Generalized projective synchronization of chaotic systems via adaptive learning control*

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In this paper, a learning control approach is applied to the generalized projective synchronisation (GPS) of different chaotic systems with unknown periodically time-varying parameters. Using the Lyapunov–Krasovskii functional stability theory, a differential-difference mixed parametric learning law and an adaptive learning control law are constructed to make the states of two different chaotic systems asymptotically synchronised. The scheme is successfully applied to the generalized projective synchronisation between the Lorenz system and Chen system. Moreover, numerical simulations results are used to verify the effectiveness of the proposed scheme.

Keywords: generalized projective synchronisation, chaotic systems, adaptive learning control, Lyapunov–Krasovskii functional

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1. Introduction

Since the idea of synchronising two identical autonomous chaotic systems under different initial conditions was first introduced in 1990 by Pecora and Carroll,[1] chaos synchronisation has been widely studied in physics, secure communication, chemical reactor, biological networks and artificial neural networks. Up to now, different types of synchronisation phenomena have been presented such as complete synchronisation (CS),[2] generalized synchronisation,[3] phase synchronisation,[4] etc. Besides many control schemes such as the OGY method,[5] delayed feedback method,[6] state observer approach,[7] adaptive control method,[8] and active control[9] approach have been employed to synchronize chaotic systems with different initial conditions.

Among all kinds of chaos synchronisation schemes, projective synchronisation, characterized by a scaling factor by which two systems synchronise proportionally, has been extensively investigated by many authors.[10−13] Because this kind of synchronisation belongs to a subclass of the generalized synchronisation, unifies different types of synchronisation phenomena such as anti-phase synchronisation and complete synchronisation, as well as a specific case of generalized synchronisation into one by a scaling factor.[14,15] In application to secure communications, this feature can be used to extend binary digital to M-ary digital communication[16] for achieving fast communication, so the research on projective synchronisation has important theoretical significance and application.

On the other hand, chaotic systems are unavoidably exposed to environments which may cause their parameters to vary within certain ranges, such as environment temperature, voltage fluctuation, mutual interference among components, etc. But parameters of some systems in practical circumstances cannot be exactly known in advance, and may drift around their nominal values. The effect of these uncertainties will destroy the synchronisation and even break it. Therefore, it is important and necessary to study synchronisation in such systems with unknown time-varying parameters. As a result, in the studies[17−19] on control and synchronisation of chaos, the problem of parametric uncertainty is a very significant and challenging one. In these research studies the common method used to solve the parametric uncertainties is the adaptive control scheme in which the unknown system parameters are updated adaptively according to certain rules. For example, in

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Refs. [17] and [18] it is assumed that the parameters of the driving system are totally uncertain or unknown to the response system, and the parameters of the response system can be different from those of the driving system. Some studies suppose that the parameters of the driving and the response systems are identical but there are also some parametric uncertainties or perturbations.[19] The learning control method[20,21] has been applied to chaotic systems in the presence of time-varying uncertainties with the uniform periodicity and the pseudo-periodic problem. Based on the adaptive iterative learning strategy, a simple time-delayed controller is proposed to stabilise unstable periodic orbits (UPOs) embedded in chaotic attractors.[22]

Motivated by the foregoing discussion, we will formulate the GPS problem of different chaotic systems for time-varying parameters with known periodicity. An adaptive learning control approach is proposed. Using the Lyapunov–Krasovskii stability theory, a differential-difference mixed parametric learning law and an adaptive learning control law are constructed to make the states of two different chaotic systems asymptotically synchronised. Some typical chaotic systems are taken as examples to illustrate our technique.

The rest of this paper is organized as follows. Section 2 gives the problem formulation. The learning control scheme is presented in Section 3. In Section 4 a simulation example is used to confirm the effectiveness of the proposed scheme. The conclusions are given in Section 5.

Notations: Throughout this paper, the notation P^{T} denotes the transpose of a vector (matrix) P, while for $x \in R^n$, the notation $\|x\| = \sqrt{(x^{\mathrm{T}}x)}$ stands for the Euclidean norm of the vector. The L_T^2 norm of a function vector $r(t), t \in [t-T, t)$ is defined as $\int_{t-T}^{t} r^{\mathrm{T}}(\tau)r(\tau)\mathrm{d}\tau$ for a finite positive constant T. For a signal $r(t)$, we say that $r(t) \in R^n$ is L_T^2 convergent if there is $\lim\limits_{t\to\infty} \int_{t-T}^{t} r^{\mathrm{T}}(\tau)r(\tau)\mathrm{d}\tau = 0$.

