Effects of orientation and shape of holes on the band gaps in water waves over periodically drilled bottoms*

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The complete band gaps (CBGs) of shallow water waves propagating over bottoms with periodically drilled holes are investigated numerically by the plane wave expansion method. Four different patterns are considered, containing triangular, square, hexagonal and circular cross-sectioned holes arranged into triangular lattices. Results show that the width of CBGs can be changed by adjusting the orientation of noncircular holes and the effect of hole shape on the width of the maximal CBGs is discussed.

Keywords: water surface wave, band gap, plane wave expansion method

PACC: 0340G, 4710, 9210H

1. Introduction

During the past decade, great progress has been made in the study of periodic structures, such as photonic crystals[1−3] and phononic crystals,[4,5] the main feature of which is that there exist band gaps in their band structures. Based on the same principle of Bragg scattering, band structures and band gaps may also exist in liquid waves propagating over periodic structures. Recently, the study on liquid surface waves propagating over topographical bottoms has become a hot topic, owing to its potential applications in ocean engineering as well as its theoretical and experimental studies.[6−23] There are some papers dealing with the band structures of water wave over one-dimensional (1D) periodic or random bottoms, including numerical calculation[9−13] and experimental observation.[10,12,13] For two-dimensional (2D) structures, Hu *et al.* investigated the shallow liquid waves propagating over bottoms with cylinders periodically mounted.[14−18] They observed the superlensing effect[17] and calculated the waves by using the multiple scattering method. Also, there are some researches on liquid surface waves over bottoms drilled with holes.[19−23] The systems with circular holes arrayed into square and triangular lattices were studied numerically[19−12] and experimentally,[19,22] in which the calculation method used was the plane wave expansion method (PWE). In addition, in Ref. [23] the band structure of water wave over periodic topography drilled by square holes in square lattice is calculated. It is found that the filling fraction and orientation of bottom-holes exert a great influence on the band gap.

However, in 2D systems with drilled bottoms the relevant studies involve only the cases of circular[19−22] and square[23] holes at the bottoms but seldom dealt with other kinds of shapes. In the present paper, we calculate the band structures of shallow water wave propagating over four kinds of periodically drilled bottoms, at which there are drilled the holes separately with regular triangular (TRI), square (SQU), hexagonal (HEX) and circular (CIR) cross-sections, arranged into triangular lattices (also referred to as hexagonal lattice in some articles). And the effects of the orientation and shape of holes on the CBGs are discussed. The research on water waves propagating over topographical bottoms may be of great importance in ocean engineering problems. Furthermore, the study of liquid waves is useful for the understanding of abstract concepts of wave phenomena.

2. Formulation and models

The problem to be discussed here is the propagation of shallow water waves. The patterns considered here are illustrated in Fig. 1, in which the lattice

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constant of the bottom-holes is denoted by a. And the depths of static water over the plane bottom and hole bottoms are h_0 and h_1, respectively, as shown in Fig. 2. According to Ref. [20], if $Kh \ll 1$, the system can be regarded as a shallow water system, here K is the wave vector. By assuming that the water is an inviscid and incompressible ideal liquid, its flow is irrotational, and the surface tension could be neglected, the shallow water surface wave equation can be expressed as

$$\nabla \cdot (h\nabla\eta) + \frac{\omega^2}{g}\eta = 0, \tag{1}$$

where $\nabla = (\partial_x, \partial_y)$, ω, $g, h(x,y)$, and $\eta(x,y,t)$ are the horizontal gradient, angular frequency, gravity acceleration (9.8 N/kg), depth of water and vertical displacement of the water surface, respectively. Equation (1) is the fundamental equation governing the small amplitude waves in shallow water, which was given first by Lamb.[20,24]

Since the systems under consideration have periodic structures, h and η in the above equation are spatially periodic. According to Bloch theory, they can be expanded in Fourier series with corresponding coefficients $H_{\boldsymbol{G}}$ and $I_{\boldsymbol{K},\boldsymbol{G}}$ as

$$\begin{aligned} h(x,y) &= \sum_{\boldsymbol{G}} H_{\boldsymbol{G}}\, \mathrm{e}^{\mathrm{i}\boldsymbol{G}\cdot\boldsymbol{r}}, \\ \eta(x,y,t) &= \mathrm{e}^{-\mathrm{i}\omega t} \sum_{\boldsymbol{G}} I_{\boldsymbol{K},\boldsymbol{G}}\, \mathrm{e}^{\mathrm{i}(\boldsymbol{K}+\boldsymbol{G})\cdot\boldsymbol{r}}, \end{aligned} \tag{2}$$

where $\boldsymbol{r} = (x,y)$, and $\boldsymbol{G} = (G_x, G_y)$ is the 2D reciprocal lattice vectors.

