ needs to satisfy $y \cdot \varphi(y) > 0$, $\varphi(y)$ can be selected in a linear form.

In this paper we will use hierarchical control scheme, which integrates the passive technique and coordination strategy to steer the coexisting attractor networks: the Newton–Leipnik chaotic system networks, where all nodes are identical, i.e. each node of the networks has the same dynamics, which is obtained by Newton and Leipnik[24] through the addition of a linear feedback to Euler's body equations in 1981, and its control problem has been investigated by several authors.[25−27]

If equation (1) has relative degree $[1, \ldots, 1]$ at $x = 0$ (i.e. $L_g h(0)$ is nonsingular) and the distribution spanned by the vector fields $g_1(x), \ldots, g_m(x)$ is involutive, then it can be represented by the so-called normal form

$$\begin{aligned} \dot{z} &= f_0(z) + p(z, y)y, \\ \dot{y} &= b(z, y) + a(z, y)u. \end{aligned} \tag{3}$$

Equation (3) may be rendered passive by a state feedback as follows:[4]

$$u = \alpha(x) + \beta(x)v. \tag{4}$$

Now we use this idea to make the following Newton–Leipnik system passive

$$\begin{aligned} \dot{x}_1 &= -\gamma x_1 + x_2 + 10x_2x_3, \\ \dot{x}_2 &= -x_1 - 0.4x_2 + 5x_1x_3, \\ \dot{x}_3 &= \sigma x_3 - 5x_1x_2. \end{aligned} \tag{5}$$

Equation (5) has two coexisting chaotic attractors when $\gamma = 0.4$ and $\sigma = 0.175$, the equilibrium points of it with the nominal parameters are

$$\begin{aligned} x_{\mathrm{e}}^{(1)} &= (0, 0, 0)^{\mathrm{T}}, \\ x_{\mathrm{e}}^{(2)} &= (0.2390, 0.0308, 0.2103)^{\mathrm{T}}, \\ x_{\mathrm{e}}^{(3)} &= (-0.2390, -0.0308, 0.2103)^{\mathrm{T}}, \\ x_{\mathrm{e}}^{(4)} &= (0.0315, -0.1224, -0.1103)^{\mathrm{T}}, \\ x_{\mathrm{e}}^{(5)} &= (-0.0315, 0.1224, -0.1103)^{\mathrm{T}}. \end{aligned}$$

All these equilibrium points are unstable. Note that the points $x_{\mathrm{e}}^{(2)}$ and $x_{\mathrm{e}}^{(3)}$ lie in the eye-centres of UA,

while $x_{\rm e}^{(4)}$ and $x_{\rm e}^{(5)}$ are situated in the eye-centres of LA, and its phase locus is shown in Fig. 1.

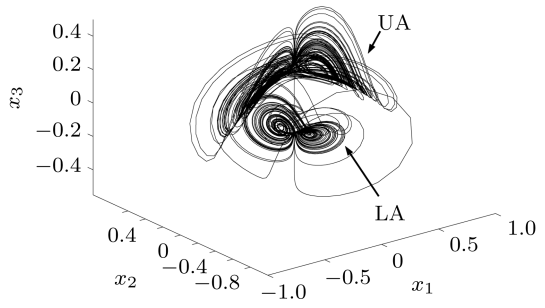


Fig. 1. The double strange attractors of the Newton–Leipnik system.

The system (5) is already in the normal form of Eq. (3), where

$$\begin{aligned}
&z_1 = x_1,\\
&z_2 = x_2,\\
&y = x_3,\\
&z = [z_1, z_2]^{\rm T},\\
&f_0(z) = [-\gamma z_1 + z_2, -z_1 - 0.4z_2]^{\rm T},\\
&p(z,y) = [10z_2, 5z_1]^{\rm T},\\
&a(z,y) = 1,\\
&b(z,y) = \sigma y - 5z_1 z_2. \qquad (6)
\end{aligned}$$

