Relative energy dissipation-induced effective attraction in granular mixture*

Jia Yan(贾 燕), Yang Xian-Qing(杨先清)†, Deng Min(邓 敏), Guo Hai-Ping(郭海萍), and Ye Jian-Lan(叶见兰)

College of Science, China University of Mining and Technology, Xuzhou 221116, *China*

(Received 1 April 2010; revised manuscript received 11 June 2010)

This paper presents the investigation of the clustering of the intruders in a vertically vibrated granular bed by means of event-driven simulations. The results indicate that the position of intruders in the vertical direction is not a key factor for their aggregation. Energy dissipation of the intruders and host particles are calculated in the process of intruder-host and host–host collisions. The relative energy dissipation of the intruders to that of the host particles is obtained. We find that clustering of the intruders happens when the relative energy dissipation is negative. The conclusion is verified when the restitution coefficient, density and diameter of the intruders are varied.

Keywords: granular mixture, energy dissipation, cluster, event-driven simulations

PACC: 8220M, 8220W, 4610

1. Introduction

Granular materials are collections of discrete macroscopic solid grains that can move independently. Despite this seeming simplicity, granular materials exhibit a series of fascinating effects due to the dissipative nature of forces acting on interacting grains, such as inelastic collisions and friction. Due to their importance in industries and ubiquitous existence in our daily lives, in the past two decades, they have attracted a large amount of research attention, and are becoming one of the most active research fields.[1−3]

Segregation phenomenon exists extensively in granular materials of different types as described in Ref. [4]. For instance, in a vibrated granular system, depending on subtle variations in physical conditions, large particles can rise to the top or sink to the bottom, or show other patterns. The first two are known as "Brazil Nut effect"(BNE) and "anti-Brazil Nut effect", respectively.[5−10] Large amounts of factors, associated with external physical conditions or particle-specific properties, can play significant roles in segregation.[11,12]

Clustering phenomenon was first observed in the system of freely cooling gas in the absence of gravity and energy input.[13] Granular gases tend to spontaneously separate into dense and dilute regions after a number of collisions. When granular gases are put into a system of two equally sized compartments separated by a wall of finite height and driven at a suitable shaking, clustering phenomenon could be investigated.[14−16]

More recently, clustering phenomenon has also been observed in sufficiently fluidized beds where the BNE has been shown to depend only on the density of intruder particles.[17] Over a wide range of vibration conditions, the movement of intruders is driven by buoyancy. In this case, the intruders with similar density to the bed are not driven to the top or bottom, but fluctuate extensively around a mean height approximately half-way up to the bed.[18,19] Simulation and experimental results show that the neutrally buoyant intruders tend to cluster.[17,20] The clustering depends only on the shock waves containing a peak of kinetic energy, produced by shaking the beds. While the shock waves propagate upwards through the bed, the kinetic energy between the intruders is different from that outside them. The gradient of the kinetic energy leads to attraction of the intruders.[21] Therefore, the relative size of the attraction and repulsion can be adjusted by the use of a non-sinusoidal wave form and the restitution coefficient between the intruder and host particle.[22,23] Both clustering and

*Project supported by the Science Fund for Creative Research Groups of the National Natural Science Foundation of China (Grant No. 50921002).

†Corresponding author. E-mail: xianqyang@yahoo.com.cn

© 2010 Chinese Physical Society and IOP Publishing Ltd

segregation concern interactions of intruder–intruder and intruder–host particles. Separation of granular particles may be applied to many industrial processes, and a completely satisfactory explanation is highly desired. Investigations of clustering phenomenon can lead to further understanding of the segregation of granular mixture.

The goal of this paper is to investigate the granular clustering by varying the restitution coefficient, density and diameter of the intruder respectively. The paper is organized as follows. Section 2 is devoted to the description of the model. In Section 3, the simulation results are analysed, and the relative energy dissipation of the intruders is introduced to explain clustering of the intruders. The conclusions are given in Section 4.