2. Problem description

Consider a class of chaotic systems with unknown time-varying parameters, described by

$$\dot{x} = Ax + f(x) + D(x)\theta(t), \tag{1}$$

where $x \in R^n$ is the state vector, $A \in R^{n\times n}$ and $f(x) : R^n \to R^n$ are the linear coefficient matrix and nonlinear part of system (1), respectively.

$$D(x) : R^n \to R^{n\times p}$$

and

$$\theta(t) = (\Theta + \Phi(t)) \in R^p$$

are the uncertain parameter vectors. Here Θ is the nominal value of $\theta(t)$ and $\Phi(t)$ is the uncertainty or disturbance. Equation (1) is considered as the drive system. The response system with a controller $u(x, y, t) \in R^n$ is introduced as follows:

$$\dot{y} = By + g(y) + u(x, y, t), \tag{2}$$

where $y \in R^n$ is the state vector, $B \in R^{n\times n}$ and $g(y) : R^n \to R^n$ are the linear coefficient matrix and a continuous nonlinear vector function, respectively, $u(x, y, t)$ is the control vector.

Remark 1 The systems studied in this paper depend linearly on the parameters. The class of nonlinear dynamical systems includes an extensive variety of chaotic systems such as Lorenz system, Rössler system, Duffing system, Chua's circuit, generalized Lorenz system, etc.

Next, we always assume that $\theta(t) \in R^p$, for all $t \in R^+$, is the unknown continuous periodic function vector with a known period T, that is to say, $\theta(t) = \theta(t - T)$.

Remark 2 Since $\theta(t) = \Theta + \Phi(t)$, obviously $\Phi(t)$ is unknown continuous periodic function vector with a known period T.

We describe the tracking error term as

$$e(t) = x - Fy, \tag{3}$$

where $F = \alpha I_{n\times n}$ is a scaling matrix (α is a nonzero constant and $I_{n\times n}$ is the identity matrix).

Therefore, the goal of control is to design and implement an appropriate controller $u(x, y, t)$ for the response system. In addition, the adaptive estimation laws of the parameters Θ and $\Phi(t)$ need to be designed respectively, such that the response system (2) could be asymptotically synchronised with the drive system (1) as follows:

$$\lim_{t\to\infty} \int_{t-T}^{t} \|e(\tau)\|^2 \,\mathrm{d}\tau = 0. \tag{4}$$

Remark 3 As we know, if equality (4) is achieved with $F = I_{n\times n}$, then complete synchronisation occurs between Eqs. (1) and (2). But, if $F = -I_{n\times n}$, then the anti-synchronisation arises.

3. Design of the learning controller

The dynamic equation of synchronisation error (3) can be easily obtained from Eqs. (1) and (2), which is expressed as follows:

$$\dot{e} = Ax + f(x) + D(x)\Theta + D(x)\Phi(t) - FBy - Fg(y) - Fu(x,y,t). \tag{5}$$

According to Eq. (5), we select the controller in the following form

$$u(x,y,t) = F^{-1}[Ke + Ax + f(x) + D(x)\hat{\Theta}(t) + D(x)\hat{\Phi}(t) - FBy - Fg(y)], \tag{6}$$

where $K \in R^{n\times n}$ is a positive-definite matrix. $\hat{\Theta}(t)$ and $\hat{\Phi}(t)$ are estimations to Θ and $\Phi(t)$, respectively.

For the constant parameter vector Θ, the differential type updating law is used

$$\dot{\hat{\Theta}}(t) = \Gamma_1 D^{\mathrm{T}}(x)e, \tag{7}$$

where $\Gamma_1 \in R^{p\times p}$ is a positive diagonal learning gain matrix.

For the period time-varying parameter vector $\Phi(t)$, the difference type updating law is employed

$$\hat{\Phi}(t) = \begin{cases} \hat{\Phi}(t-T) + \Gamma_2 D^{\mathrm{T}}(x)e & t \in [T,\infty), \\ \Gamma_0(t) D^{\mathrm{T}}(x)e & t \in [0,T), \\ 0 & t \in [-T,0), \end{cases} \tag{8}$$

where $\Gamma_2 \in R^{p\times p}$ is a positive diagonal gain matrixes, and $\Gamma_0(t) \in R^{p\times p}$ is a continuous, positive, diagonal gain matrix for the first period $[0,T)$ satisfying $\Gamma_0(0) = 0, \Gamma_0(T) = \Gamma_2$, and each element of $\Gamma_0(t)$ is chosen to be strictly increasing. The purpose of choosing $\Gamma_0(t)$ is to ensure the continuity of $\hat{\Phi}(t)$ at the instants $T = iT$ $(i = 1, 2, \ldots)$, when the updating law (8) is used.