Choose a storage function candidate

$$V(z,y) = W(z) + \frac{1}{2}y^2, \qquad (7)$$

where $W(z) = (z_1^2 + z_2^2)/2$ is Lyapunov function with $W(0) = 0$. The zero dynamics of expression (6) describes the internal dynamics, which is consistent with external constraint $y = 0$, i.e.,

$$\dot{z} = f_0(z). \qquad (8)$$

Consider Eq. (8), since γ and σ are positive, we have

$$\begin{aligned}
\frac{\rm d}{{\rm d}t}W(z) &= [z_1, z_2][-\gamma z_1 + z_2, -z_1 - 0.4z_2]^{\rm T}\\
&= -\gamma z_1^2 - 0.4z_2^2 \le 0. \qquad (9)
\end{aligned}$$

The zero dynamics of the Newton–Leipnik system is Lyapunov-stable, i.e., the Newton–Leipnik system is minimum phase. The derivative of $V(z,y)$ along the trajectory of Eq. (6) is

$$\begin{aligned}
\frac{\rm d}{{\rm d}t}V(z,y) &\le \frac{\partial}{\partial z}W(z)\dot{z} + y\dot{y}\\
&= \frac{\partial}{\partial z}W(z)f_0(z) + \frac{\partial}{\partial z}W(z)p(z,y)y\\
&\quad + [b(z,y) + a(z,y)u]y. \qquad (10)
\end{aligned}$$

As the Newton–Leipnik system is a minimum phase, we have

$$\frac{\partial}{\partial z}W(z)f_0(z) \le 0.$$

Then equation (10) becomes

$$\frac{\rm d}{{\rm d}t}V(z,y) \le \frac{\partial}{\partial z}W(z)p(z,y)y + [b(z,y) + a(z,y)u]y.$$

If we select the feedback control Eq. (4) and consider Eq. (6), then we have

$$\begin{aligned}
u &= a^{-1}(z,y)\left[-b^{\rm T}(z,y) - \frac{\partial W}{\partial z}p(z,y) - \lambda y + v\right]\\
&= -\sigma y - 10z_1 z_2 - \lambda y + v, \qquad (11)
\end{aligned}$$

where λ is a positive constant and $v = -\varphi(y)$ s.t. $y\varphi(y) > 0$. The above inequality can be rewritten as

$$\frac{\rm d}{{\rm d}t}V(z,y) \le vy - \lambda y^2. \qquad (12)$$

Then, taking integration for both sides of inequality (12), we have

$$\begin{aligned}
&V(z,y) - V(z_0,y_0)\\
&\le \int_0^t v(\tau)y(\tau){\rm d}\tau - \lambda\int_0^t y^2(\tau){\rm d}\tau. \qquad (13)
\end{aligned}$$

For $V(z,y) \ge 0$, let $\mu = V(z_0,y_0)$, then we obtain

$$\begin{aligned}
\int_0^t v(\tau)y(\tau){\rm d}\tau + \mu &\ge \lambda\int_0^t y^2(\tau){\rm d}\tau + V(z,y)\\
&\ge \lambda\int_0^t y^2(\tau){\rm d}\tau\\
&= \lambda\int_0^t y^{\rm T}(\tau)y(\tau){\rm d}\tau.
\end{aligned}$$

It satisfies the passive definition (2b). Therefore, equation (5) has been rendered output strict passive (OSP) under Eq. (11).

Lemma 1[28] If system (1) has been made passive, then there exists a C^1 scalar storage function $V: R^n \to R$, $V(x) \ge 0$, $\forall x \backslash \{0\}$, and $S(x) \ge 0$ such that

$$\begin{aligned}
&L_f V(x) = -S(x),\\
&L_g V(x) = h^{\rm T}(x), \qquad (14)
\end{aligned}$$

where $L_f V(x) = (\partial V^{\rm T}/\partial x)f(x)$ and $L_g V(x) = (\partial V^{\rm T}/\partial x)g(x)$.

