Cooperation influenced by the correlation degree of two-layered complex networks in evolutionary prisoner's dilemma games*

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An evolutionary prisoner's dilemma game is investigated on two-layered complex networks respectively representing interaction and learning networks in one and two dimensions. A parameter q is introduced to denote the correlation degree between the two-layered networks. Using Monte Carlo simulations we studied the effects of the correlation degree on cooperative behaviour and found that the cooperator density nontrivially changes with q for different payoff parameter values depending on the detailed strategy updating and network dimension. An explanation for the obtained results is provided.

Keywords: prisoner's dilemma game, two-layered complex networks, cooperation
PACC: 0250, 0175, 0650

1. Introduction

Cooperative behaviour among selfish individuals is ubiquitous in natural and social systems.[1] It is, however, difficult to explain why individuals should cooperate. Evolutionary game theory can describe the characteristic interactions among individuals.[2−5] Its most prominent application is the quest for the origins and evolution of cooperation.[5,6] In the traditional prisoner's dilemma game (PDG), defecting individuals always have a higher fitness than cooperators. Cooperation is not an evolutionary stable strategy.[5] Hence, during the last decades, several prominent mechanisms including kin selection,[7] direct and indirect reciprocity,[8−11] network (or spatial) reciprocity,[12−14] and group selection,[15] have been proposed to explain the emergence and persistence of cooperation in various types of games.

Recently, complex networks have provided a natural framework for describing the population structure, and much attention has been attracted to the study of evolutionary games on complex networks or in structured population[13,16−29] by considering the fact that who-meets-whom is determined by spatial relationships or underlying networks. It has been shown that, spatial games (i.e. network reciprocity) can lead to cooperation in the absence of any strategic complexity.[5,12−14,30] The sites on networks are occupied by individuals who are cooperators or defectors, and can interact and collect payoffs from their neighbours (interaction neighbours) by playing PDG.[6,13,14,17,30] Also, according to the payoff difference between the individual and its randomly chosen neighbour (learning neighbour), the individual possibly updates its strategy by learning the chosen neighbour's strategy.[30]

In most studies, the network composed of interaction neighbours (INs) is the same as that composed of learning neighbours (LNs). However, in real systems, the INs and LNs are always different.[31,32] Therefore, it is natural to ask whether the difference between the IN and LN has some constructive influences to the evolution of cooperation. In a recent paper, Ifti *et al.*[31] studied the continuous PDG when the IN and LN are different in the lattice topology and found that, when the neighbourhood sizes for "interacting" and "learning" differ by more than 0.5, cooperation is not sustainable.[31] The PDG on two-layered networks, where the players can follow only one of the two pure strategies and the LNs for the individuals are inhomogeneous, has been studied by Wu *et al.*[32] They found that cooperation is modified substantially in a way resembling a coherence–resonance-like behaviour when the number of learning edges is increased.[32]

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We ask how the difference between IN and LN influences the evolution of cooperation when the players can follow only one of the two pure strategies. In this paper, we study the effects of the correlation degree between LN and IN on the cooperative behaviour by introducing a tunable parameter q. We find that the correlation degree q has a remarkable effect on the evolution of cooperation, i.e. the cooperator density $\rho_{\rm C}$ changing with q represents several different behaviours which depend on the network dimension d, the cost c of the cooperative behaviour, and the detailed updating (synchronous or asynchronous). We suggest that these behaviours should be due to the effects of the correlation degree on the fraction of cooperator–cooperator links in the system.

