Spin-polarization-dependent transport in a quantum dot array coupled with an Aharonov–Bohm ring*

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In this paper the quantum transport in a dot-array coupled with an Aharonov–Bohm (AB) ring is investigated via single-band tight-binding Hamiltonian. It is shown that the output spin current is a periodic function of the magnetic flux in the quantum unit Φ_0. The resonance positions of the total transmission probability do not depend on the size of the AB ring but the electronic spectrum. Moreover, the persistent currents in the AB ring is also spin-polarization dependent and different from the isolated AB ring where the persistent current is independent of spin polarization.

Keywords: spin-polarization, transmission, AB ring, current

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1. Introduction

Progress in the nanofabrication of quantum devices has enabled us to study the electron transport through a mesoscopic ring in a controllable way. Many interesting quantum phenomena have been predicted and measured in the presence of a magnetic flux, for example, the Aharonov–Bohm (AB) oscillations of the conductance and persistent current.[1−4] It is reported that AB effect in the energy spectra of closed rings has been observed by magnetotransport experiments.[5] Recently, considerable attention has been paid to the complex electronic device consisting of quantum dots, quantum wires and mesoscopic rings,[6−11] where the size of structure is small compared with the coherence length and therefore the transmission probability plays an essential role in studying the quantum transport through the system. For example, we can obtain the conductance of the devices from the transmission probability with the help of the Laudauer–Büticker formula $G = (2e^2/h)T$, where T is the transmission probability. Various methods have been developed for numerical calculations of the transmission probability T, such as mode-matching method,[12−16] recursive Green's function technique[17−24] and the scattering matrix.[25]

The quantum transport through a mesoscopic AB ring coupled with reservoirs was first studied by Büttiker, in which the reservoirs acted as a source of electrons and an inelastic scatterer.[26] The persistent current induced by a magnetic flux was obtained with the applied electrostatic potential[27] and a series of rings attached to a wire was also discussed.[28] All these studies are based on solutions of the single-electron Schrödinger equation or the tilting-binding model for a lattice ring.[29−32]

In this paper, we consider a system where an AB lattice ring is coupled to a quantum dot (QD) array with two magnetic fields applied to two separated dots of the array. The first field leads to the energy level splitting, while the second field, of which the direction forms an angle θ with the first one, leads to the spin flipping and thus the transmission probability as well as the current depends on the spin-polarization. And the persistent currents in the AB lattice ring will be also spin-polarization dependent due to the coupling with the QD array. The persistent spin current in the AB ring is, perhaps, the most important observation in the system considered. We will show how the persistent spin-current can be controlled by the magnetic flux and the size of the lattice ring. Additionally, the spin-current in output terminal is also discussed.

2. Model and formulation

The system under consideration is explained schematically in Fig. 1, where a lattice ring consist-

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ing of N QDs is connected to a QD array by tunnel coupling. The QD array is denoted by the lattice sites $-N_{\rm L}, \cdots -1, 0, 1, \ldots, N_{\rm R}$ with two ideal wires placed on the left- and right-hand sides of spanning lattice sites $-\infty, \ldots, -(N_{\rm L}+2), -(N_{\rm L}+1)$ and sites $N_{\rm R}+1, N_{\rm R}+2, \ldots, \infty$, respectively. The first magnetic field pointing to the z direction is applied to site $-N_{\rm L}$, and the other to the site $N_{\rm R}$, forming an angle θ with the z axis. In the single-band tight-binding approximation, the Hamiltonian of this system can be written as