In order to explain Lemma 1 and also make it clear enough, we take Newton–Leipnik system as an example. When control action (11) is added in its

third equation, Newton–Leipnik system (5) can be turned into

$$\begin{aligned}\dot{x}_1 &= -\gamma x_1 + x_2 + 10x_2x_3,\\ \dot{x}_2 &= -x_1 - 0.4x_2 + 5x_1x_3,\\ \dot{x}_3 &= -15x_1x_2 - \lambda x_3 - \varphi(x_3).\end{aligned} \tag{15}$$

Notice that we replace z_1, z_2 and y with x_1, x_2 and x_3. Considering Lyapunov function $V(x) = (x_1^2+x_2^2+x_3^2)/2$, we have

$$\begin{aligned}L_fV(x) &= \frac{\partial V^{\mathrm{T}}}{\partial x}f(x)\\ &= [x_1\ x_2\ x_3]\begin{bmatrix}-\gamma x_1 + x_2 + 10x_2x_3\\ -x_1 - 0.4x_2 + 5x_1x_3\\ -15x_1x_2 - \lambda x_3 - \varphi(x_3)\end{bmatrix}\\ &= -\gamma x_1^2 - 0.4x_2^2 - \lambda x_3^2 - x_3\varphi(x_3)\\ &= -S(x),\end{aligned}$$

and

$$\begin{aligned}L_gV(x) &= \frac{\partial V^{\mathrm{T}}}{\partial x}g(x)\\ &= [x_1\ x_2\ x_3]\begin{bmatrix}0\\0\\1\end{bmatrix}\\ &= x_3\\ &= h^{\mathrm{T}}(x).\end{aligned}$$

In this way, we show Lemma 1 clearly.

3. Network including coexisting attractors

In this section, we take each single coexisting attractor dynamic model (which has been made passive) as an agent. For $i = 1, \ldots, N$ each agent dynamics can be written as

$$\begin{aligned}\dot{x}_i &= f_i(x_i) + g_i(x_i)\bar{u}_i,\\ y_i &= h_i(x_i).\end{aligned} \tag{16}$$

Assumption 1 Let directed graph G represent the network whose nodes correspond to the coexisting attractors (16); and it is assumed to be strongly connected and balanced.

Let $B = (b_{ij})_{N\times N}$ denote the inter-coupling among the nodes of the network with coexisting attractors. In this model, it is required that the coupling coefficients satisfy $\sum_{j=1}^{N} b_{ij} = 0$ to ensure that the coupling is diffusive. If there is a connection between node i and node j ($j \neq i$), then $b_{ij} = b_{ji} = 1$; otherwise $b_{ij} = b_{ji} = 0$. According to Assumption 1, for G is connected, then $B = B^{\mathrm{T}}$ is an irreducible real symmetric matrix.

Definition 2 Suppose that we have a network with N identical agents expressed as Eq. (16). In the absence of communication delays, the agents' outputs are said to be synchronized if equation (17) holds

$$\lim_{t\to\infty} |y_i(t) - y_j(t)| = 0 \qquad \forall i, j = 1, \ldots, N. \tag{17}$$

We analyse the case when the communication topology is fixed, i.e., the information graph does not change with time. Suppose that the agents are coupled by using the control

$$\bar{u}_i = \sum_{j\in N_i}(y_j - y_i), \quad \forall i, j = 1, \ldots, N, \tag{18}$$

where N_i is the set of agents transmitting their outputs to the i-th agent.

Theorem 1 Consider the dynamical system described by Eq. (16) with control (18). Then under Assumption 1, the nonlinear system (16) will be globally stabilized by control (18) and the outputs of the agents can be synchronized.

Proof Consider a positively defined Lyapunov function for the N agent system

$$V = 2(V_1 + V_2 + \ldots + V_N), \tag{19}$$

where V_i is the storage function for agent i. The derivative of this Lyapunov function along trajectories of system (16) is

$$\begin{aligned}\dot{V} &= 2\sum_{i=1}^{N}(L_{f_i}V_i + (L_{g_i}V_i)\bar{u}_i)\\ &= 2\sum_{i=1}^{N}(-S_i(x_i) + y_i^{\mathrm{T}}\bar{u}_i)\\ &= -2\sum_{i=1}^{N}S_i(x_i) + 2\sum_{i=1}^{N}\sum_{j\in N_i}y_i^{\mathrm{T}}(y_j - y_i)\\ &= -2\sum_{i=1}^{N}S_i(x_i) - 2\sum_{i=1}^{N}\sum_{j\in N_i}y_i^{\mathrm{T}}y_i + 2\sum_{i=1}^{N}\sum_{j\in N_i}y_j^{\mathrm{T}}y_i.\end{aligned} \tag{20}$$

