Epitaxy surface effect on the first-order phase transition properties in a ferroelectric thin film*

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By modifying the interchange interactions and the transverse fields on the epitaxy surface layer, this paper studies the phase transition properties of an n-layer ferroelectric thin film by the Fermi-type Green's function technique based on the transverse Ising model with a four-spin interaction. The special attention is given to the effect of the epitaxy surface layer on the first-order phase transition properties in the parameter space constructed by the ratios of the bulk transverse field and the bulk four-spin interaction to the bulk two-spin interaction with the framework of the higher-order decoupling approximation to the Fermi-type Green's function. The results show that the first-order phase transition properties will be changed significantly due to the modification of interchange interaction and transverse field parameters on the epitaxy surface layer. The dependence of the first-order phase transition properties on the thickness of ferroelectric thin films is also discussed.

Keywords: ferroelectric thin film, phase diagram

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1. Introduction

Surface and size effects on phase transitions in ferroelectric thin films have been the subject of growing interest in recent years with the rapid development in the preparation technology and practical application of ferroelectric materials.[1] Depending on the sample size and the surface preparation, the Curie temperature and the polarization can be significantly altered in finite size ferroelectric materials. On the theoretical treatment, the Landau phenomenological theory[2−7] and the transverse Ising model[8−15] have been maturely developed for the investigation of the second-order phase transition properties in ferroelectric thin films due to the surface modification. However, the researches on the first-order phase transition properties in ferroelectric thin films are relatively much less than those on the second-order phase transition. Scott *et al.*[16] as well as Tan *et al.*[17] and Lin *et al.*[18] had studied the ferroelectric thin film with a first-order phase transition by the phenomenological theory. Many researchers had used the transverse Ising model with a four-spin interaction to study the first-order phase transition properties in ferroelectric thin films. With the usual mean-field approximation to the transverse Ising model with a four-spin interaction, the first-order phase transition in ferroelectric thin films[19,20] or ferroelectric superlattices[21,22] was systematically studied. Using the Green's function technique, Wang *et al.*[23] and Wesselinowa[24] investigated the first-order phase transition properties in order–disorder ferroelectrics and ferroelectric thin films respectively. Adopting the higher-order approximation to the Fermi-type Green's function, Teng and Sy[25] studied the first-order phase transition properties in the parameter space with respect to the ratios of the transverse field and the four-spin interaction to the two-spin interaction for ferroelectric thin films with the uniform surface and bulk parameters. However, the effect of interchange interactions and transverse fields on the epitaxy surface layer on the first-order phase transition properties has not been calculated numerically.

In the present work, interchange interactions and transverse fields on the epitaxy surface layer for an

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n-layer ferroelectric thin film are assumed to be different from those in the bulk layers. The effect of various interaction parameters on the epitaxy surface layer on the first-order phase transition properties of the system will be studied in the parameter space with respect to the ratios of the bulk transverse field and the bulk four-spin interaction to the bulk two-spin interaction as the function of thickness parameter n by a higher-order decoupling approximation to the Fermi-type Green's function. The dependence of the first-order phase transition properties on the interchange interaction and the transverse field on the epitaxy surface layer as well as the thickness of the ferroelectric thin film is obtained. Simultaneously, the second-order and the paraelectric phase transition properties are also discussed.

2. The model and the Green's function

The Hamiltonian of the transverse field Ising model with a four-spin interchange interaction term is[25,26]

$$H = -\sum_i \Omega_i S_i^x - \sum_{\langle i,j\rangle} J_{ij}^{(2)} S_i^z S_j^z - \sum_{\langle i,j,k,l\rangle} J_{ijkl}^{(4)} S_i^z S_j^z S_k^z S_l^z, \tag{1}$$

where Ω_i is the transverse field, S_i^x and S_i^z are the x and z components of pseudospin-1/2 operator at site i, $J_{ij}^{(2)}$ and $J_{ijkl}^{(4)}$ are the two-spin and four-spin interchange interaction, $\sum_{\langle i,j\rangle}$ and $\sum_{\langle i,j,k,l\rangle}$ run over only the nearest-neighbouring pairs and round-a-plaquette spins, respectively. In order to distinguish the surface layers from the bulk layers, transverse fields and interaction parameters are defined as