$$H = H_0 + H_{\rm R} + H_{\rm I}, \tag{1}$$

$$\begin{aligned}
H_0 =& \sum_{n=-N_{\rm L}+1,\sigma}^{N_{\rm R}-1} \varepsilon_n d^{\dagger}_{n,\sigma} d_{n,\sigma} - \sum_{n=-N_{\rm L},\sigma}^{N_{\rm R}} (t_{n,n+1}(\sigma) d^{\dagger}_{n+1,\sigma} d_{n,\sigma} + {\rm h.c}) + (\varepsilon_{-N_{\rm L}} + \sigma g \mu_B B) d^{\dagger}_{-N_{\rm L},\sigma} d_{-N_{\rm L},\sigma} \\
&+ (\varepsilon_{N_{\rm R}} + \sigma g \mu_{B'} B' \cos\theta) d^{\dagger}_{N_{\rm R},\sigma} d_{N_{\rm R},\sigma} + g \mu_{B'} B' \sin\theta (d^{\dagger}_{N_{\rm R},\sigma} d_{N_{\rm R},-\sigma} + {\rm h.c}) \\
&+ \sum_{n\le -(N_{\rm L}+1),\sigma} \varepsilon_{\rm L} a^{\dagger}_{n,\sigma} a_{n,\sigma} - \sum_{n\le -(N_{\rm L}+2),\sigma} (t_{n,n+1}(\sigma) a^{\dagger}_{n+1,\sigma} a_{n,\sigma} + {\rm h.c}) \\
&+ \sum_{n\ge N_{\rm R}+1,\sigma} \varepsilon_{\rm R} a^{\dagger}_{n,\sigma} a_{n,\sigma} - \sum_{n\ge N_{\rm R}+2,\sigma} (t_{n,n+1}(\sigma) a^{\dagger}_{n+1,\sigma} a_{n,\sigma} + {\rm h.c}) \\
&- V_{{\rm L},\sigma} (a^{\dagger}_{-N_{\rm L}-1} a_{-N_{\rm L}} + {\rm h.c}) - V_{{\rm R},\sigma} (a^{\dagger}_{N_{\rm R}+1} a_{N_{\rm R}} + {\rm h.c}), \\
H_{\rm R} =& \sum_{j=1}^{N} \varepsilon_j c^{\dagger}_{j,\sigma} c_{j,\sigma} - \sum_{l=1}^{N-1} (V_{\rm c}(\sigma) c^{\dagger}_{j+1,\sigma} c_{j,\sigma} + {\rm h.c}) - V_{\rm c}(\sigma) ({\rm e}^{{\rm i}2\pi\alpha} c^{\dagger}_{1,\sigma} c_{N,\sigma} + {\rm h.c}), \\
H_{\rm I} =& -V_0(\sigma)(c^{\dagger}_{1,\sigma} d_{0,\sigma} + {\rm h.c}),
\end{aligned} \tag{2}$$

where H_0 is the Hamiltonian of the QD array with two ideal wires placed on the left- and right-hand sides; $H_{\rm R}$ is for the AB lattice-ring; $H_{\rm I}$ describes the interaction between the QD array and the AB ring; ε_n with $-N_{\rm L} \le n \le N_{\rm R}$, denotes the energy eigenvalue of the n-th QD in the QD array; $\varepsilon_{\rm R}$ and $\varepsilon_{\rm L}$ are the on-site energy on the right-hand-side and the left-hand-side leads respectively; $d^{\dagger}_{n,\sigma}$ (with $\sigma = \uparrow, \downarrow$) is the creation operator of the electrons with spin index σ at the n-th QD, whith $-\sigma$ denoting the opposite spin polarization with respect to σ; $a^{\dagger}_{n,\sigma}$ the creation operator of electrons with spin σ at lattice n in the leads, $d_{n,\sigma}$ and $a_{n,\sigma}$ are the corresponding annihilation operators; $t_{n,n+1}(\sigma)$ is the hopping integral between the nearest QDs, defined as $t_{n,n+1}(\sigma) = \langle n+1| H |n\rangle = t$ which is independent of spin polarization, except for $n = -N_{\rm L} - 1$ and $n = N_{\rm R}$, where the QD array is connected to the ideal wires; $V_{{\rm L},\sigma}$ and $V_{{\rm R},\sigma}$ are the hopping integrals. When $n = -N_{\rm L}$ and $N_{\rm R} - 1$, we have $t_{-N_{\rm L},-N_{\rm L}+1}(\sigma) = t_{N_{\rm R}-1,N_{\rm R}}(\sigma) = t'_{\sigma}$ which is spin-dependent. In the expression of $H_{\rm R}$, ε_j with $1 \le j \le N$, denotes the energy eigenvalues on the j-th QD in the AB ring. $c^{\dagger}_{j,\sigma}$ (with $\sigma = \uparrow, \downarrow$) is the creation operator with spin index σ at the j-th QD and $c_{j,\sigma}$ is the corresponding annihilation operators. $V_{\rm c}(\sigma)$ is the corresponding hopping integral. The magnetic flux is measured in terms of the quantum flux unit $\Phi_0 = hc/e$ as $\alpha = \Phi/\Phi_0$. In this work, we adopt the singular gauge of the vector potential associated with the magnetic flux, i.e., AB flux, which affects only the hopping coupling between the first and the last sites of the ring. $V_0(\sigma)$ is the tunneling coupling between the AB ring and the QD array.[33]