As the information exchange graph is balanced, we have

$$2\sum_{i=1}^{N}\sum_{j\in N_i}y_i^{\mathrm{T}}y_i = \sum_{i=1}^{N}\sum_{j\in N_i}y_i^{\mathrm{T}}y_i + \sum_{i=1}^{N}\sum_{j\in N_i}y_j^{\mathrm{T}}y_j. \tag{21}$$

And, therefore, it follows that

$$\dot{V} = -2\sum_{i=1}^{N} S_i(x_i) - \sum_{i=1}^{N}\sum_{j\in N_i} (y_i - y_j)^{\mathrm{T}}(y_i - y_j) \leq 0. \quad (22)$$

Thus the system is globally stable and all signals are bounded. Consider the set $E = \left\{x_i \in R^{n\times 1}, i = 1, \ldots, N | \dot{V} \equiv 0\right\}$, which is characterized by all trajectories such that $\{S_i(x_i) \equiv 0, (y_i - y_j)^{\mathrm{T}}(y_i - y_j) \equiv 0, \forall j \in N_i, \forall i = 1, 2, \ldots, N\}$. LaSalle's invariance principle[29] and strong connectivity of the network then imply output synchronization of system (16).

4. Examples and results

In order to explain the hierarchical control mechanism further, three examples of co-existing attractor networks corresponding to star-, ring- and chain-connected topologies are studied.

4.1. Star-connected network

The star-connected coexisting attractor network is shown in Fig. 2, the nodes are labeled as 1 to 5. Here we have $N = 5$.

The inter-coupling matrix B is

$$B = \begin{bmatrix} -4 & 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix}. \quad (23)$$

Thus, the inter-controls corresponding to network nodes can be listed as follows:

$$\begin{aligned} \bar{u}_1 &= \sum_{j\in N_1} (y_j - y_1) \\ &= (y_2 - y_1) + (y_3 - y_1) + (y_4 - y_1) + (y_5 - y_1); \\ \bar{u}_2 &= \sum_{j\in N_2} (y_j - y_2) = y_1 - y_2; \\ \bar{u}_3 &= \sum_{j\in N_3} (y_j - y_3) = y_1 - y_3; \\ \bar{u}_4 &= \sum_{j\in N_4} (y_j - y_4) = y_1 - y_4; \\ \bar{u}_5 &= \sum_{j\in N_5} (y_j - y_4) = y_1 - y_5. \end{aligned} \quad (24)$$

The intra-control of network nodes can be expressed as

$$\begin{aligned} u_i &= -\sigma y_i - 10 z_{i1} z_{i2} - \lambda y_i + v_i, \quad (i = 1, \ldots, 5) \\ &= -\sigma x_{i3} - 10 x_{i1} x_{i2} - \lambda x_{i3} + v_i, \end{aligned} \quad (25)$$

where λ is selected as 2 througout this paper. Output synchronization at the manifold of origin is illustrated in Fig. 3(a).

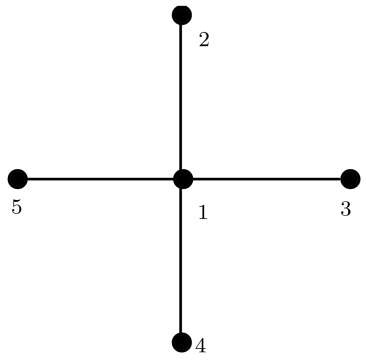


Fig. 2. Star-connected coexisting attractors network.