$$\Omega_i = \begin{cases} \Omega_{\rm s}, & i \text{ on the surface,} \\ \Omega_{\rm b}, & i \text{ in the bulk,} \end{cases} \tag{2}$$

$$J_{ij}^{(2)} = \begin{cases} J_{\rm s}^{(2)}, & i,j \text{ on the surface,} \\ J_{\rm b}^{(2)}, & i,j \text{ in the bulk,} \end{cases} \tag{3}$$

$$J_{ijkl}^{(4)} = \begin{cases} J_{\rm s}^{(4)}, & i,j,k,l \text{ on the surface,} \\ J_{\rm b}^{(4)}, & i,j,k,l \text{ in the bulk.} \end{cases} \tag{4}$$

According to the Hamiltonian of the transverse Ising model with a four-spin interchange interaction, the higher-order equations of motion for the Fermi-type Green's function can be obtained, and consequently the polarization $\langle S^z\rangle$ and the transverse component $\langle S^x\rangle$ for an n-layer ferroelectric thin film can be given by[25]

$$\begin{aligned} \langle S_k^z\rangle &= (\sigma/2\omega_{nk})\tanh(\omega_{nk}/2k_{\rm B}T), \\ \langle S_k^x\rangle &= (\Omega_k/2\omega_{nk})\tanh(\omega_{nk}/2k_{\rm B}T), \end{aligned} \tag{5}$$

where

$$\sigma = \sum_j J_{ij}^{(2)} \langle S_j^z\rangle + \sum_{jkl} J_{ijkl}^{(4)} \langle S_j^z\rangle \langle S_k^z\rangle \langle S_l^z\rangle. \tag{6}$$

Obviously, equation (5) is a transcendental equation for the polarization $\langle S^z\rangle$ and the transverse component $\langle S^x\rangle$ with respect to the interchange interaction, transverse field, thickness, and temperature. For simplicity, we assume that the thin film has a simple cubic symmetrical structure, therefore $S_k^z = S_{n+1-k}^z$, $S_k^x = S_{n+1-k}^x$, $\omega_{n,k} = \omega_{n,n+1-k}$, $\Omega_1 = \Omega_n = \Omega_{\rm s}$, and $\Omega_k = \Omega_{\rm b}$ $(2 \le k \le n-1)$. Here the epitaxy surface is taken into account, therefore the eigenfrequencies are obviously different from those of the uniform ferroelectric thin film.[25] In this way, the eigenfrequencies for an n-layer ferroelectric thin film can be given out in terms of general expressions:

(i) for the film with one layer,

$$\begin{aligned} \omega_{11}^2 = {} & \Omega_{\rm s}^2 + 4J_{\rm s}^{(2)}\left\{\frac{J_{\rm s}^{(2)}}{4} + 3J_{\rm s}^{(2)}(S_1^z)^2 + \left[\frac{1}{2}(S_1^z)^2 + 2(S_1^z)^4\right] J_{\rm s}^{(4)} + \Omega_{\rm s} S_1^x\right\} \\ & + 4J_{\rm s}^{(4)}\left\{\frac{J_{\rm s}^{(2)}}{2}(S_1^z)^2 + 2J_{\rm s}^{(2)}(S_1^z)^4 + \left[\frac{1}{64} + \frac{1}{2}(S_1^z)^4 + (S_1^z)^6\right] J_{\rm s}^{(4)} + 3\Omega_{\rm s}(S_1^z)^2 S_1^x\right\}; \end{aligned} \tag{7}$$