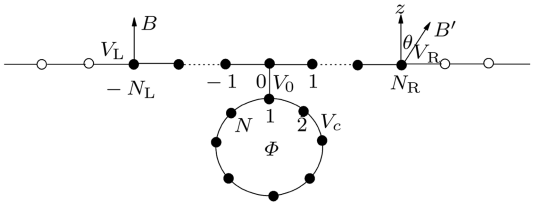


Fig. 1. QD array with an attached AB ring.

The eigenstates of the whole Hamiltonian can be written as

$$|\Psi\rangle = \sum_{n=-\infty,\sigma}^{\infty} C_{n,\sigma} a^{\dagger}_{n,\sigma} |0\rangle + \sum_{j=1,\sigma}^{N} b_{j,\sigma} c^{\dagger}_{j,\sigma} |0\rangle . \tag{3}$$

The expansion coefficients, $\{C_{n,\sigma}\}$, satisfy the recurring equation

$$(E - \varepsilon_n) C_{n,\sigma} + t_{n-1,n}(\sigma) C_{n-1,\sigma} + t_{n,n+1}(\sigma) C_{n+1,\sigma} = 0, \ n \ne -N_{\rm L}, 0, N_{\rm R}, \tag{4}$$

$$
\begin{aligned}
EC_{-N_{\rm L},\sigma} &= (\varepsilon_{-N_{\rm L}} + \sigma g\mu_B B)C_{-N_{\rm L},\sigma} - t_{-N_{\rm L}-1,-N_{\rm L}}(\sigma)C_{-N_{\rm L}-1,\sigma} - t_{-N_{\rm L},-N_{\rm L}+1}(\sigma)C_{-N_{\rm L}+1,\sigma}, \quad n = -N_{\rm L}, \\
EC_{N_{\rm R},\sigma} &= (\varepsilon_{N_{\rm R}} + \sigma g\mu_{B'}B'\cos\theta)C_{N_{\rm R},\sigma} + B'\sin\theta C_{N_{\rm R},-\sigma} - t_{N_{\rm R}-1,N_{\rm R}}(\sigma)C_{N_{\rm R}-1,\sigma} \\
&\quad - t_{N_{\rm R},N_{\rm R}+1}(\sigma)C_{N_{\rm R}+1,\sigma}, \quad n = N_{\rm R}, \\
EC_{0,\sigma} &= \varepsilon_0 C_{0,\sigma} - V_0(\sigma)b_{1,\sigma} - t_{-2,-1}(\sigma)C_{-1,\sigma} - t_{0,1}(\sigma)C_{1,\sigma}, \quad n = 0,
\end{aligned} \tag{5}
$$