Remark 4 The adaptation law is a difference-type pointwise integration over the period $[t-T, t]$, which takes the place of the differential-type adaptation law. The fundamental idea of the pointwise integration based periodic updating can be explained as follows.[21] Since $\Phi(t) = \Phi(t-T)$, for any time instant $iT \le t \le (i+1)T$, where $i \in Z_+$, we have $\Phi(t) = \Phi(t-T) = \cdots = \Phi(t-iT)$. This means that $\Phi(t)$ can be treated as a "constant" when it is sampled at any interval which is the integer multiple of the period T. The periodic updating law (8) in the interval of one period T is a difference-type integrator for such a "constant".

Lemma 1 $\hat{\Phi}(t), \forall t \in [0,\infty)$ is of continuity.

Proof Continuous property of $\hat{\Phi}(t)$ for the first period $[0,T)$ is obvious. Hence, we only need to focus on $t \in [iT,(i+1)T)$, $i = 2,3,\ldots$. When $i = 2$, we have

$$\begin{aligned}
\lim_{t\to 2T^-} \hat{\Phi}(t) &= \lim_{t\to 2T^-} [\Gamma_0(t-T) D^{\mathrm{T}}(x(t-T))e(t-T) \\
&\quad + \Gamma_2 D^{\mathrm{T}}(x(t))e(t)] \\
&= \Gamma_2 D^{\mathrm{T}}(x(T))e(T) + \Gamma_2 D^{\mathrm{T}}(x(2T))e(2T), \\
\lim_{t\to 2T^+} \hat{\Phi}(t) &= \lim_{t\to 2T^+} [\hat{\Phi}(t-2T) + \Gamma_2 D^{\mathrm{T}}(x(t-T)) \\
&\quad \times e(t-T) + \Gamma_2 D^{\mathrm{T}}(x(t))e(t)] \\
&= \lim_{t\to 2T^+} [\Gamma_0(t-2T) D^{\mathrm{T}}(x(t-2T)) \\
&\quad \times e(t-2T) + \Gamma_2 D^{\mathrm{T}}(x(t-T))e(t-T) \\
&\quad + \Gamma_2 D^{\mathrm{T}}(x(t))e(t)] \\
&= \Gamma_2 D^{\mathrm{T}}(x(T))e(T) + \Gamma_2 D^{\mathrm{T}}(x(2T))e(2T).
\end{aligned}$$

Obviously, we have $\lim_{t\to 2T^-} \hat{\Phi}(t) = \lim_{t\to 2T^+} \hat{\Phi}(t)$. Therefore, $\hat{\Phi}(t)$ is continuous for two periods $[T,2T)$. Similarly, the continuity of $\hat{\Phi}(t), t \in [iT,(i+1)T), i = 2,3,\ldots$, can be ensured.

Thereby, equation (5) can be formulated as

$$\dot{e} = -Ke + D(x)\tilde{\Phi}(t) + D(x)\tilde{\Theta}(t), \tag{9}$$

where $\tilde{\Theta}(t) = \Theta - \hat{\Theta}(t), \tilde{\Phi}(t) = \Phi(t) - \hat{\Phi}(t)$.

The convergence property of the proposed adaptive learning control method is summarized in the following theorem.

Theorem 1 For a given scaling matrix F and any initial conditions $x(0)$ and $y(0)$, there are the control law (6) and two parameter updating laws (7) and (8) that warrant the asymptotical convergence of the tracking error $e(t)$ in L_T^2 norm, that is to say, $\lim_{t\to\infty} \int_{t-T}^{t} \|e(\tau)\|^2 \,\mathrm{d}\tau = 0$.

Proof To facilitate the convergence analysis, let $V(t, e(t), \tilde{\Theta}(t), \tilde{\Phi}(t)) = V(t)$, we choose a Lyapunov–Krasovskii functional as follows:

$$V(t) = \begin{cases} \dfrac{1}{2}e^{\mathrm{T}}e + \dfrac{1}{2}\tilde{\Theta}^{\mathrm{T}}(t)\Gamma_1^{-1}\tilde{\Theta}(t) + \dfrac{1}{2}\displaystyle\int_0^t \tilde{\Phi}^{\mathrm{T}}(\tau)\Gamma_2^{-1}\tilde{\Phi}(\tau)\mathrm{d}\tau, & t \in [0,T), \\ \dfrac{1}{2}e^{\mathrm{T}}e + \dfrac{1}{2}\tilde{\Theta}^{\mathrm{T}}(t)\Gamma_1^{-1}\tilde{\Theta}(t) + \dfrac{1}{2}\displaystyle\int_{t-T}^t \tilde{\Phi}^{\mathrm{T}}(\tau)\Gamma_2^{-1}\tilde{\Phi}(\tau)\mathrm{d}\tau, & t \in [T,\infty). \end{cases} \tag{10}$$

Firstly, we will prove that the finiteness of $V(t)$ in $[0,T)$. Secondly, we will prove the asymptotical convergence of $e(t)$.