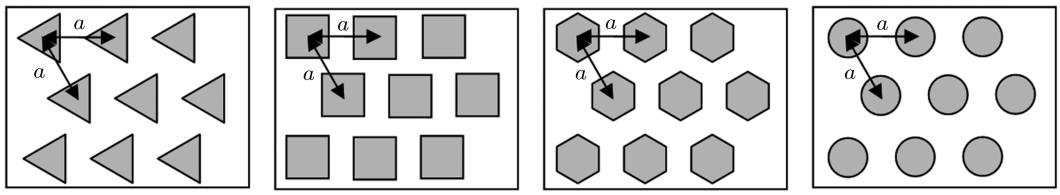


Fig. 1. Top views of drilled bottom patterns. Panels (a), (b), (c) and (d) show TRI, SQU, HEX and CIR holes, respectively.

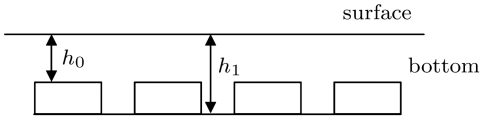


Fig. 2. Side views of shallow water wave propagating over a drilled bottom where h_0 and h_1 represent the depths of static water over the plane bottom and hole bottoms, respectively.

Substituting expression (2) into Eq. (1), we obtain the eigenvalue equation[19–23]

$$\sum_{\boldsymbol{G}'} \left[(\boldsymbol{G}+\boldsymbol{K})\cdot(\boldsymbol{G}'+\boldsymbol{K}) H_{\boldsymbol{G}-\boldsymbol{G}'} - \frac{\omega^2}{g} \right] I_{\boldsymbol{K},\boldsymbol{G}'} = 0, \tag{3}$$

in which the Fourier coefficient $H_{\boldsymbol{G}}$ can be expressed as

$$H_{\boldsymbol{G}} = \begin{cases} h_1 f + h_0(1-f) & \text{for} \quad \boldsymbol{G} = 0, \\ (h_1 - h_0) P_{\boldsymbol{G}} & \text{for} \quad \boldsymbol{G} \neq 0, \end{cases} \tag{4}$$

where $P_{\boldsymbol{G}} = (1/S)\int_{\text{hole}} \mathrm{e}^{-\mathrm{i}\boldsymbol{G}\cdot\boldsymbol{r}}\,\mathrm{d}\boldsymbol{r}$ is the structure function, determined by the shape of holes completely,[5,20,23] f is the filling factor, namely, the area-ratio of the hole to the unit cell.

It is supposed that the TRI, SQU and HEX holes can be rotated around their sites in clockwise direction starting from the orientation indicated in Figs. 1(a), 1(b) and 1(c). Then, after rotating through angle θ, the reciprocal lattice vector $\boldsymbol{G}$ above can be replaced with $\boldsymbol{G}'$ as shown below:[5,23]

$$\begin{aligned} G'_x &= G_x \cos\theta + G_y \sin\theta, \\ G'_y &= -G_x \sin\theta + G_y \cos\theta. \end{aligned} \tag{5}$$

3. Results and discussion

The parameters of the shallow water system adopted here are as follows: lattice constant a = 22.0 mm, depths of water over the plane bottom h_0 = 0.2 mm and over the hole bottoms h_1 = 2.2 mm, that

is to say, the depth of drilled holes is $h_1 - h_0 = 2.0$ mm, which accords with those in Refs. [19], [21] and [23]. By the PWE method, 961 plane waves are used in the numerical calculation. Compared with the results obtained by using more than one thousand plane waves, our result is found to have a convergence accuracy less than 1% for the lowest ten bands.