while the coefficients, $\{b_{j,\sigma}\}$, satisfy the equation

$$
\begin{aligned}
&(E-\varepsilon_j)b_{j,\sigma} + V_{\rm c}(\sigma)b_{j-1,\sigma} + V_{\rm c}(\sigma)b_{j+1,\sigma} = 0, \quad j = 2,\ldots,N-1, \\
&(E-\varepsilon_N)b_{N,\sigma} + V_{\rm c}(\sigma)b_{N-1,\sigma} + V_{\rm c}(\sigma)\,{\rm e}^{-{\rm i}2\pi\varphi}b_{1,\sigma} = 0, \\
&(E-\varepsilon_1)b_{1,\sigma} + V_{\rm c}(\sigma)\,{\rm e}^{{\rm i}2\pi\varphi}b_{N,\sigma} + V_{\rm c}(\sigma)b_{2,\sigma} = -V_0(\sigma)a_{0,\sigma}.
\end{aligned} \tag{6}
$$

In terms of the Gramer principle, we can obtain

$$
b_{1,\sigma} = \frac{D_2(\varepsilon)}{D(\varepsilon)}, \tag{7}
$$

where $D(\varepsilon) = \det(\varepsilon I - H_{\rm R})$, and $D_2(\varepsilon)$ is given by

$$
D_2(\varepsilon) = \det\begin{bmatrix}
E-\varepsilon_1 & -V_0(\sigma)a_{0,\sigma} & 0 & \ldots & V_{\rm c}(\sigma)\,{\rm e}^{{\rm i}2\pi\varphi} \\
V_{\rm c}(\sigma) & 0 & V_{\rm c}(\sigma) & \ldots & 0 \\
0 & 0 & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & E-\varepsilon_{N-1} & V_{\rm c}(\sigma) \\
V_{\rm c}(\sigma)\,{\rm e}^{-{\rm i}2\pi\varphi} & \ldots & 0 & V_{\rm c}(\sigma) & E-\varepsilon_N
\end{bmatrix}. \tag{8}
$$

Then we can rewrite Eqs. (6) and (7) as a matrix equation

$$
\begin{bmatrix} C_{n+1,\sigma} \\ C_{n,\sigma} \\ C_{n+1,-\sigma} \\ C_{n,-\sigma} \end{bmatrix} = M(n,E)\begin{bmatrix} C_{n,\sigma} \\ C_{n-1,\sigma} \\ C_{n,-\sigma} \\ C_{n-1,-\sigma} \end{bmatrix}, \tag{9}
$$

where $M(n,E)$ is called the scattering matrix which links the expansion coefficient vector $(C_{n+1,\sigma}, C_{n,\sigma}, C_{n+1,-\sigma}, C_{n,-\sigma})^{\rm T}$ to the vector $(C_{n,\sigma}, C_{n-1,\sigma}, C_{n,-\sigma}, C_{n-1,-\sigma})^{\rm T}$. Thus it is also called the transfer matrix determined by

$$
M(n,E) = \begin{bmatrix}
-\frac{E-\varepsilon'_n}{t_{n,n+1}(\sigma)} & -\frac{t_{n-1,n}(\sigma)}{t_{n,n+1}(\sigma)} & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & -\frac{E-\varepsilon'_n}{t_{n,n+1}(-\sigma)} & -\frac{t_{n-1,n}(-\sigma)}{t_{n,n+1}(-\sigma)} \\
0 & 0 & 1 & 0
\end{bmatrix} = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix}, \ n \neq N_{\rm R},
$$