Step 1 let us confirm the finiteness property of $V(t)$ for the first period $[0,T)$. From the system dynamics (1) and (2) and the proposed control laws (6)–(8), it can be seen that the right-hand side of Eqs. (1) and (2) are continuous with respect to all arguments. In light of the existence theorem of differential equation,[23] equation (5) has a solution in an interval $[0,T_1) \subset [0,T)$, with $0 < T_1 \leq T$. Therefore, the boundedness of $V(t)$ over $[0,T_1)$ can be guaranteed and we need only focus on the interval $[T_1,T)$.

For any $t \in [T_1,T)$, the time derivative of $V(t)$ for $t \in [T_1,T)$ is given by

$$\dot{V}(t) = e^{\mathrm{T}}\dot{e} + \tilde{\Theta}^{\mathrm{T}}(t)\Gamma_1^{-1}\dot{\tilde{\Theta}}(t) + \frac{1}{2}\tilde{\Phi}^{\mathrm{T}}(t)\Gamma_2^{-1}\tilde{\Phi}(t), \quad t \in [T_1,T). \tag{11}$$

Taking the first term on the right-hand side of $\dot{V}(t)$, Eq. (9), we obtain

$$e^{\mathrm{T}}\dot{e} = -e^{\mathrm{T}}Ke + e^{\mathrm{T}}D(x)\tilde{\Theta}(t) + e^{\mathrm{T}}D(x)\tilde{\Phi}(t). \tag{12}$$

Using the parametric updating law (7) we have

$$\tilde{\Theta}^{\mathrm{T}}(t)\Gamma_1^{-1}\dot{\tilde{\Theta}}(t) = -\tilde{\Theta}^{\mathrm{T}}(t)\Gamma_1^{-1}\dot{\hat{\Theta}}(t) = -\tilde{\Theta}^{\mathrm{T}}(t)D^{\mathrm{T}}(x)e. \tag{13}$$

Let us focus on the third term on the right-hand side of Eq. (11). Since $\Gamma_0(t)$ is a diagonal element and each diagonal element is strictly increasing in $[0,T)$, $\Gamma_2^{-1} \leq \Gamma_0^{-1} < \infty$ is ensured in the interval $[T_1,T)$, and

$$\begin{aligned}\frac{1}{2}\tilde{\Phi}^{\mathrm{T}}(t)\Gamma_2^{-1}\tilde{\Phi}(t) &\leq \frac{1}{2}\tilde{\Phi}^{\mathrm{T}}(t)\Gamma_0^{-1}\tilde{\Phi}(t)\\ &= \frac{1}{2}[\Phi^{\mathrm{T}}(t)\Gamma_0^{-1}(\Phi(t)-\hat{\Phi}(t)) - 2\hat{\Phi}^{\mathrm{T}}(t)\Gamma_0^{-1}\tilde{\Phi}(t) + \hat{\Phi}^{\mathrm{T}}(t)\Gamma_0^{-1}\tilde{\Phi}(t)]\\ &\leq \frac{1}{2}[\Phi^{\mathrm{T}}(t)\Gamma_0^{-1}\Phi(t) - 2e^{\mathrm{T}}D(x)\tilde{\Phi}(t)].\end{aligned} \tag{14}$$

Substituting Eqs. (12)–(14) into Eq. (11) yields

$$\dot{V}(t) \leq -e^{\mathrm{T}}Ke + \frac{1}{2}\Phi^{\mathrm{T}}(t)\Gamma_0^{-1}\Phi(t) \leq \frac{1}{2}\Phi^{\mathrm{T}}(t)\Gamma_0^{-1}\Phi(t). \tag{15}$$

Note that $\Phi(t)$ is periodic, thus it is bounded. The boundedness of $\Phi(t)$ leads to the boundedness of $\dot{V}(t)$. As $V(T_1)$ is bounded, $\forall t \in [T_1,T)$. The finiteness of $V(t)$ is obvious.