(ii) for the film with two layers,

$$\omega_{21}^2 = \Omega_{\rm s}^2 + 4J_{\rm s}^{(2)}\left\{\frac{J_{\rm s}^{(2)}}{4} + 3J_{\rm s}^{(2)}(S_1^z)^2 + J_{\rm b}^{(2)}(S_1^z)^2 + \left[\frac{1}{2}(S_1^z)^2 + 2(S_1^z)^4\right] J_{\rm s}^{(4)}\right.$$

$$+\left[\frac{1}{4}(S_1^z)^2+3(S_1^z)^4\right]J_{\rm b}^{(4)}+\Omega_{\rm s}S_1^x\Bigg\}+J_{\rm b}^{(2)}\left\{\frac{J_{\rm b}^{(2)}}{4}+4J_{\rm s}^{(2)}(S_1^z)^2+4(S_1^z)^4J_{\rm s}^{(4)}+(S_1^z)^2J_{\rm b}^{(4)}+\Omega_{\rm s}S_1^x\right\}$$
$$+4J_{\rm s}^{(4)}\left\{\frac{J_{\rm s}^{(2)}}{2}(S_1^z)^2+2J_{\rm s}^{(2)}(S_1^z)^4+J_{\rm b}^{(2)}(S_1^z)^4+\left[\frac{1}{64}+\frac{1}{2}(S_1^z)^4+(S_1^z)^6\right]J_{\rm s}^{(4)}\right.$$
$$\left.+\left[\frac{1}{2}(S_1^z)^4+2(S_1^z)^6\right]J_{\rm b}^{(4)}+3\Omega_{\rm s}(S_1^z)^2S_1^x\right\}$$
$$+4J_{\rm b}^{(4)}\left\{\frac{J_{\rm s}^{(2)}}{4}(S_1^z)^2+3J_{\rm s}^{(2)}(S_1^z)^4+\frac{J_{\rm b}^{(2)}}{4}(S_1^z)^2+\left[\frac{1}{2}(S_1^z)^4+2(S_1^z)^6\right]J_{\rm s}^{(4)}\right.$$
$$\left.+\left[\frac{1}{64}+\frac{3}{4}(S_1^z)^4\right]J_{\rm b}^{(4)}+\Omega_{\rm s}(S_1^z)^2S_1^x+2\Omega_{\rm s}(S_1^z)^2S_1^x\right\};\tag{8}$$

(iii) for the film with n layers ($n\geq 3$),

$$\omega_{n1}^2=\Omega_{\rm s}^2+4J_{\rm s}^{(2)}\left\{\frac{J_{\rm s}^{(2)}}{4}+3J_{\rm s}^{(2)}(S_1^z)^2+J_{\rm b}^{(2)}S_1^zS_2^z+\left[\frac{1}{2}(S_1^z)^2+2(S_1^z)^4\right]J_{\rm s}^{(4)}\right.$$
$$\left.+\left[\frac{1}{4}(S_2^z)^2+3(S_1^z)^2(S_2^z)^2\right]J_{\rm b}^{(4)}+\Omega_{\rm s}S_1^x\right\}$$
$$+J_{\rm b}^{(2)}\left\{\frac{J_{\rm b}^{(2)}}{4}+4J_{\rm s}^{(2)}S_1^zS_2^z+4(S_1^z)^3S_2^zJ_{\rm s}^{(4)}+S_1^zS_2^zJ_{\rm b}^{(4)}+\Omega_{\rm b}S_2^x\right\}$$
$$+4J_{\rm s}^{(4)}\left\{\frac{J_{\rm s}^{(2)}}{2}(S_1^z)^2+2J_{\rm s}^{(2)}(S_1^z)^4+J_{\rm b}^{(2)}(S_1^z)^3S_2^z+\left[\frac{1}{64}+\frac{1}{2}(S_1^z)^4+(S_1^z)^6\right]J_{\rm s}^{(4)}\right.$$
$$\left.+\left[\frac{1}{2}(S_1^z)^2(S_2^z)^2+2(S_1^z)^4(S_2^z)^2\right]J_{\rm b}^{(4)}+3\Omega_{\rm s}(S_1^z)^2S_1^x\right\}$$
$$+4J_{\rm b}^{(4)}\left\{\frac{J_{\rm s}^{(2)}}{4}(S_2^z)^2+3J_{\rm s}^{(2)}(S_1^z)^2(S_2^z)^2+\frac{J_{\rm b}^{(2)}}{4}S_1^zS_2^z+\left[\frac{1}{2}(S_1^z)^2(S_2^z)^2+2(S_1^z)^4(S_2^z)^2\right]J_{\rm s}^{(4)}\right.$$
$$\left.+\left[\frac{1}{64}+\frac{3}{4}(S_1^z)^2(S_2^z)^2\right]J_{\rm b}^{(4)}+\Omega_{\rm s}(S_2^z)^2S_1^x+2\Omega_{\rm b}S_1^zS_2^zS_2^x\right\},\tag{9}$$