2. Model

We consider the evolutionary PDG with pure strategists (C and D) on two-layered graphs, where the lower layer denotes the interaction network, and the upper layer represents the learning network. For simplicity, we consider fixed interaction network and variable learning network. For one-dimensional case, the interaction network is a ring under periodic boundary conditions of $N = 40000$ nodes. Each node is connected to its $2K$ ($K = 2$) nearest neighbours as its interaction neighbours. The two-dimensional interaction network is a square lattice of $N = 200 \times 200$ vertices under periodic boundary conditions. Four nearest neighbours of each player are their interaction neighbours. And the learning neighbours for one- and two-dimensional networks are obtained by replacing one by one the interaction neighbour of each player by a randomly selected individual from the system with a probability q, in which duplicate learning links are forbidden. The parameter q ($0 \leq q \leq 1$) denotes the correlation degree between the IN and LN. For $q = 0$ the IN and LN are identical (strongly correlated), while for $q = 1$, they are independent of each other (completely uncorrelated). When $0 < q < 1$, on interaction network, each player has four interaction neighbours composed of their four nearest neighbours, and on the learning network, each player has four learning neighbours composed of $4(1-q)$ nearest neighbours and $4q$ randomly selected individuals.

In the PDG, a cooperator pays a cost c ($c > 0$ and c is a small value), for another individual to receive a benefit b ($b > c$), and a defector pays no cost and cannot obtain any benefits. Thus the reward for mutual cooperation is $R = b - c$, the sucker's payoff $S = -c$, the punishment for mutual defection is $P = 0$, and the temptation to defect is $T = b$ ($1 < b < 2$). Following previous studies,[6] we set R to 1.0 so that there is only one free parameter c left in the game. At each round of the game, every player plays PDG with its four nearest neighbours on the interaction network, and collects payoffs according to the payoff matrix. The total payoff of a certain player is the sum of all its interactions. After each round of the game, the players are allowed to inspect their learning neighbours' payoffs and strategies, and according to the comparison, determine one of their strategies to be adopted in the next round. Following previous studies,[6,30] the evolution of the present system is governed by strategy adoptions between randomly chosen player i and one of its learning neighbours j. Namely, in the next round of the game, the player i will adopt its randomly selected learning neighbour j's strategy with a probability W dependent on their payoff difference $(U_i - U_j)$ as

$$W = \frac{1}{1 + \exp\left[(U_i - U_j)/\kappa\right]}, \tag{1}$$

where κ characterizes the noise introduced to permit irrational choices. $\kappa = 0$ and $\kappa \to \infty$ denote the completely deterministic and completely random selection of the neighbour's strategy, respectively. In all our following studies, we simply fix the value of κ at 0.1.

Using Monte Carlo (MC) simulations, we study the key quantity of cooperator density $\rho_{\rm C}$ in the steady state. Initially, the two strategies of C and D are randomly distributed among the individuals with equal probability 1/2. The above models were simulated with both synchronous and asynchronous updating. Eventually, the system reaches a dynamic equilibrium state. The simulation results were obtained by averaging over the last 2000 MC time steps of the total 52000. Each data point results from an average over 10 realizations.

3. Results

First we investigate the influence of the correlation degree between IN and LN in the case of one dimension on the evolution of cooperation. In Fig. 1, we show the average fraction $\rho_{\rm C}$ of cooperators in the equilibrium as a function of the cost c of cooperative behaviour for several different values of the correlation degree q. We can see from Fig. 1 that cooperator density $\rho_{\rm C}$ decreases monotonically with c, because

large cost c of cooperative behaviour benefits defection. For synchronous strategy-updating (Fig. 1(a)), if the cost c is sufficiently small ($c \leq 0.01$), the cooperation level of the system is higher for too strong and too weak correlation than that for middle level correlation, i.e., moderate correlation degree between LN and IN can essentially inhibit the cooperation. Whereas for larger values of c ($c \geq 0.018$), the highest level of cooperation occurs at the moderate correlation degree, and too strong and too weak correlations can facilitate the defection, i.e., moderate correlation degree between LN and IN can remarkably enhance the cooperation. For the case of $0.01 < c < 0.014$, the cooperator density $\rho_{\rm C}$ decreases monotonically with the values of the correlation degree q, i.e., the stronger the correlation is, the higher the cooperation level would be. When $0.014 \leq c < 0.018$, $\rho_{\rm C}$ first decreases with q and then it increases monotonically after reaching its minimum value, and then it decreases monotonically after reaching its maximum value. For asynchronous strategy-updating (Fig. 1(b)), when c is large ($c > 0.026$), the evolution of cooperation is similar to the case of large c for synchronous strategy-updating, however when $c < 0.023$, the cooperator density $\rho_{\rm C}$ decreases monotonically with q. For the moderate level ($0.023 \leq c \leq 0.026$), $\rho_{\rm C}$ first decreases with q and then it increases monotonically after reaching its minimum value, and then it decreases monotonically after reaching its maximum value.