The hole orientation affects the band gap in water surface wave considerably, just as those in photonic and phononic crystals.[5] Figure 3 shows the band structures of HEX hole system with filling factor $f = 0.70$ at orientation angle $\theta = 0°$ (Fig. 3(a)) and $\theta = 30°$ (Fig. 3(b)). We can see clearly that both have large CBGs between the first and the second bands. The band gaps in Figs. 3(a) and 3(b) are within ranges of 2.02–3.32 Hz and 2.29–3.19 Hz, whose widths are 1.30 Hz and 0.90 Hz. And it is obvious that the former produces a larger band gap.

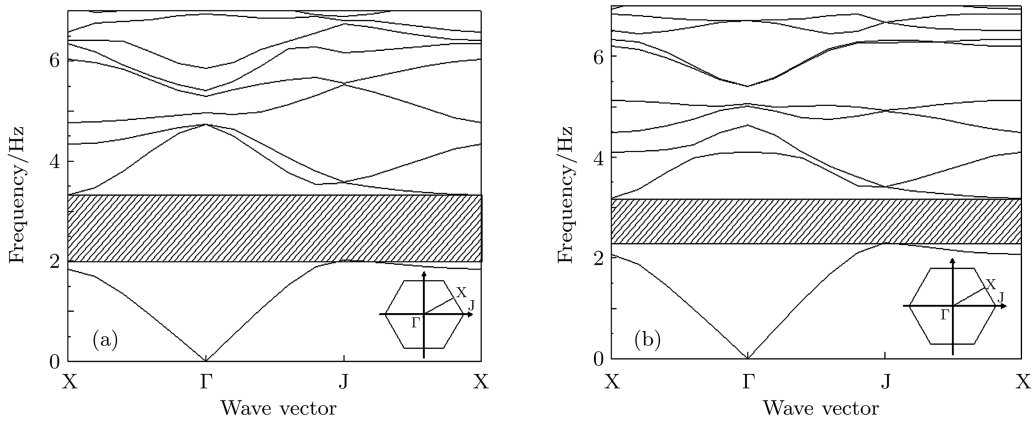
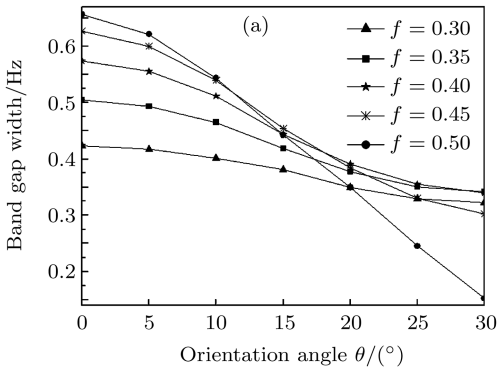


Fig. 3. Band structures of HEX holes system with $f = 0.70$. Panels (a) and (b) are for $\theta = 0°$ and $\theta = 30°$, respectively. The inset shows the first Brillouin zoom.

Then, we come to consider the dependence of CBGs on orientation angle θ for the three noncircular hole systems (in Fig. 4). Since the states of the systems are periodic when rotating the holes, the orientation angles are limited only within the non-repeated range.[5,23] Figure 4(a) is for the case of TRI holes, where the five curves correspond to different values of filling factor f , i.e. $f = 0.30$, 0.35, 0.40, 0.45, and 0.50. It is clear that the CBGs approach the maximal width at orientation angle $\theta = 0°$, and decrease significantly with the value of θ increasing, then reach the minima at $\theta = 30°$. The larger the value of f is, the more remarkable the change of width with the value of θ increasing will be. Figure 4(b) represents the systems of SQU holes for $f = 0.400$, 0.425, 0.450, 0.475, and 0.500, from which it follows that the width of band gap increases monotonically with the value of θ increasing and peaks at $\theta = 15°$. Relatively, the band gap width changes a little because of a small non-repeated θ span. Figure 4(c) shows the case of HEX holes when $f = 0.50$, 0.55, 0.60, 0.65, and 0.70, in which the curves decrease monotonically with the value of θ increasing, just similar to those in Fig. 4(a).