$$\omega_{nk}^2(k\geq 2)$$
$$=\Omega_k^2+4J_{\rm b}^{(2)}\left\{\frac{J_{\rm b}^{(2)}}{4}+3J_{\rm b}^{(2)}(S_k^z)^2+J_{\rm b}^{(2)}S_k^zS_{k-1}^z+J_{\rm b}^{(2)}S_k^zS_{k+1}^z\right.$$
$$\left.+\left[\frac{1}{2}(S_k^z)^2+2(S_k^z)^4+\frac{1}{4}(S_{k-1}^z)^2+\frac{1}{4}(S_{k+1}^z)^2+3(S_k^z)^2(S_{k-1}^z)^2+3(S_k^z)^2(S_{k+1}^z)^2\right]J_{\rm b}^{(4)}+\Omega_kS_k^x\right\}$$
$$+J_{\rm b}^{(2)}\left\{\frac{J_{\rm b}^{(2)}}{4}+4J_{\rm b}^{(2)}S_{k-1}^zS_k^z+J_{\rm b}^{(2)}S_{k-1}^zS_{k+1}^z\right.$$
$$\left.+\left[S_{k-1}^zS_k^z+4S_{k-1}^zS_k^z(S_{k+1}^z)^2+4S_{k-1}^z(S_k^z)^3\right]J_{\rm b}^{(4)}+\Omega_{k-1}S_{k-1}^x\right\}$$
$$+J_{\rm b}^{(2)}\left\{\frac{J_{\rm b}^{(2)}}{4}+4J_{\rm b}^{(2)}S_{k+1}^zS_k^z+J_{\rm b}^{(2)}S_{k-1}^zS_{k+1}^z\right.$$
$$\left.+\left[S_{k+1}^zS_k^z+4S_{k+1}^zS_k^z(S_{k-1}^z)^2+4S_{k+1}^z(S_k^z)^3\right]J_{\rm b}^{(4)}+\Omega_{k+1}S_{k+1}^x\right\}$$
$$+4J_{\rm b}^{(4)}\left\{\frac{J_{\rm b}^{(2)}}{2}(S_k^z)^2+2J_{\rm b}^{(2)}(S_k^z)^4+J_{\rm b}^{(2)}(S_k^z)^3S_{k-1}^z+J_{\rm b}^{(2)}(S_k^z)^3S_{k+1}^z\right.$$