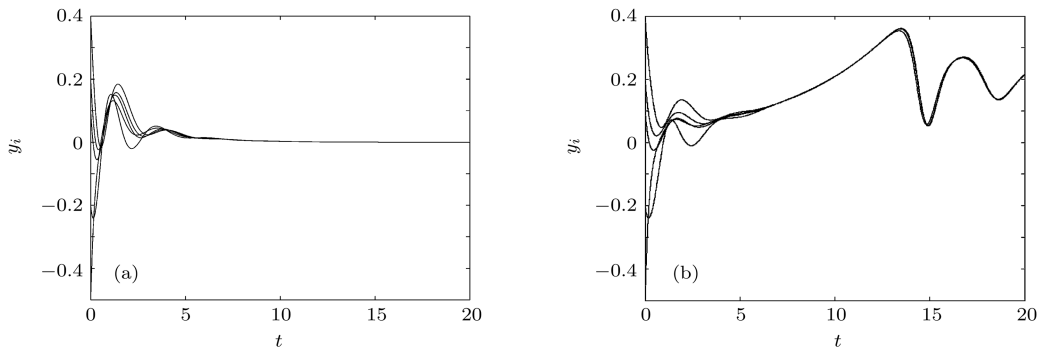


Fig. 3. (a) Output synchronization at the manifold of origin of star-connected network; (b) Output results of star-connected network without intra control.

The initial outputs of the coexisting agents are given as

$$\begin{bmatrix} y_{10} & y_{20} & y_{30} & y_{40} & y_{50} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} -0.5 & -0.2 & 0.1 & 0.2 & 0.4 \end{bmatrix}^{\mathrm{T}}.$$

Remark 1 In order to explain the superiority of the hierarchical control policy to the other ones, we also give Fig. 3(b), where we use inter-control only, i.e we do not employ intra-control, in this case, each node is solely suffered from linear negative feedback specified by the outputs of its neighbours and the output of its own. It is easy to find that we cannot direct the network outputs to the origin manifold and we cannot ensure persistent outputs synchronization in networks with coexisting attractors either.

If we want to drive the coexisting-attractor network to synchronize at the other non-zero manifold, extra reference must be injected into the network, which acts as targeting output shift; for example, with extra reference signal $y_r = 0.5310$, the coexisting-attractor networks can be synchronized at the manifold of 0.2103. This corresponds to the third coordinate of the equilibrium points of the upper attractor[23] (see Fig. 4).

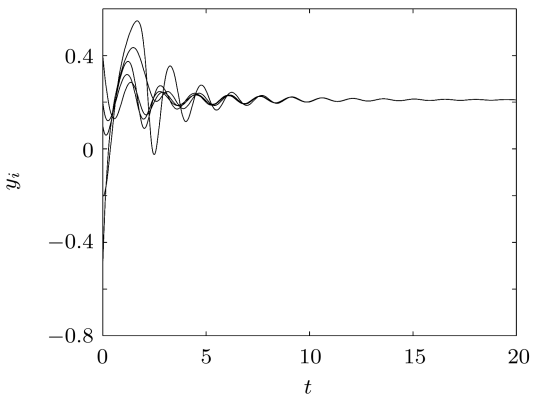


Fig. 4. Output synchronization at the manifold of 0.2103 of star-connected network.

4.2. Ring-connected network

The ring-connected coexisting-attractor network is depicted in Fig. 5. For simplicity, we only display the inter-coupling-matrix and the results obtained here.

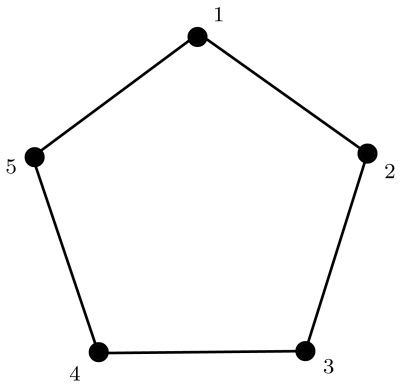


Fig. 5. Ring-connected coexisting-attractor network.

$$B = \begin{bmatrix} -2 & 1 & 0 & 0 & 1 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 1 & 0 & 0 & 1 & -2 \end{bmatrix}. \tag{26}$$

The corresponding output synchronization results are shown in Figs. 6 and 7.