Step 2 Let us compute the difference of $V(t)$ over one period for $T \in [T,\infty)$, which is

$$\begin{aligned}\Delta V(t) &= V(t) - V(t-T)\\ &= \int_{t-T}^{t}\left\{e^{\mathrm{T}}\dot{e} + \tilde{\Theta}^{\mathrm{T}}(\tau)\Gamma_1^{-1}\dot{\tilde{\Theta}}(\tau) + \frac{1}{2}[\tilde{\Phi}^{\mathrm{T}}(\tau)\Gamma_2^{-1}\tilde{\Phi}(\tau) - \tilde{\Phi}^{\mathrm{T}}(\tau-T)\Gamma_2^{-1}\tilde{\Phi}(\tau-T)]\right\}\mathrm{d}\tau.\end{aligned} \tag{16}$$

Using the learning updating law (8), we obtain the periodic property $\Phi(t) = \Phi(t-T)$ and the relationship

$$(a-b)^{\mathrm{T}}\Gamma(a-b) - (a-c)^{\mathrm{T}}\Gamma(a-c) = (c-b)^{\mathrm{T}}\Gamma[2(a-b) + (b-c)],$$

where $a,b,c \in R^p, \Gamma \in R^{p\times p}$. Moreover, the following relation can be obtained

$$\frac{1}{2}[\tilde{\Phi}^{\mathrm{T}}(t)\Gamma_2^{-1}\tilde{\Phi}(t) - \tilde{\Phi}^{\mathrm{T}}(t-T)\Gamma_2^{-1}\tilde{\Phi}(t-T)] = -e^{\mathrm{T}}D(x)\tilde{\Phi}(t) - \frac{1}{2}e^{\mathrm{T}}D(x)\Gamma_2 D^{\mathrm{T}}(x)e. \tag{17}$$

Substituting Eqs. (12), (13), and (17) into Eq. (16), we can attain

$$\Delta V(t) = \int_{t-T}^{t}\left[-e^{\mathrm{T}}Ke - \frac{1}{2}e^{\mathrm{T}}D(x)\Gamma_2 D^{\mathrm{T}}(x)e\right]\mathrm{d}\tau \leq -\lambda_{\min}(K)\int_{t-T}^{t}e^{\mathrm{T}}e\,\mathrm{d}\tau, \tag{18}$$

where $\lambda_{\min}(K)$ is the minimum eigenvalue of the matrix K.

For any $t \in [iT, (i+1)T]$, denoting $t_0 = t - iT$, where $t_0 \in [0, T)$, and $i = 1, 2, \ldots$, clearly, when $t \to \infty, i \to \infty$; applying Eq. (18) repeatedly, we have

$$V(t) = V(t_0) + \sum_{j=0}^{i-1} \Delta V(t - jT). \tag{19}$$

Since $T_0 \in [0, T)$, according to Eq. (19), we obtain

$$\lim_{t\to\infty} V(t) = V(t_0) - \lambda_{\min}(K) \lim_{i\to\infty} \sum_{j=0}^{i-1} \int_{t-(j+1)T}^{t-jT} e^{\mathrm{T}} e \mathrm{d}\tau.$$

Consider the positiveness of $V(t)$, and $V(t_0)$ bounded in the interval $[0, T)$. According to the convergence theorem of the sum of series, the tracking error $e(t)$ converges to zero asymptotically in L_T^2 norm. That is to say, we have $\lim_{t\to\infty} \int_{t-T}^{t} \|e(\tau)\|^2 d\tau = 0$. This completes the proof. □

Remark 5 Note that scaling matrix F has no effect on $\dot{V}(t)$. Thus one can change the scaling matrix arbitrarily during control without worrying about the control robustness.

4. Numerical simulations

In this section, we employ an example to illustrate the effectiveness of the proposed adaptive learning control method.

Example Consider the generalized projective synchronisation between Lorenz system

$$\begin{cases} \dot{x}_1 = \theta_1(t)(x_2 - x_1), \\ \dot{x}_2 = -x_1 x_3 - x_2 + \theta_2(t) x_1, \\ \dot{x}_3 = x_1 x_2 - \theta_3(t) x_3, \end{cases} \tag{20}$$

and Chen system

$$\begin{cases} \dot{y}_1 = 35(y_2 - y_1) + u_1, \\ \dot{y}_2 = -y_1 y_3 + 28 y_2 - 7 y_1 + u_2, \\ \dot{y}_3 = y_1 y_2 - 3 y_3 + u_3. \end{cases} \tag{21}$$