2. Description of the system

We adopt an event-driven algorithm[24,25] to simulate a system of $N_{\rm s} = 1000$ small grains (host particles) and $N_{\rm l} = 11$ large grains (intruders). The density and diameter of the host particle are denoted by $\rho_{\rm s}$ and $D_{\rm s}$ respectively. Here $\rho_{\rm s} = 1$ g/mm^2 and $D_{\rm s} = 2$ mm. The density and diameter of the intruder $\rho_{\rm l}$ and $D_{\rm l}$ can be changed. The present work focuses on the case $N_{\rm s} \gg N_{\rm l}$, i.e., the intruder model system. The particles are restricted to move in x–y plane, with gravity acceleration g acting in the negative y direction. They are enclosed in a rectangular box with height $L_y = 400$ mm in y direction and width $L_x = 200$ mm in x direction with periodic boundary conditions.

The system is subject to sinusoidal excitation of the container base with amplitude A and frequency f, i.e., the height of the base varies as $y = A\sin(2\pi ft)$. The base has a maximum acceleration with respect to gravity $\Gamma = A\omega^2/g$, where $\omega = 2\pi f$. The particles are fluidized by colliding with the vibrating base that oscillates with high frequency and low amplitude. Here the vibration parameters $A = 1$ mm and $f = 30$ Hz are selected.

Particles are modeled as smooth inelastic hard disks with different restitution coefficients characterizing their inelastic collisions. The restitution coefficient for intruder–intruder collisions is $e_{\rm l}$, for host–host collisions is $e_{\rm s}$, and for intruder–host collisions is denoted by $e_{\rm ls}$. To facilitate the research, we assume $e_{\rm s}$ is fixed at 0.9 and $e_{\rm l} = e_{\rm ls} = e$. We mainly investigate effects of restitution coefficient e on clustering of intruders.

3. Simulation results

The horizontal aggregation of eleven intruders in the granular bed is studied by changing the intruder–intruder and intruder–host coefficients of restitution. To clarity the cluster extent, we introduce an aggregation parameter

$$\Phi = \frac{1}{N(N-1)} \sum_{i}^{N} \sum_{j \neq i}^{N} |r_i - r_j|$$

as defined in Ref. [22]. The quantity r_i is the position of the intruder from the centre of the bed and N is the number of the intruders. Φ means a measure of the average distance between two intruders. It has recently been shown that neutrally buoyant intruders in a two-dimensional granular bed attract each other over a distance of up to five intruder diameters.[17] In this consideration, if Φ is smaller than five intruder diameters, the intruders cluster, otherwise they will not aggregate.

First, we study effects of restitution coefficient on clustering of intruders, as shown in Figs. 1 and 2. In the process of simulations, the eleven intruders are placed randomly in the bed, but do not be overlapped. The aggregation parameter Φ is measured when the bed is vibrating and the height of the base is $y = 0$. To obtain accurate results, simulation results are calculated by averaging over 300 seconds and 10 different initial configurations of the bed when the system is in the steady state after initial 200 seconds. Figure 1 indicates that the aggregation parameter Φ decreases with the increase of e when the intruder density $\rho_{\rm l} = 1$ g/mm^2 and the diameter $D_{\rm l} = 6$ mm. Figure 2 shows typical configurations of the granular bed for different values of e. For $e = 0.9$ and $e = 0.85$, the intruders form a tightly bound cluster and remain bunched over the period of the measurement. For $e = 0.8$, the tendency for the intruders to cluster is much reduced. However, for $e = 0.75$, there is no tendency for the intruders to cluster. Comparing Fig. 1 with Fig. 2, it is clear that the intruders attract to each other when the aggregation parameter is smaller than five intruder diameters ($\Phi = 30$) corresponding to the cases of $e = 0.9$ and $e = 0.85$. But when the aggregation parameter is larger than five intruder diameters for $e = 0.8$ and $e = 0.75$, the intruders wander in the bed. And no cluster is observed.