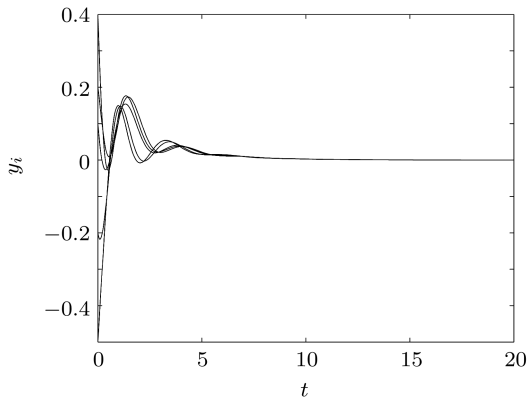


Fig. 6. Output synchronization at the manifold of origin of ring connected network.

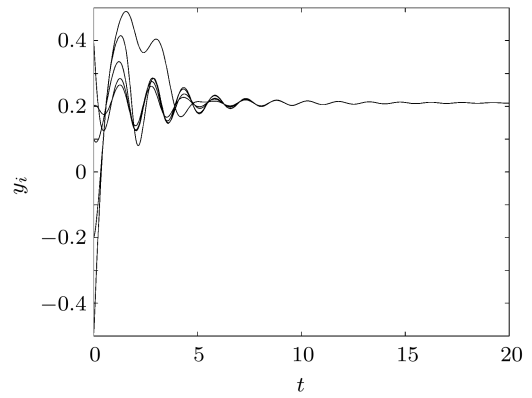


Fig. 7. Output synchronization at the manifold of 0.2103 of ring-connected network.

4.3. Chain-connected network

The chain-connected coexisting-attractor network is given in Fig. 8. Due to the same reason as for the ring-connected case, we here only give the inter-coupling-matrix and the results obtained.

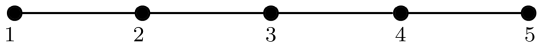


Fig. 8. Chain-connected coexisting-attractor network.

$$B = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}. \quad (27)$$

The corresponding output synchronization results are illustrated in Figs. 9 and 10.

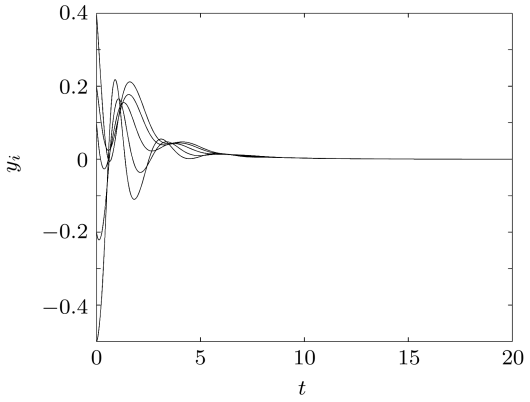


Fig. 9. Output synchronization at the manifold of origin of chain-connected network.

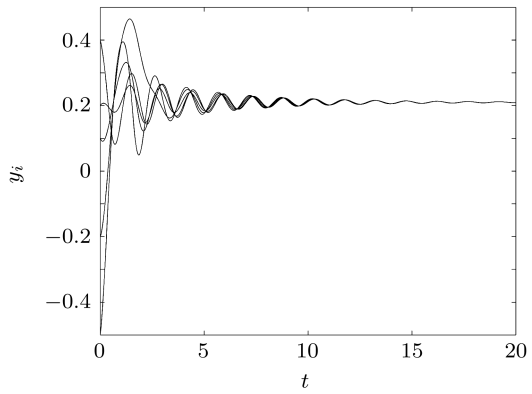


Fig. 10. Output synchronization at the manifold of 0.2103 of chain-connected network.

5. Conclusions

Coexisting-attractor networks' output synchronization has been studied and discussed in this paper. Unlike the existing methods, a hierarchical control strategy is used here, which can be divided into two different parts: one is the intra-control for rendering node dynamics passive and the other is the inter-control, which works in a distributed way, emphasizing the coordination of the agents. Hierarchical control can be equivalently regarded as the synergy of intra-control and inter-control. Then, output synchronization becomes natural. Due to its freedom from limits of linearization, the results are global, which is vitally important for nonlinear systems. In addition, the method used here pays impartial attention to the two fundamental factors of networks: nodes dynamics and network topology. This paves the way for the harmonious manipulation of networks to some extent.

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