Comparing systems (20) and (21) with Eqs. (1) and (2), we obtain

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad f(x) = \begin{pmatrix} 0 \\ -x_1 x_3 \\ x_1 x_2 \end{pmatrix},$$

$$D(x) = \begin{bmatrix} x_2 - x_1 & 0 & 0 \\ 0 & x_1 & 0 \\ 0 & 0 & -x_3 \end{bmatrix},$$

$$B = \begin{bmatrix} -35 & 35 & 0 \\ -7 & 28 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \quad g(y) = \begin{pmatrix} 0 \\ -y_1 y_3 \\ y_1 y_2 \end{pmatrix},$$

and the unknown uncertain parameter $\theta(t) = [10 + 0.2\sin t \quad 28 + 0.2\cos t \quad 8/3 - 0.2\sin t]^{\mathrm{T}}$ is continuous periodic function vector with known periods $T = 2\pi$. If the system parameters are chosen to be $\theta_1(t) = 10$, $\theta_2(t) = 28$, $\theta_3(t) = 8/3$, then Lorenz system has a chaotic attractor. The behaviour of the Lorenz system with $\theta(t) = [10 + 0.2\sin t \quad 28 + 0.2\cos t \quad 8/3 - 0.2\sin t]^{\mathrm{T}}$ is shown in Fig. 1. We obtain the nominal value of the parameter vector $\Theta = [10 \quad 28 \quad 8/3]^{\mathrm{T}}$. And hence we have $\Phi(t) = \theta(t) - \Theta = [0.2\sin t \quad 0.2\cos t \quad -0.2\sin t]^{\mathrm{T}}$. The initially estimated value of the unknown parameter Φ is $\hat{\Phi}(0) = [0 \quad 0 \quad 0]^{\mathrm{T}}$. The initial states of the drive system and response system are chosen as $x(0) = [0 \quad 0.1 \quad 1]^{\mathrm{T}}$ and $y(0) = [0.1 \quad 0.1 \quad 0.2]^{\mathrm{T}}$, respectively. Taking the scaling matrix to be $F = 2I_{3\times 3}$, we choose $K = \mathrm{diag}[10 \quad 10 \quad 10]$, $\Gamma_1 = \mathrm{diag}[25 \quad 10 \quad 0.11], \Gamma_2 = \mathrm{diag}[0.55 \quad 1.1 \quad 0.09]$ and $\Gamma_0(t) = (t/T)\Gamma_2$. According to Theorem1, it is concluded that the controller u along with the parameter updating law given by Eqs. (6), (7) and (8) will achieve the generalized projective synchronisation between Lorenz system and Chen system. This is verified by the simulation results shown in Fig. 2. Furthermore, figure 3 depicts the time evolution of the controllers, and figure 4 shows the evolution of the estimated time-varying parameters. Figure 5 displays the evolution of the estimated constant parameters.

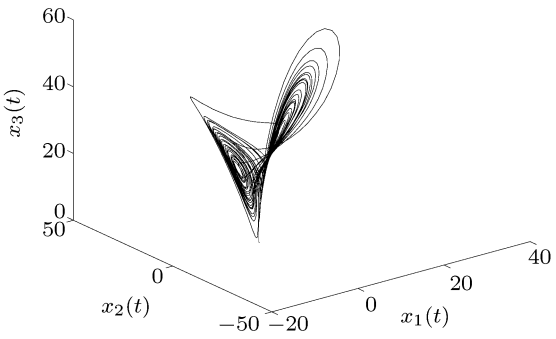


Fig. 1. Chaotic behaviour of Lorenz system with $\theta(t) = [10 + 0.2\sin t \quad 28 + 0.2\cos t \quad 8/3 - 0.2\sin t]^{\mathrm{T}}$.