$$
M(N_{\rm R},E) = \begin{bmatrix}
-\frac{E-\varepsilon'_{N_{\rm R}}}{t_{N_{\rm R},N_{\rm R}+1}(\sigma)} & -\frac{t_{N_{\rm R}-1,N_{\rm R}}(\sigma)}{t_{N_{\rm R},N_{\rm R}+1}(\sigma)} & \frac{g\mu_{B'}B'\sin\theta}{t_{N_{\rm R},N_{\rm R}+1}(\sigma)} & 0 \\
1 & 0 & 0 & 0 \\
\frac{g\mu_{B'}B'\sin\theta}{t_{N_{\rm R},N_{\rm R}+1}(-\sigma)} & 0 & -\frac{E-\varepsilon'_{N_{\rm R}}}{t_{N_{\rm R},N_{\rm R}+1}(-\sigma)} & -\frac{t_{N_{\rm R}-1,N_{\rm R}}(-\sigma)}{t_{N_{\rm R},N_{\rm R}+1}(-\sigma)} \\
0 & 0 & 1 & 0
\end{bmatrix}, \quad (n = N_{\rm R}), \tag{10}
$$

where $\varepsilon'_{(n,0)}$ is the effective on-site energy given by

$$
\begin{aligned}
&\varepsilon'_n = \varepsilon_n, \ n \neq -N_{\rm L}, 0, N_{\rm R}, \quad \varepsilon'_{-N_{\rm L}} = \varepsilon_{-N_{\rm L}} + \sigma g\mu_B B, \ n = -N_{\rm L}, \\
&\varepsilon'_0 = \varepsilon_0 - \frac{D_2(\varepsilon)}{D_N(\varepsilon)}V_0(\sigma)^2, \ n = 0, \quad \varepsilon'_{N_{\rm R}} = \varepsilon_{N_{\rm R}} + \sigma g\mu_{B'}B'\cos\theta, \ n = -N_{\rm R}.
\end{aligned} \tag{11}
$$

We assume that the electron in the left lead is described by a plane wave of unity amplitude and a reflection amplitude r_σ and in the right lead by a transmission amplitude t_σ. Taking this into account we can have

$$C_{n,\sigma} = \mathrm{e}^{\mathrm{i}K_\sigma^{\mathrm{L}} na} + r_\sigma \mathrm{e}^{-\mathrm{i}K_\sigma^{\mathrm{L}} na}, \quad n \le -(N_{\mathrm{L}}+1),$$
$$C_{n,\sigma} = t_\sigma \mathrm{e}^{\mathrm{i}K_\sigma^{\mathrm{L}} na}, \quad n \ge N_{\mathrm{R}}+1.$$

It can be shown that the transmission amplitude t_σ is related to the incident coefficients via the equation

$$\begin{bmatrix} t_\sigma \\ 0 \\ t_{-\sigma} \\ 0 \end{bmatrix} = T(E) \begin{bmatrix} 1 \\ r_\sigma \\ 1 \\ r_{-\sigma} \end{bmatrix}, \tag{12}$$