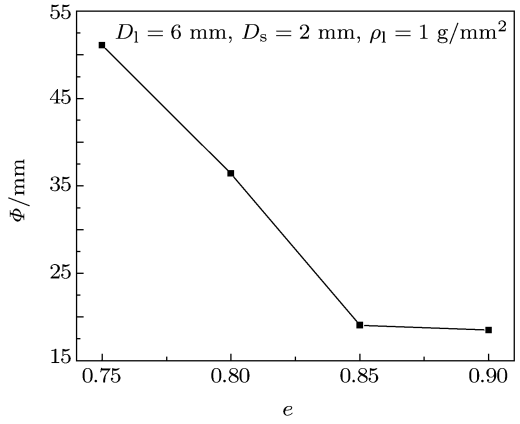


Fig. 1. The aggregation parameter Φ (in units of radius of host particles) as a function of the restitution coefficient e for the intruder diameter $D_l = 6$ mm, and density $\rho_l = 1$ g/mm^2. Φ is measured twice every period of vibration of the container basis. To obtain accurate results, simulation results are calculated by averaging over 300 seconds and 10 different initial configurations of the bed when the system is in the steady state after initial 200 seconds.

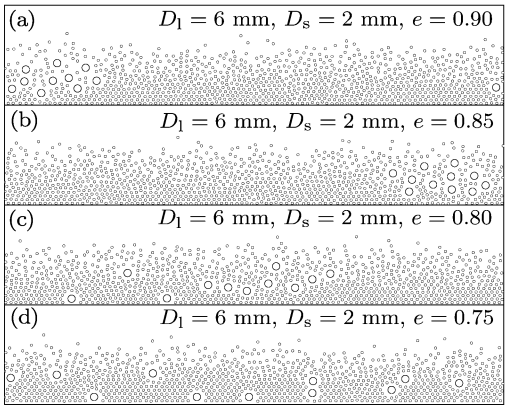


Fig. 2. Four typical configurations of a bed for different values of e: (a) $e = 0.9$, (b) $e = 0.85$, (c) $e = 0.8$, and (d) $e = 0.75$.

In Fig. 3, we plot the ratio of $\langle h_l \rangle / \langle h_s \rangle$ as a function of restitution coefficient e when the intruder density $\rho_l = 1$ g/mm^2 and diameter $D_l = 6$ mm. $\langle h_l \rangle$ and $\langle h_s \rangle$ are the time-averaged vertical positions of the intruders and the host particles, respectively. As shown in Fig. 3, the average position of the intruders in the vertical direction decreases as e becomes smaller. These results are not unexpected as described in Refs. [26] and [27], where it is argued that the segregation is produced when the particles have different restitution coefficients. As we know a key factor for the intruders to cluster is that the intruders must locate around the mid-bed height in the vertical direction.[17] For intruders with small restitution coefficient, they sink to the bottom, and the reverse Brazil Nut effect comes out, so clustering becomes impossible. We propose an assumption that intruders with small restitution coefficient would cluster if their average height is about half of the bed.

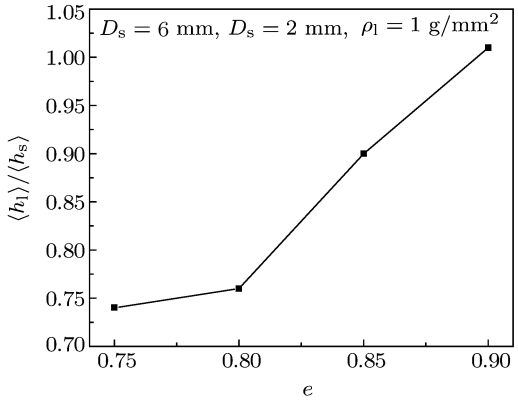


Fig. 3. The ratio of $\langle h_l \rangle / \langle h_s \rangle$ as a function of the restitution coefficient e for the intruder diameter $D_l = 6$ mm, and density $\rho_l = 1$ g/mm^2.

To compensate the descending of intruders with smaller restitution coefficients, we try to verify their density. Figure 4 denotes the ratio of $\langle h_l \rangle / \langle h_s \rangle$ as a function of the intruder density ρ_l when the restitution coefficient $e = 0.75$. From Fig. 4, we can find that the reducing of the intruder density does raise the height of the intruders. As the intruder density $\rho_l = 0.9$, the ratio $\langle h_l \rangle / \langle h_s \rangle$ is around 1, the intruders locate near the middle of the bed on the average. With further decreasing of the intruder density, the intruders can rise to the top of the bed. The conclusion is consistent with the results of Ref. [18]. Therefore, figures 3 and 4 demonstrate that both the restitution coefficient and buoyancy control vertical segregation for fluidized beds.