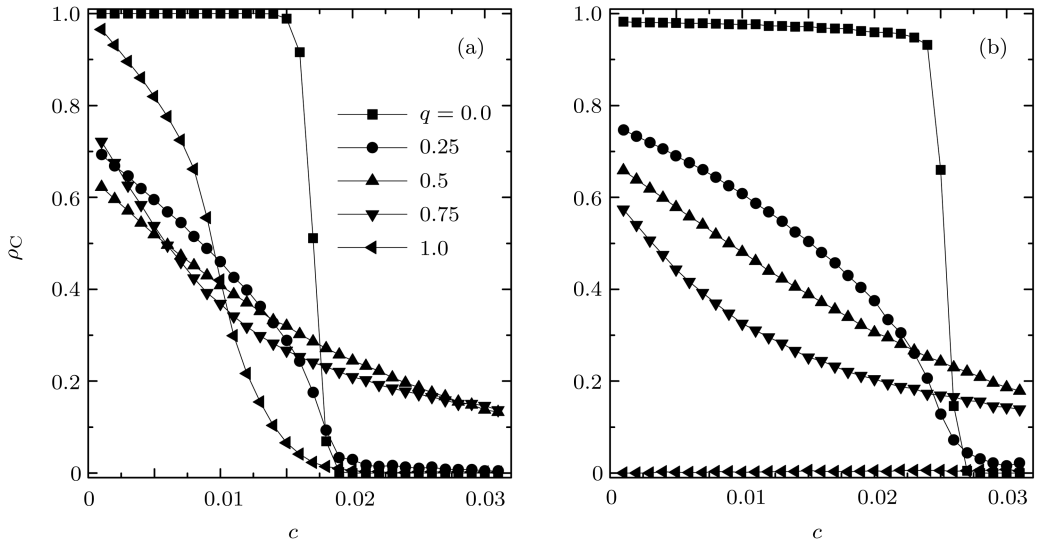


Fig. 1. Average fraction of cooperators $\rho_{\rm C}$ vs. the cost of cooperative behaviour c for several values of q in the case of one-dimensional networks. (a) and (b) respectively correspond to the results for synchronous and asynchronous strategy updating.

In Fig. 2, we plot the average fraction $\rho_{\rm C}$ of cooperators as a function of the parameter q for several different values of c by using the synchronous (Fig. 2(a)) and asynchronous (Fig. 2(b)) updating. We can see from this figure that our previous findings correctly re-emerge, i.e., the correlation degree q between LN and IN has a nontrivial influence on the evolution of cooperation, which depends strongly on the cost c of cooperative behaviour (see the curves for $c = 0.005, 0.012, 0.015, 0.02$ in Fig. 2(a), and the curves for $c = 0.005, 0.025, 0.028$ in Fig. 2(b)), and on the synchronous and asynchronous timing of strategy-updating events (see the curves for $c = 0.005, 0.012, 0.015, 0.02, 0.025$ in Figs. 2(a) and 2(b)).

Now we would study the case for two-dimensional networks. Figure 3 shows the variation of $\rho_{\rm C}$ with the cost c for several different values of q. For synchronous updating (see Fig. 3(a)), the cooperator density $\rho_{\rm C}$ increases monotonically with q for small c ($c < 0.008$), i.e., the weaker the correlation between LN and IN is, the larger the cooperator density $\rho_{\rm C}$ would be. While for large values of c ($c \geq 0.008$), too strong and too weak correlation remarkably inhibit the cooperation, and the highest level of cooperation occurs at the moderate correlation degree. For asynchronous updating (Fig. 3(b)), for large values of c ($c \geq 0.019$), the evolution of cooperation is similar to the case for synchronous updating, while for small c ($c < 0.019$), the cooperator density $\rho_{\rm C}$ decreases monotonically with q, which is contrary to the results for synchronous updating. We can conclude

from these results that the variation of cooperation with q displays several different behaviours, which depends on the cost c and the synchronous or asynchronous updating.