$$+\left[\frac{1}{64}+\frac{1}{2}(S_k^z)^4+\frac{1}{2}(S_k^z)^2(S_{k-1}^z)^2+\frac{1}{2}(S_k^z)^2(S_{k+1}^z)^2+(S_k^z)^6+2(S_k^z)^4(S_{k-1}^z)^2+2(S_k^z)^4(S_{k+1}^z)^2\right]J_{\rm b}^{(4)}$$
$$+3\Omega_k(S_k^z)^2S_k^x\Bigg\}+4J_{\rm b}^{(4)}\Bigg\{\frac{J_{\rm b}^{(2)}}{4}(S_{k-1}^z)^2+3J_{\rm b}^{(2)}(S_k^z)^2(S_{k-1}^z)^2+\frac{J_{\rm b}^{(2)}}{4}S_k^zS_{k-1}^z+J_{\rm b}^{(2)}S_k^z(S_{k-1}^z)^2S_{k+1}^z$$
$$+\left[\frac{1}{64}+\frac{5}{4}(S_k^z)^2(S_{k-1}^z)^2+2(S_k^z)^4(S_{k-1}^z)^2+\frac{1}{4}(S_{k+1}^z)^2(S_{k-1}^z)^2+3(S_k^z)^2(S_{k-1}^z)^2(S_{k+1}^z)^2\right]J_{\rm b}^{(4)}$$
$$+\Omega_k(S_{k-1}^z)^2S_k^x+2\Omega_{k-1}S_k^zS_{k-1}^zS_{k-1}^x\Bigg\}$$
$$+4J_{\rm b}^{(4)}\Bigg\{\frac{J_{\rm b}^{(2)}}{4}(S_{k+1}^z)^2+3J_{\rm b}^{(2)}(S_k^z)^2(S_{k+1}^z)^2+\frac{J_{\rm b}^{(2)}}{4}S_k^zS_{k+1}^z+J_{\rm b}^{(2)}S_k^z(S_{k+1}^z)^2S_{k-1}^z$$
$$+\left[\frac{1}{64}+\frac{5}{4}(S_k^z)^2(S_{k+1}^z)^2+2(S_k^z)^4(S_{k+1}^z)^2+\frac{1}{4}(S_{k+1}^z)^2(S_{k-1}^z)^2+3(S_k^z)^2(S_{k-1}^z)^2(S_{k+1}^z)^2\right]J_{\rm b}^{(4)}$$
$$+\Omega_k(S_{k+1}^z)^2S_k^x+2\Omega_{k+1}S_k^zS_{k+1}^zS_{k+1}^x\Bigg\}. \tag{10}$$

3. Numerical results and discussion

The polarizations $\langle S_{k-1}^z\rangle$, $\langle S_k^z\rangle$, $\langle S_{k+1}^z\rangle$ and $\langle S_{k-1}^x\rangle$, $\langle S_k^x\rangle$, $\langle S_{k+1}^x\rangle$ can be solved out by recurrence algorithms in accordance with the transcendental equations (5)–(10). First, the curves of the temperature T vs. the polarization $\langle S^z\rangle$ for different parameters can be calculated numerically.[25] According to Refs. [23] and [25], when $\left(\partial^2T/\partial\langle S^z\rangle^2\right)_{\langle S^z\rangle=0}>0$ and $\left(\partial^2T/\partial\langle S^z\rangle^2\right)_{\langle S^z\rangle\neq0}<0$, the system will be in a first-order phase transition; when $\left(\partial^2T/\partial\langle S^z\rangle^2\right)_{\langle S^z\rangle=0}<0$, the system will be in a second-order phase transition. Therefore, the phase transition properties undergone in a thin film can be determined. In the following we will investigate the phase transition properties of an n-layer ferroelectric thin film and inspect the internal relations among the interaction parameters. In order to facilitate our discussion, the parameter space with the ratios of the bulk transverse field and the bulk four-spin interaction to the bulk two-spin interaction will be constructed. The parameter space is divided into three different regions, namely, the first-order ferroelectric phase region (FE(1st)), the second-order ferroelectric phase region (FE(2nd)) and the paraelectric phase region (PE). For simplicity, the thickness n of the ferroelectric thin film varies from 3 to 5.