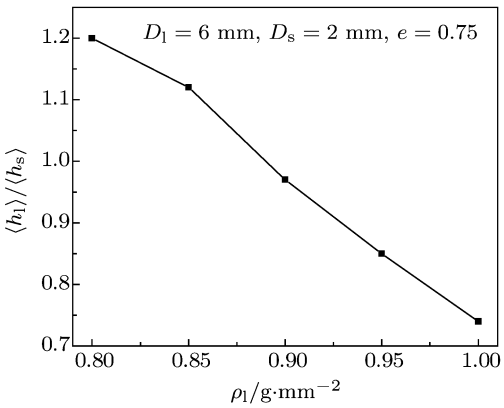


Fig. 4. The ratio $\langle h_l \rangle / \langle h_s \rangle$ as a function of the intruder density ρ_l for the intruder diameter $D_l = 6$ mm, and the restitution coefficient $e = 0.75$.

Figure 5 shows the parameter Φ as a function of the intruder density ρ_l when the restitution coefficient

$e = 0.75$ and the intruder diameter $D_\mathrm{l} = 6$ mm. In Fig. 5, the value of the parameter Φ fluctuates around 50 mm when the intruder density varies. It is obviously shown that the cluster does not appear even if the height ratio $\langle h_\mathrm{l}\rangle/\langle h_\mathrm{s}\rangle$ is around 1. Therefore, the relative average height of the intruders is not a determining factor for the intruders to cluster.

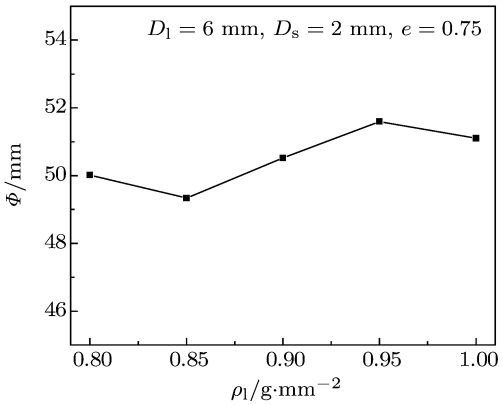


Fig. 5. The aggregation parameter Φ as a function of the intruder density ρ_l for $D_\mathrm{l} = 6$ mm and $e = 0.75$.

In the granular system, particles lose energy while they are colliding with other particles due to inelastic collisions. When the two particles lose little energy in the process of collisions, they will be relatively active and push each other far away, as if there is an effective repulsive force between them. However, if their energy dissipation is large, they will lack vitality, and seem to be effectively attracted to each other. Therefore, if the energy dissipation for the intruder–host particle collisions is smaller than that for the host–host particle collisions, the intruders can be effectively pushed away by the host particles and tend to form one cluster. To describe the energy dissipation quantitatively, the energy dissipation $\eta = (\Delta E_\mathrm{l} - \Delta E_\mathrm{s})/\Delta E_\mathrm{l}$ is introduced. $\Delta E_\mathrm{l} = m_\mathrm{l}(v_\mathrm{l0}^2 - v_\mathrm{l}^2)/2$ is the average energy loss of the intruder in the process of the intruder–host collision, where m_l is the mass of the intruder, v_l0 and v_l are the velocities before and after the collision separately. $\Delta E_\mathrm{s} = m_\mathrm{s}(v_\mathrm{s0}^2 - v_\mathrm{s}^2)/2$ is the average energy loss of one host particle in the process of the host–host collision. For the host particle, the meanings of m_s, v_s0 and v_s are similar to those for the intruder. The parameters ΔE_l and ΔE_s are obtained by averaging over 280 seconds. The term $\Delta E_\mathrm{l} - \Delta E_\mathrm{s}$ denotes the relative energy dissipation of the intruder to that of the host particle when they collide with one host particle respectively. To get a non-dimensional parameter, we divide the term by ΔE_l.