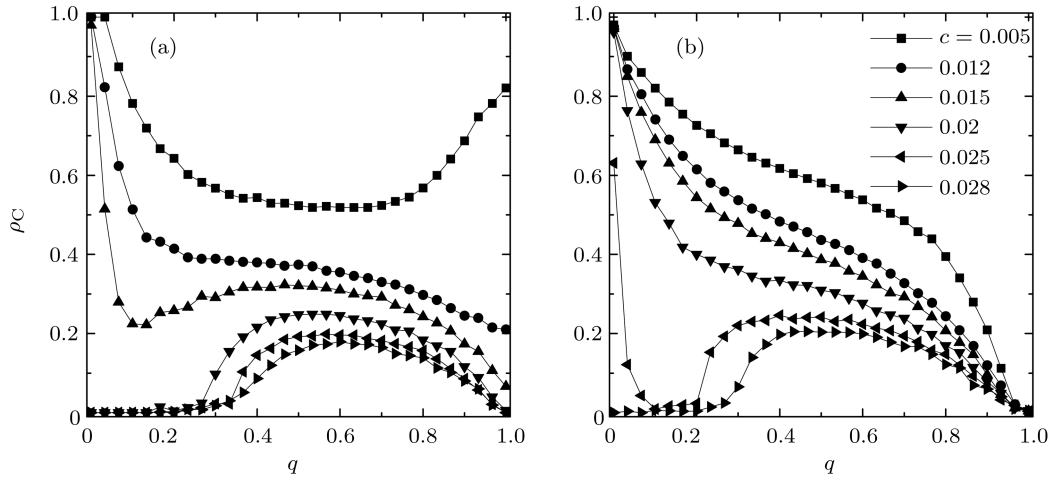


Fig. 2. Average fraction of cooperator ρ_C versus the parameter q for several different values of the cost c of cooperative behaviour in the case of one-dimensional networks. (a) and (b) respectively correspond to the results for synchronous and asynchronous strategy updating.

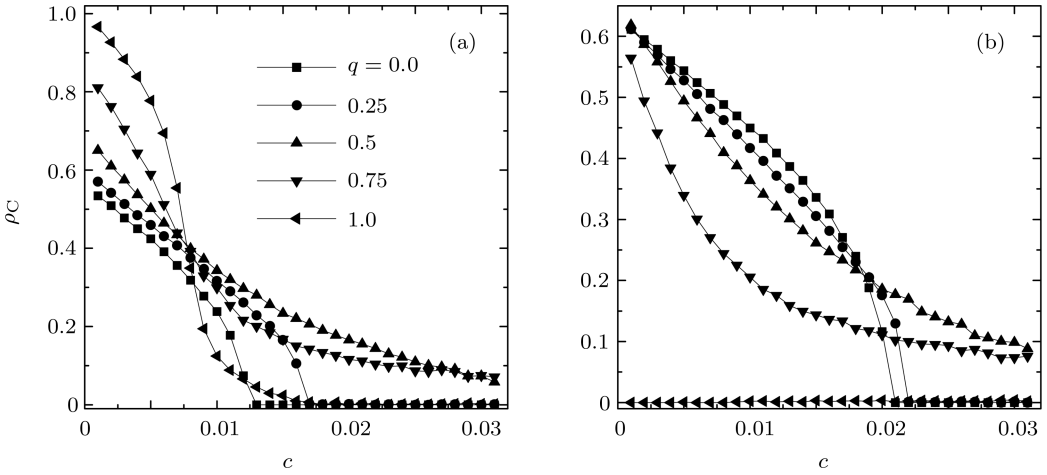


Fig. 3. Average cooperator density ρ_C versus the cost of cooperative behaviour c for several values of q in the case of two-dimensional networks. (a) and (b) respectively correspond to the results for synchronous and asynchronous strategy updating.