It should be pointed out that for the TRI and HEX (SQU) systems, at the orientation angles of $\theta = 0°$ and $\theta = 30°$ ($\theta = 0°$ and $\theta = 15°$), the space groups constituted are of the highest symmetry among those at other general orientations.[5] It can be seen that both the maximal and the minimal CBGs appear under the highest symmetry at any given filling factor. However, we should notice that the above special orientation angles may be unable to reach when the filling factor is sufficiently large enough because of geometry limit. The maximal or minimal CBGs may appear at the close-packed orientation angles.



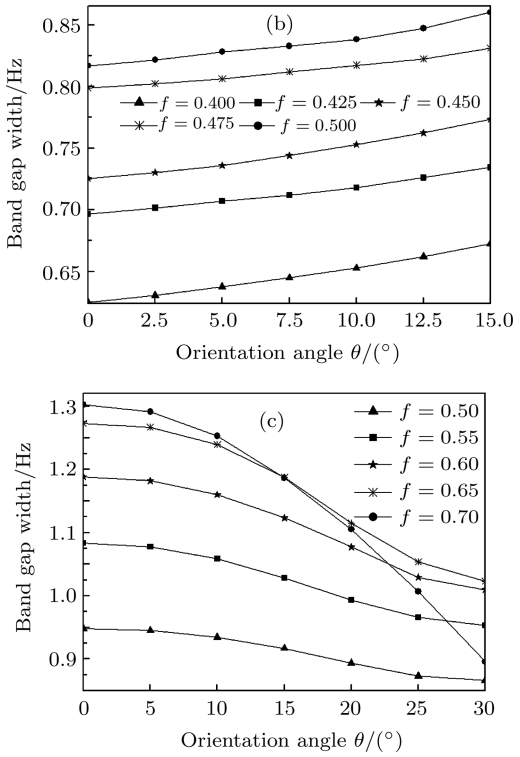


Fig. 4. Dependences of band gap width on orientation angle (θ) for the three noncircular hole systems. Panels (a), (b) and (c) show the cases of TRI, SQU and HEX holes, respectively.

As discussed above, the maximal CBGs at any given filling factor can be achieved at $\theta = 0^\circ$ ($\theta = 15^\circ$, $\theta = 0^\circ$) or at the close-packed orientation for TRI (SQU, HEX) systems. Next we plot the curves for the maximum gap width against f for the above four patterns in Fig. 5. All the curves increase steadily and then decrease gradually with the value of f increasing. At a relatively small value of f the effect due to the different shapes of holes is so small that the four curves almost superposed together. As the value of f increases, the four curves are separated from each other gradually. The curves from top to down present the systems of HEX, CIR, SQU and TRI holes, respectively. The peaks of curves or the largest CBGs produced in the sequence of decrease are for HEX, CIR, SQU and TRI patterns. It is worth while to notice that the allowed maximal filling factors due to the limitation on geometric structures for HEX, CIR, SQU and TRI holes in triangular lattice are 1.0, 0.906, 0.866 and 0.667, respectively. It seems that the larger the maximal filling factor achieved is, the wider the band gap produced will be.

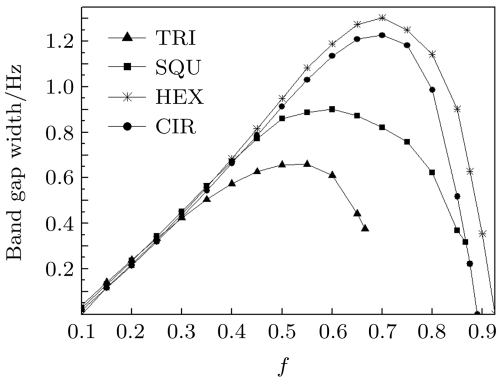


Fig. 5. Curves for widths of maximal CBGs at any given filling factor versus filling factor (f) for the four patterns.

4. Conclusions

The CBGs of shallow water surface waves propagating over 2D periodically drilled bottoms are studied numerically using the PWE method. The drilled patterns include four different shapes of holes (with TRI, SQU, HEX and CIR cross-sections) arranged into triangular lattices. Results show that for each noncircular drilled pattern, the width of CBG can be changed by adjusting the orientation of the holes. Furthermore, we find that the largest CBG in any given pattern rises with the allowed maximal filling factor, that is to say, for the triangular lattice the pattern with HEX holes gives the largest CBG. These results will conduce to finding large CBGs in water surface waves.

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