Figure 6 shows how relative energy dissipation of the intruders η evokes clustering of intruders as the restitution coefficients for host–host and intruder–host vary when the intruder density $\rho_\mathrm{l} = 1$ g/mm^2 and diameter $D_\mathrm{l} = 6$ mm. The values of η decrease with the increase of the restitution coefficient e as shown in Fig. 6. For example, when the restitution coefficient $e = 0.75$ or 0.8, the value of η is positive. But when $e = 0.85$ or 0.9, the value of η is negative. Comparing Fig. 1 with Fig. 6, we can find that when $\eta > 0$, the intruders cannot form cluster. However, when $\eta < 0$, the intruders can congregate.

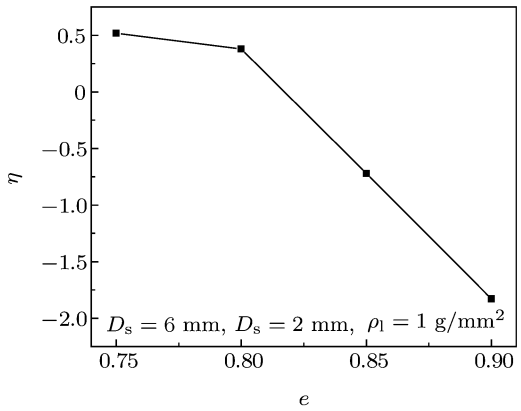


Fig. 6. The relative energy dissipation of the intruders η as a function of the restitution coefficient e for $N_\mathrm{l} = 1$, $D_\mathrm{l} = 6$ mm and $\rho_\mathrm{l} = 1$ g/mm^2.

For fluidized beds, particles collide with each other frequently. In the process of the intruder–host and host–host collisions, when $\eta > 0$, the energy loss of the intruders ΔE_l is larger than that of the host particles ΔE_s. The host particles can effectively attract the intruder and gather around the intruders. Thus the cluster of the intruders cannot be observed. However, when $\eta < 0$, the energy loss of the intruders ΔE_l is smaller than that of the host particles ΔE_s. In this case, the effectively attractive force between the host particles makes the intruder cluster possible. And once the intruders get together, they will never move apart.

The results of Fig. 5 can also be explained by the relative energy dissipation of the intruders η, as shown in Fig. 7 where is shown η as a function of the intruder density ρ_l when $e = 0.75$ and $D_\mathrm{l} = 6$ mm. In Fig. 7, the values of η are positive, and increase with the increase of the intruder density ρ_l. According to the above discussions of η, the cluster of the intruder is not obtained. Especially, when the intruder density $\rho_\mathrm{l} = 0.9$ g/mm^2, the height of the intruders is about half of the bed (shown in Fig. 4), the intruders cannot

gather together.

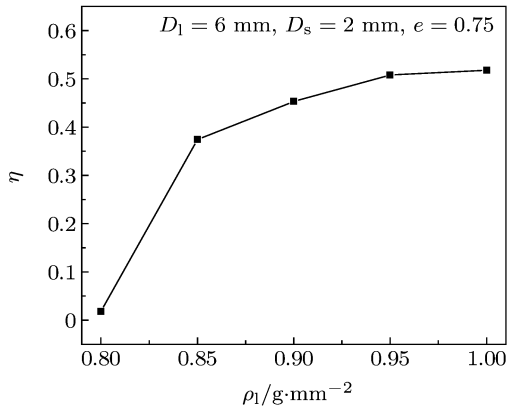


Fig. 7. The relative energy dissipation η as a function of the intruder density ρ_l for $N_l = 1$, $D_l = 6$ mm and $e = 0.75$.

To confirm the effects of the restitution coefficient on the effective attraction between particles, we calculate the probability distribution $P(r)\mathrm{d}r$ of finding the host particles at a distance between r and $r + \mathrm{d}r$ from the intruder. In simulations, we adopt one intruder system, i.e., $N_l = 1$. The distribution curves of $P(r)$ are calculated by averaging over data measured during 300 seconds after the system is in the steady state. The results are shown in Fig. 8.