To investigate this in detail, we plot ρ_C versus q for different values of c by using synchronous and asynchronous updating (see Fig. 4). For synchronous updating (Fig. 4(a)), variation of the fraction ρ_C of cooperators with the correlation degree q represents two different behaviours depending on the values of c, in which re-emerge the results in Fig. 3(a). For asynchronous updating (Fig. 4(b)), there appear three different behaviours for the variation of cooperation with q: when c is small, ρ_C decreases monotonically with q, which is different from the results for synchronous updating (see the curves for $c = 0.005, 0.012, 0.015, 0.02$ in Figs. 4(a) and 4(b)). For c not too large, the changing tendency of ρ_C with q appears nonmonotonically, i.e., ρ_C first decreases with q, then it increases with q, and after that it decreases monotonically with q again, which has not been observed for synchronous updating. And for large value of c, the variation of ρ_C with q represents the same behaviours for the two types of updating. Once again, these indicate definitely that the cooperative behaviour has strong dependence on the updating rule and the cost c.

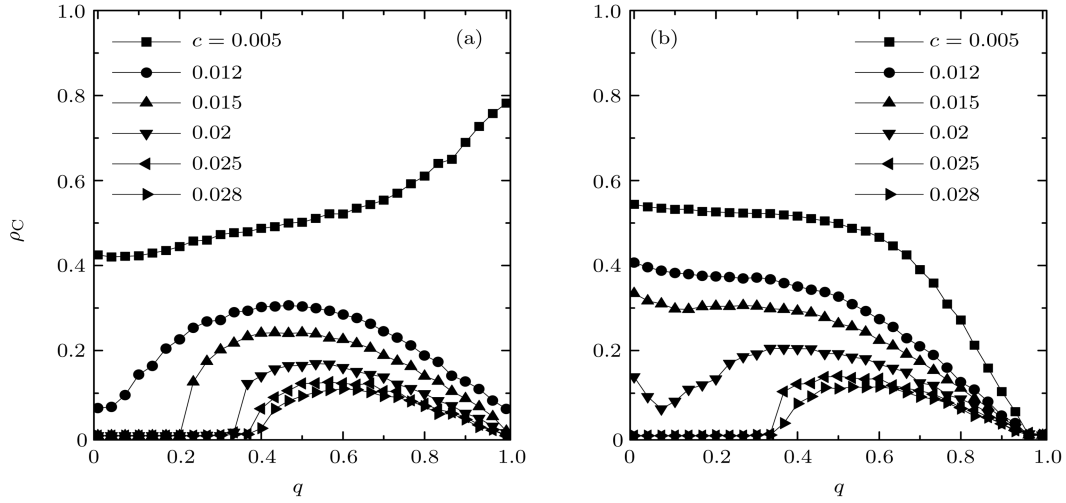


Fig. 4. Average cooperator density ρ_C versus the parameter q for several different values of the cost c of cooperative behaviour in the case of two-dimensional networks. (a) and (b) respectively correspond to the results for synchronous and asynchronous strategy updating.

In addition, by comparing the results for one-dimensional networks with that for two-dimensional networks, we can find that the evolution of cooperation with the parameter q also depends on the network dimension (see the curves for $c = 0.005, 0.012, 0.015$ in Figs. 2(a) and 4(a); the curves for $c = 0.02, 0.025$ in Figs. 2(b) and 4(b)). Thus, we can conclude from the above results that the change of ρ_C with q displays several different behaviours dependent on the cost c of cooperative behaviour, and also on whether the updating is synchronous or asynchronous, and the dimension of the networks.