Figure 1 shows the effect of the two-spin interaction of the epitaxy surface $J_{\rm s}^{(2)}/J_{\rm b}^{(2)}$ on various phase transition regions in the parameter space ($\Omega_{\rm b}/J_{\rm b}^{(2)}$ vs. $J_{\rm b}^{(4)}/J_{\rm b}^{(2)}$) for the ferroelectric thin film with a certain thickness n, i.e., the curves of $\Omega_{\rm b}/J_{\rm b}^{(2)}$ vs. $J_{\rm b}^{(4)}/J_{\rm b}^{(2)}$ for different $J_{\rm s}^{(2)}/J_{\rm b}^{(2)}$. It can be found that the first-order ferroelectric phase region (FE(1st)) is below the circle line, the second-order ferroelectric phase region (FE(2nd)) is on the left of the circle line and below the solid line, and the paraelectric phase transition region (PE) is above the solid and circle lines. Figure 1 indicates how various phase transition regions depend sensitively on the two-spin interaction of the epitaxy surface $J_{\rm s}^{(2)}/J_{\rm b}^{(2)}$. For a group of fixed parameters on the epitaxy surface $J_{\rm s}^{(2)}/J_{\rm b}^{(2)}$, $J_{\rm s}^{(4)}/J_{\rm b}^{(2)}$, $\Omega_{\rm s}/J_{\rm b}^{(2)}$, the features of the three different phase transition regions are distinct from those of the uniform surface and bulk parameter space.[25] The larger the two-spin interaction on the epitaxy surface $J_{\rm s}^{(2)}/J_{\rm b}^{(2)}$ is, the larger the range of the ferroelectric phase transition region (FE) is. The ferroelectric phase transition region (FE) includes the first-order ferroelectric phase region (FE(1st)) and the second-order ferroelectric phase region (FE(2nd)). Therefore, the range of the first-order ferroelectric phase transition region (FE(1st)) is increasing with the increase of the two-spin interaction on the epitaxy surface $J_{\rm s}^{(2)}/J_{\rm b}^{(2)}$. Meanwhile, figure 1 shows how the range of the second-order ferroelectric phase transition region (FE(2nd)) relies on the two-spin interaction of the epitaxy surface $J_{\rm s}^{(2)}/J_{\rm b}^{(2)}$. In addition, it indicates that a larger $J_{\rm b}^{(4)}/J_{\rm b}^{(2)}$ may result in a system undergoing a first-order phase transition while changing temperature, but a much larger ratio will result in the system being always paraelectric. It shows that such a much larger round-a-plaquette interaction $J_{\rm b}^{(4)}$ than pair interaction $J_{\rm b}^{(2)}$ could result in the disappearance of a long-range order.

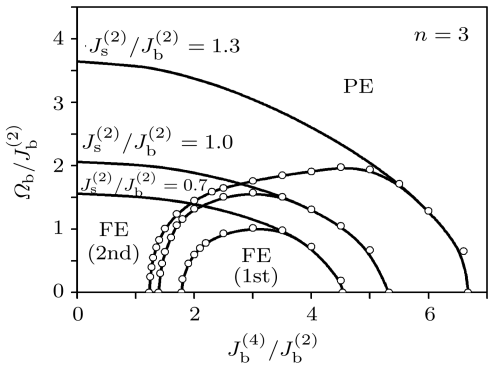


Fig. 1. Effect of the epitaxy surface $J_s^{(2)}/J_b^{(2)}$ on various phase transition regions. All curves are for $J_s^{(4)}/J_b^{(2)} = 1.0$, $\Omega_s/J_b^{(2)} = 1.5, n = 3$.

Figure 2 shows the effect of the transverse field of the epitaxy surface $\Omega_s/J_b^{(2)}$ on various phase transition regions in the parameter space ($\Omega_b/J_b^{(2)}$ vs. $J_b^{(4)}/J_b^{(2)}$) for the ferroelectric thin film with a certain thickness n, i.e., the curves of $\Omega_b/J_b^{(2)}$ vs. $J_b^{(4)}/J_b^{(2)}$ for different $\Omega_s/J_b^{(2)}$. It indicates how various phase transition regions depend sensitively on the transverse field of the epitaxy surface $\Omega_s/J_b^{(2)}$. The larger the transverse field of the epitaxy surface $\Omega_s/J_b^{(2)}$ is, the smaller the range of the ferroelectric phase transition region (FE). The range of the first-order ferroelectric phase transition region (FE(1st)) is decreasing with the increase of the transverse field of the epitaxy surface $\Omega_s/J_b^{(2)}$. Similar to the results for the two-spin interaction of the epitaxy surface $J_s^{(2)}/J_b^{(2)}$, figure 2 also illustrates how the range of the second-order ferroelectric phase transition region (FE(2nd)) relies on the transverse field of the epitaxy surface $\Omega_s/J_b^{(2)}$. It implies that the effect of increasing transverse field on the epitaxy surface $\Omega_s/J_b^{(2)}$ is the same as that of decreasing the two-spin interaction on the epitaxy surface $J_s^{(2)}/J_b^{(2)}$.