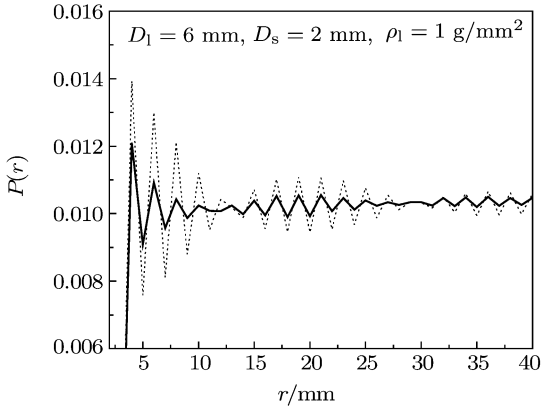


Fig. 8. The probability density $P(r)$ for host particle around an intruder as a function of r. The solid line shows the data for $e = 0.9$, while the dot line shows the data for $e = 0.75$. The intruder diameter $D_l = 6$ mm, and the intruder density $\rho_l = 1$ g/mm^2.

For separations greater than 40 host–particle radius the probability is roughly constant, indicating the distribution of the host particles is uniform. For smaller separations the oscillations in $P(r)$ are exhibited and indicate correlation effects mediated by the host particles. In Fig. 8, we observe that the values of the probability $P(r)$ for the restitution coefficient $e = 0.75$ are obviously larger than those for $e = 0.9$. For smaller restitution coefficient, more host particles can gather around the intruder. The situation demonstrates that the effective attraction between the intruder and host particle becomes stronger with the decreasing of the restitution coefficient.

The effect of changing the intruder diameter on the clustering of the intruders is also investigated, the results are shown in Fig. 9. Figure 9 indicates the relations of the relative energy dissipation η with the diameter D_l. For the diameter of the intruder $D_l = 8$ mm, the relative energy dissipation η is negative. However, when $D_l = 4$ mm and $D_l = 6$ mm, the values of η are both positive. According to the above discussion, we can deduce that the cluster of the intruders is obtained only for the case of $D_l = 8$ mm. The results can be verified by the parameter Φ, as shown in the inset of Fig. 9. The parameter Φ decreases with the increase of the intruder diameter D_l. For the case of $D_l = 8$ mm, the value of the aggregation parameter Φ is about 30, and smaller than five intruder diameters, corresponding to the congregation of the intruders. But for the cases of $D_l = 4$ mm and $D_l = 6$ mm, larger values of Φ demonstrate that the intruders do not aggregate. Therefore, figure 9 can further indicate that the relative energy dissipation of the intruder is the key factor for clustering.

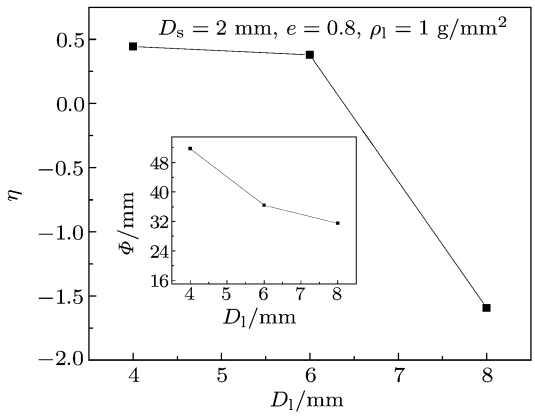


Fig. 9. The relative energy dissipation η as a function of the intruder diameter D_l for $N_l = 1$. The inset shows the aggregation parameter Φ as a function of D_l. The restitution coefficient $e = 0.8$, and the intruder density $\rho_l = 1$ g/mm^2.

4. Conclusions

We use an event-driven algorithm to investigate the collection of the intruder particles in a two-dimensional granular system by varying the restitution coefficient, density and diameter of the intruders.

The results indicate that the intruders can form a cluster when the restitution coefficient e is large enough. However, for a small restitution coefficient, the cluster vanishes with the intruders straying horizontally in the bed, but we find that the mean height of the intruders decreases with the decreasing of the restitution coefficient when the bed is highly vibro-fluidized. Experimental results indicate that the intruders aggregate when they remain on average at mid-bed height.[17] To avoid the effects of the intruder height on the cluster, we make the intruders half way up to the bed by reducing the density of the intruders. Under the circumstances the cluster is not observed yet. Therefore, the relative average height of the intruders is not a determining factor for the intruders to cluster.