where the transfer matrix $T(E)$ is given by

$$\begin{aligned} T(E) = & \begin{bmatrix} \mathrm{e}^{-\mathrm{i}K_\sigma^{\mathrm{R}}(N_{\mathrm{R}}+1)a} & 0 & 0 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}K_\sigma^{\mathrm{R}}(N_{\mathrm{R}}+1)a} & 0 & 0 \\ 0 & 0 & \mathrm{e}^{-\mathrm{i}K_{-\sigma}^{\mathrm{R}}(N_{\mathrm{R}}+1)a} & 0 \\ 0 & 0 & 0 & \mathrm{e}^{\mathrm{i}K_{-\sigma}^{\mathrm{R}}(N_{\mathrm{R}}+1)a} \end{bmatrix} \\ & \times \begin{bmatrix} \mathrm{e}^{\mathrm{i}k_\sigma^{\mathrm{R}} a} & \mathrm{e}^{-\mathrm{i}k_\sigma^{\mathrm{R}} a} & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & \mathrm{e}^{\mathrm{i}k_{-\sigma}^{\mathrm{R}} a} & \mathrm{e}^{-\mathrm{i}k_{-\sigma}^{\mathrm{R}} a} \\ 0 & 0 & 1 & 1 \end{bmatrix}^{-1} \prod_{n=N_{\mathrm{R}}+1}^{-(N_{\mathrm{L}}+1)} M(n,E) \\ & \times \begin{bmatrix} \mathrm{e}^{\mathrm{i}k_\sigma^{\mathrm{L}} a} & \mathrm{e}^{-\mathrm{i}k_\sigma^{\mathrm{L}} a} & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & \mathrm{e}^{\mathrm{i}k_{-\sigma}^{\mathrm{L}} a} & \mathrm{e}^{-\mathrm{i}k_{-\sigma}^{\mathrm{L}} a} \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{e}^{-\mathrm{i}K_\sigma^{\mathrm{L}}(N_{\mathrm{L}}+1)a} & 0 & 0 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}K_\sigma^{\mathrm{L}}(N_{\mathrm{L}}+1)a} & 0 & 0 \\ 0 & 0 & \mathrm{e}^{-\mathrm{i}K_{-\sigma}^{\mathrm{L}}(N_{\mathrm{L}}+1)a} & 0 \\ 0 & 0 & 0 & \mathrm{e}^{\mathrm{i}K_{-\sigma}^{\mathrm{L}}(N_{\mathrm{L}}+1)a} \end{bmatrix}. \end{aligned} \tag{13}$$

So from Eq. (10) we can obtain the reflection amplitudes $r_\uparrow$ and $r_\downarrow$ as

$$r_\uparrow = \frac{-\left(\dfrac{T_{21}}{T_{22}} + \dfrac{T_{23}}{T_{22}}\right) + \dfrac{T_{24}}{T_{22}}\left(\dfrac{T_{41}}{T_{44}} + \dfrac{T_{43}}{T_{44}}\right)}{1 - \dfrac{T_{24}}{T_{22}}\dfrac{T_{42}}{T_{44}}},$$
$$r_\downarrow = \frac{-\left(\dfrac{T_{41}}{T_{44}} + \dfrac{T_{43}}{T_{44}}\right) + \dfrac{T_{42}}{T_{44}}\left(\dfrac{T_{21}}{T_{22}} + \dfrac{T_{23}}{T_{22}}\right)}{1 - \dfrac{T_{24}}{T_{22}}\dfrac{T_{42}}{T_{44}}}, \tag{14}$$

and the transmission amplitudes $t_\uparrow$ and $t_\downarrow$

$$t_\uparrow(E) = T_{11} + T_{12}r_\uparrow + T_{13} + T_{14}r_\downarrow,$$
$$t_\downarrow(E) = T_{31} + T_{32}r_\uparrow + T_{33} + T_{34}r_\downarrow. \tag{15}$$

While the spin-polarization-dependent reflection and transmission probabilities are

$$R_\sigma(E) = \frac{1}{2}\left|r_\sigma\right|^2,$$
$$T_\sigma(E) = \frac{1}{2}\frac{V_{\mathrm{R},\sigma}}{V_{\mathrm{L},\sigma}}\left|t_\sigma\right|^2. \tag{16}$$

The total transmission probability and the reflection probability are determined by

$$T_{\mathrm{tot}}(E) = T_\uparrow(E) + T_\downarrow(E),$$
$$R_{\mathrm{tot}}(E) = R_\uparrow(E) + R_\downarrow(E). \tag{17}$$