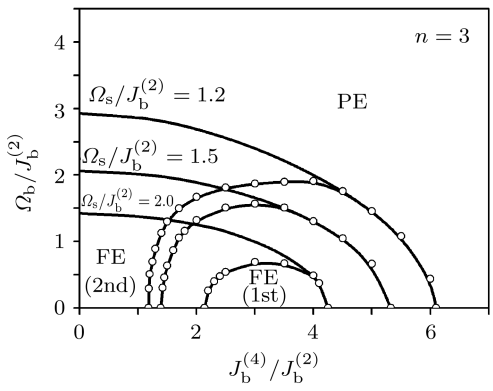


Fig. 2. Effect of the epitaxy surface $\Omega_s/J_b^{(2)}$ on various phase transition regions. All curves are for $J_s^{(2)}/J_b^{(2)} = 1.0$, $J_s^{(4)}/J_b^{(2)} = 1.0, n = 3$.

Figure 3 shows the dependence of various phase transition regions on the thickness n of the ferroelectric thin film. The larger the thickness n is, the larger range of the ferroelectric phase transition region (FE). The smaller the thickness n is, the larger range of the paraelectric phase transition region (PE) is. This is similar to the situation for the uniform surface and bulk parameter space.[25] However, by considering the epitaxy surface effects of interchange interaction and the transverse field, the common value of the demarcation line intersecting with the horizontal coordinate axis $J_b^{(4)}/J_b^{(2)}$ does not exist for the different thickness n. The larger the thickness n is, the larger the intersecting value of the demarcation line with the horizontal coordinate axis $J_b^{(4)}/J_b^{(2)}$ is. It indicates that the ranges of the first-order ferroelectric phase transition region (FE(1st)) enlarges with the increase of the thickness n of a ferroelectric thin film.

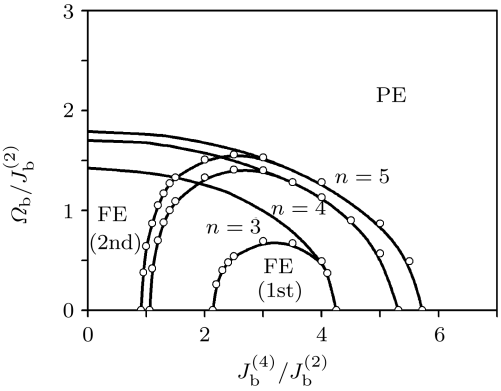


Fig. 3. The dependence of various phase transition regions on the thickness n. All curves are for $J_s^{(2)}/J_b^{(2)} = 1.0$, $J_s^{(4)}/J_b^{(2)} = 1.0$, $\Omega_s/J_b^{(2)} = 2.0$.

4. Conclusions

In conclusion, based on the transverse Ising model with a four-spin interaction, the phase transition properties of the n-layer ferroelectric thin film with an epitaxy surface layer have been studied under the Fermi-type Green's function technique. Taking into account the interchange interaction and the transverse field on the epitaxy surface layer, we have investigated systematically the first-order phase transition properties in the parameter space with respect to the ratios of the bulk transverse field and the bulk four-spin interaction to the bulk two-spin interaction. The results indicate that the first-order phase transition properties depend sensitively on the spin interchange interaction and the transverse field on the epitaxy surface layer as well as the thickness of the ferroelectric thin film. Meanwhile, it has also been studied how the epitaxy

surface parameters and the thickness of a ferroelectric thin film affect ranges of the second-order ferroelectric phase transition (FE(2nd)) and the paraelectric phase transition (PE).

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