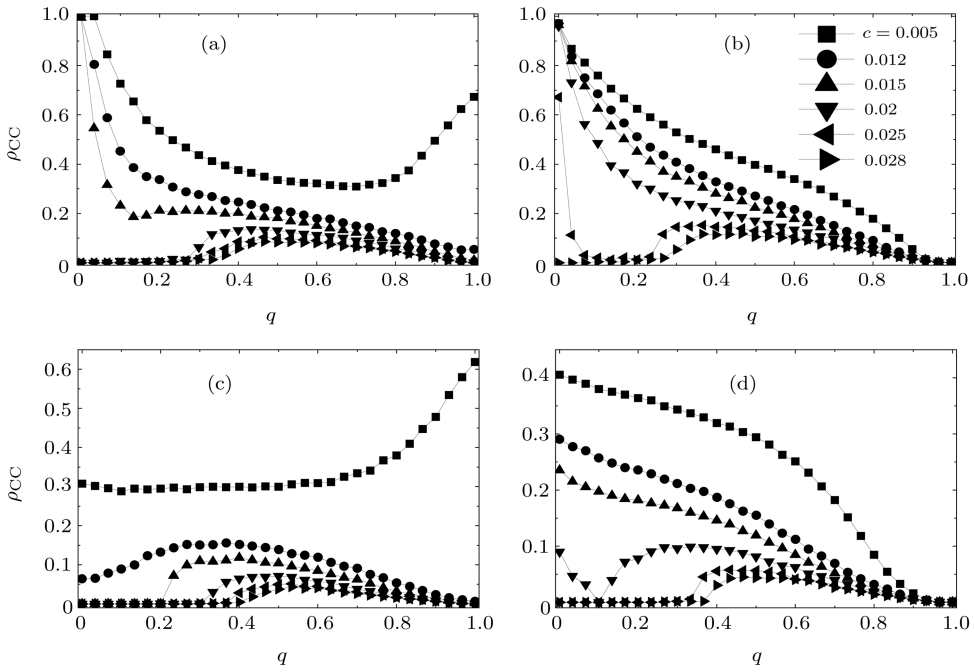


Fig. 5. Average cooperator–cooperator link density ρ_{CC} versus the parameter q for several different values of the cost c of cooperative behaviour: for (a) synchronous updating in one-dimensional networks, (b) asynchronous updating in one-dimensional networks, (c) synchronous updating in two-dimensional networks, and (d) asynchronous updating in two-dimensional networks.

To explain the above results, we investigate the average fraction ρ_{CC} of links with two cooperator ends (cooperator–cooperator link density) in the equilibrium. Figure 5 presents the variation of ρ_{CC} with the parameter q for several different values of c. Figures 5(a) and 5(c) are for synchronous updating respectively in one-

and two-dimensional networks, and figures 5(b) and 5(d) are for asynchronous updating respectively in one- and two-dimensional networks. Comparing the results in Fig. 5 with the above results in Figs. 2 and 4, we find that the cooperator density ρ_C and the fraction ρ_{CC} of cooperator–cooperator links changing with the parameter q display qualitatively the same behaviours. We know that for spatially structured populations where individuals interact and compete only within a limited neighbourhood, cooperators can form clusters and thus individuals along the boundary can outweigh their losses against defectors by gains from interactions within the cluster.[6] The introduction of the correlation degree between LN and IN clearly affects the level of cooperator–cooperator links which have positive correlation to the cooperator density. Therefore, the cooperative behaviours can be changed accordingly. We argue that these effects are general for both one- and two-dimensional networks and independent of the detailed strategy updating.

4. Conclusions

In summary, we have studied the effects of differences between the interaction and learning networks on the evolution of cooperation by introducing the correlation degree q. The results show that correlation degree has nontrivial influences on cooperative behaviours, i.e., the average cooperator density ρ_C varying with the parameter q displays several different behaviours, which are related to the payoff parameter c, to the network dimension d, and to the detailed updating. We suggest that the reason for these behaviours should be the introduction of the correlation degree between LN and IN which clearly affects the fraction of cooperator–cooperator links and which is positively correlated to the fraction of cooperators in the system. Thus, the effect of the correlation degree is a correlated factor in game dynamics, which plays an important role in the evolution of cooperation.

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