To further investigate the factor which determines whether the intruders cluster or not, we introduced the relative energy dissipation η of the intruders in the process of the intruder–host and host–host particle collisions. The η demonstrates the relative energy loss of intruder to that of host particle when they collide with one host particle respectively. The results show that in the case of $\eta > 0$, the energy loss of the intruder is larger than that of the host particle. Effective attraction exists between the host particle and intruder. Therefore the intruders tend to approach the host particles, and cannot aggregate together. However when $\eta < 0$, the energy loss of the intruder is smaller than that of the host particle. The intruders can be effectively pushed away by the host particles and tend to form one cluster. We calculate the values of η in the process of collision by changing the restitution coefficient, density and the diameter of the intruders. The results are consistent with the conclusions we draw that the energy dissipation induces clustering for granular mixture.

References

[1] Jaeger H M, Nagel S R and Behringer R P 1996 *Rev. Mod. Phys.* **68** 1259

[2] Kadanoff L P 1999 *Rev. Mod. Phys.* **71** 435

[3] Aranson I S and Tsimring L S 2006 *Rev. Mod. Phys.* **78** 641

[4] Kudroll A 2004 *Rep. Prog. Phys.* **67** 209

[5] Duran J, Rajchenbach J and Clement E 1993 *Phys. Rev. Lett.* **70** 2431

[6] Knight J B, Jaeger H M and Nagel S R 1993 *Phys. Rev. Lett.* **70** 3728

[7] Shinbrot T and Muzzio F J 1998 *Phys. Rev. Lett.* **81** 4365

[8] Hong D C, Quinn P V and Luding S 2001 *Phys. Rev. Lett.* **86** 3423

[9] Mobius M E, Lauderdale B E, Nagel S R and Jaeger H M 2001 *Nature* (London) **414** 270

[10] Yan X, Shi Q, Hou M, Lu K and Chan C K 2003 *Phys. Rev. Lett.* **91** 014302-1

[11] Hu M B, Kong X Z, Wu Q S and Wu Y H 2005 *Chin. Phys.* **14** 1844

[12] Xia J H, You Y W, Wang P P, Wang W L and Liu C S 2010 *Chin. Phys.* B **19** 056404

[13] Goldhirsch I and Zanetti G 1993 *Phys. Rev. Lett.* **70** 1619

[14] Eggers J 1999 *Phys. Rev. Lett.* **83** 5322

[15] Weele van der K, Meer van der D, Versluis M and Lohse D 2001 *Europhys. Lett.* **53** 328

[16] Meer van der D, Weele van der K and Lohse D 2002 *Phys. Rev. Lett.* **88** 174302-1

[17] Sanders D A, Swift M R, Bowley R M and King P J 2004 *Phys. Rev. Lett.* **93** 208002-1

[18] Huerta D A and Ruiz-Suarez J C 2004 *Phys. Rev. Lett.* **92** 114301-1

[19] Yang X Q, Zhou K, Qiu K and Zhao Y M 2006 *Phys. Rev.* E **73** 031305-1

[20] Sanders D A, Swift M R, Bowley R M and King P J 2006 *Europhys. Lett.* **73** 349

[21] Lui L T, Swift M R, Bowley R M and King P J 2008 *Phys. Rev.* E **77** 020301-1

[22] Sanders D A, Swift M R, Bowley R M and King P J 2006 *Appl. Phys. Lett.* **88** 264106

[23] Lui L T, Swift M R, Bowley R M and King P J 2007 *Phys. Rev.* E **75** 051303-1

[24] Lubachevsky B D 1991 J. *Comput. Phys.* **94** 255

[25] Herrmann H J and Luding S 1998 *Continuum Mech. Themodyn.* **10** 189

[26] Brito R, Enriquez H, Godoy S and Soto R 2008 *Phys. Rev.* E **77** 061301-1

[27] Serero D, Goldhirsch I, Noskowick S H and Tan M L 2006 *J. Fluid Mech.* **554** 237