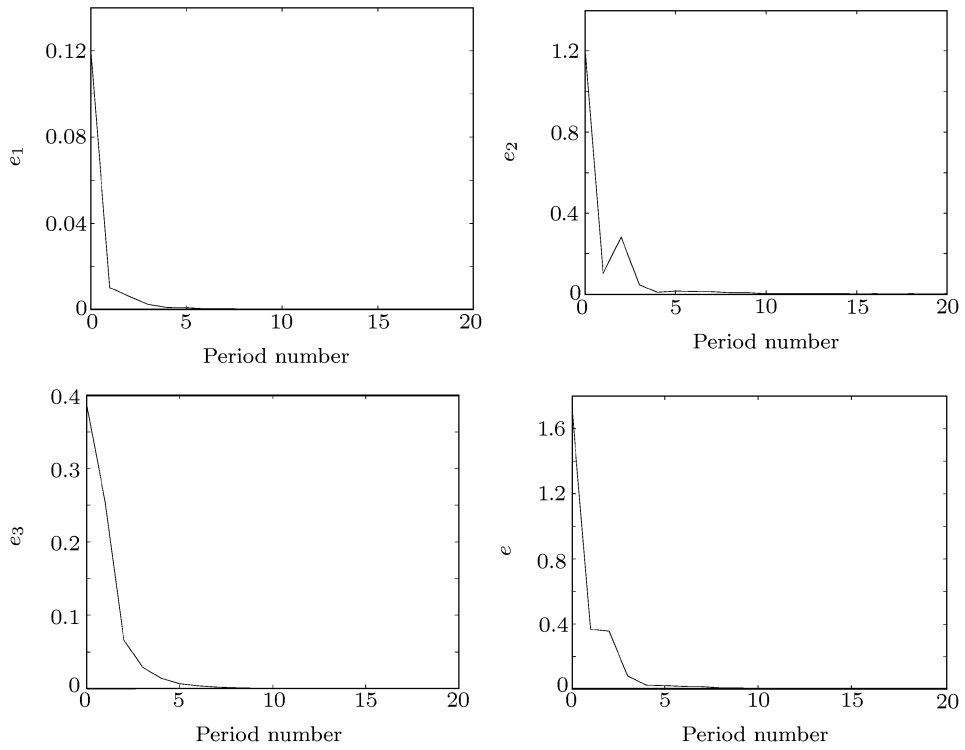


Fig. 2. Time evolution of the tracking errors in L_T^2 norm.

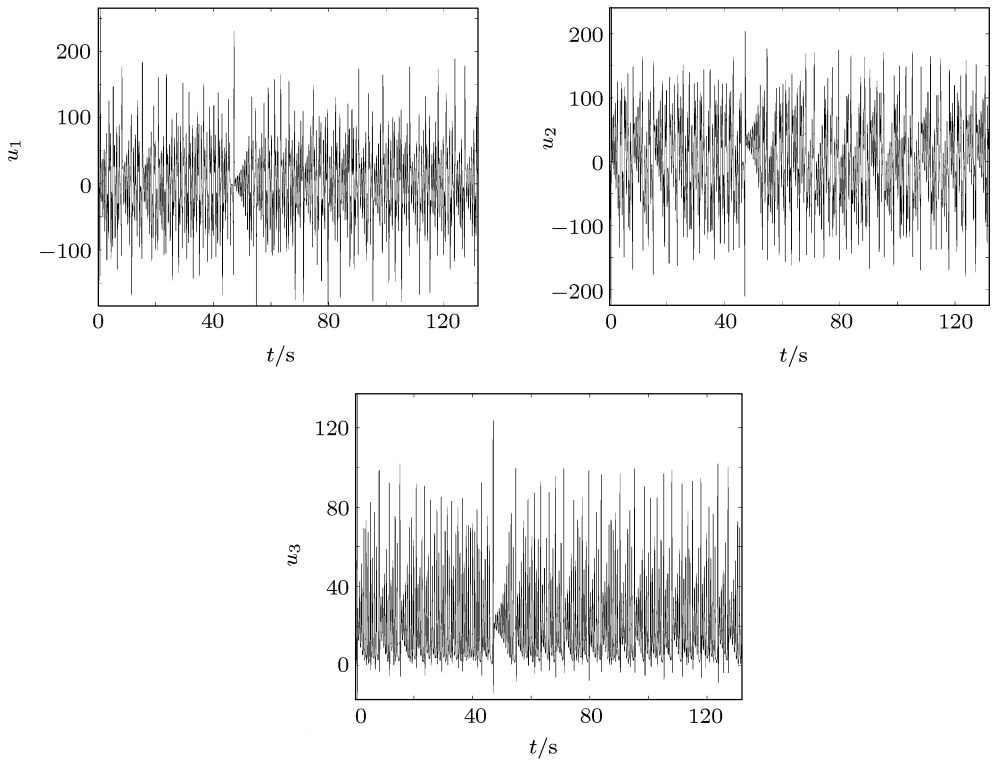


Fig. 3. Time evolution of the controllers.

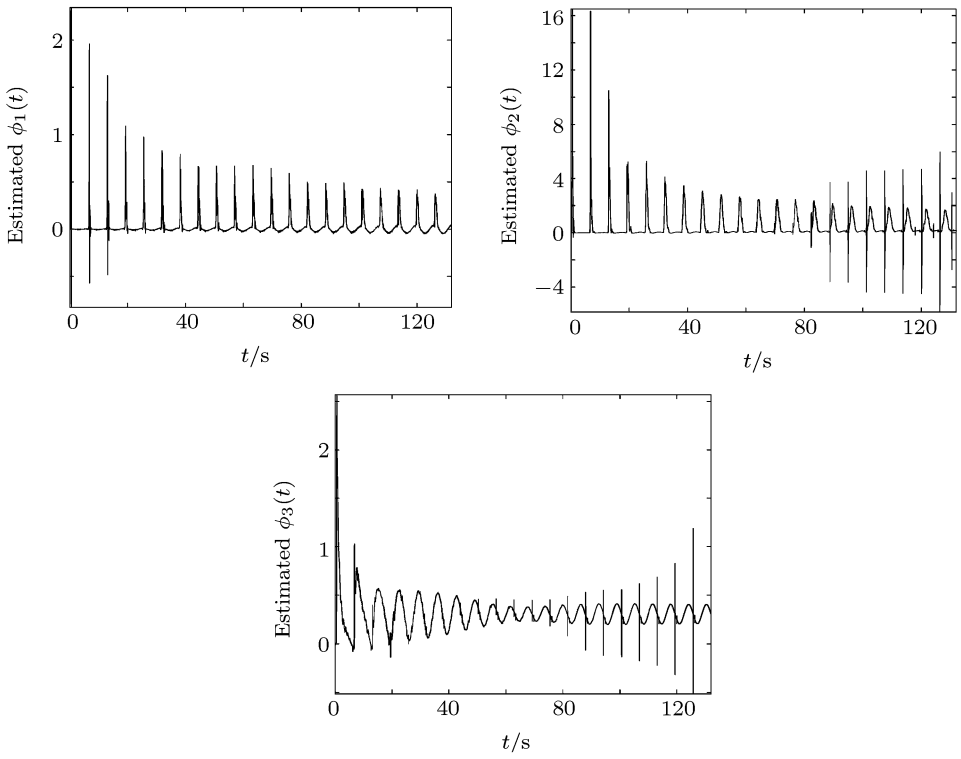


Fig. 4. Time evolution of the estimated time-varying parameters.

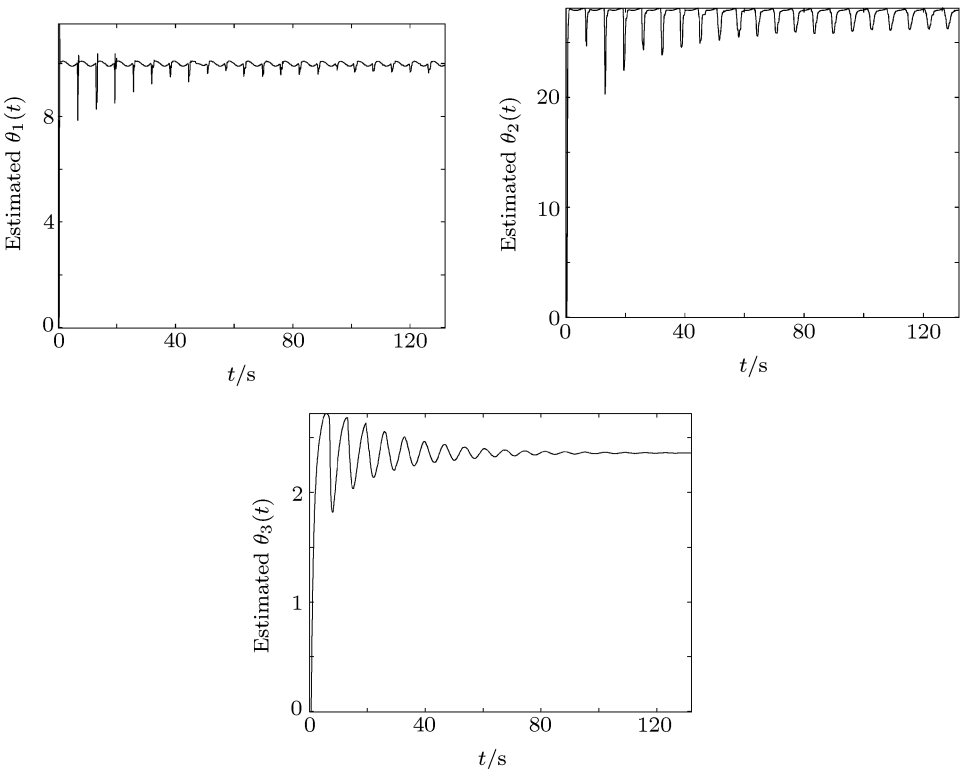


Fig. 5. Time evolution of the estimated constant parameters.

5. Conclusions

In this paper, we have proposed a learning control method for GPS of two different chaotic systems with unknown periodic time varying parameters. Based on the Lyapunov–Krasovskii stability theory, the adaptive learning controller and the differential-difference mixed parametric learning law are obtained for global stability and asymptotic synchronisation of the tracking error dynamics between the drive and the response systems. The proposed controller has been successfully applied to GPS between the Lorenz system and Chen system. The effectiveness of the approach is confirmed through theoretical analysis and numerical simulations.

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