And the spin current is related to the net spin-polarization transmission probability such that

$$\Delta T(E) = T_\uparrow(E) - T_\downarrow(E). \tag{18}$$

The persistent current generated by the AB flux,[34] J_σ, depends on the spin-polarization due to the coupling with the dot array and can be evaluated by

$$J_\sigma = \frac{2eV_{\mathrm{c}}(\sigma)}{\hbar}\,\mathrm{Im}(b_{j+1,\sigma}^* b_{j,\sigma}) \tag{19}$$

for any pair of adjacent sites, for example $j = 1$. And $b_{1,\sigma}$ is given by Eq. (7), similarly, $b_{2,\sigma}$ is given by

$$b_{2,\sigma} = \frac{D_3(\varepsilon)}{D(\varepsilon)},$$

where $D_3(\varepsilon)$ is given by

$$D_3(\varepsilon) = \det \begin{bmatrix} E-\varepsilon_1 & V_{\rm c}(\sigma) & -V_0(\sigma)a_{0,\sigma} & \dots & V_{\rm c}(\sigma)\,{\rm e}^{{\rm i}2\pi\varphi} \\ V_{\rm c}(\sigma) & E-\varepsilon_2 & 0 & \dots & 0 \\ 0 & 0 & \dots & \dots & \dots \\ \dots & \dots & \dots & E-\varepsilon_{N-1} & V_{\rm c}(\sigma) \\ V_{\rm c}(\sigma)\,{\rm e}^{-{\rm i}2\pi\varphi} & \dots & 0 & V_{\rm c}(\sigma) & E-\varepsilon_N \end{bmatrix}.$$

While we can obtain $a_{0,\sigma}$ through Eqs. (9) and (12). Thus we obtain the total persistent current J as

$$J = J\uparrow + J\downarrow, \tag{20}$$

and the net spin-current ΔJ is

$$\Delta J = J\uparrow - J\downarrow. \tag{21}$$

It is clear that when $D_2(\varepsilon) = 0$ (or $D_3(\varepsilon) = 0$) and $D(\varepsilon) \neq 0$, the spin-current vanishes, and on the other hand when $D(\varepsilon) = 0$ it oscillates with the magnetic flux in the period Φ_0.

3. Numerical result and discussion

In our model, the generation of spin current in the output electrode can be understood as the fact that a force applied to the spin $\boldsymbol{S}$ is induced by the spatially inhomogeneous magnetic field, namely $\boldsymbol{F} = g\mu_{\rm b}\boldsymbol{S}\cdot\bigtriangledown\boldsymbol{B}$ (where $\mu_{\rm b}$ is the Bohr magneton, g is the spin Lande g-factor), which will remove the degeneracy of spin-polarization in the transport and thus leads to the relative shifts of resonant peaks of the transmission spectra between the spin-up and spin-down components of electron. The spin current appears when the transmission spectrum of one spin component reaches a maximum while the spectrum of the other spin component is a minimum.

For simplicity in the actual calculation, we assume that the hopping integrals t_σ, t'_σ, $V_{{\rm L},\sigma}$, $V_{{\rm R},\sigma}$, $V_{\rm c}(\sigma)$ and $V_0(\sigma)$ are all independent of the spin index, we have $t_\sigma = V_{\rm c}(\sigma) = V_0(\sigma) = t$, $t'_\sigma = t' = 0.8t$; and $V_{{\rm L},\sigma} = V_{{\rm R},\sigma} = 0.5t$. Figure 2 shows the numerical results of the relative transmission probabilities between the spin-up and the spin-down components, i.e., the net spin transition, as a function of electronic energy E. The on-site energies of all QDs (ε_n with $-N_{\rm L} \leq n \leq N_{\rm R}$, as well as $\varepsilon_{\rm L}$ and $\varepsilon_{\rm R}$) are set to be the same value of $2t$, and such parameters are chosen that $g\mu_B B = g\mu_{B'}B' = 1t$ and the second magnetic field is vertical to the first one, i.e., $\theta = \pi/2$. Figure 2(a) is for the case of the magnetic flux number $\alpha = 0.3$, while figure 2(b) is for $\alpha = 0.5$ with $N = 6$ (solid line) and $N = 7$ (dash line) respectively. It can be seen that the net spin transition is modulated strongly by the magnetic flux α and the number of the QDs in the ring. In some energy ranges (for example, when the energy of transmitted electron is defined as being from $1.28t$ to $1.92t$), and the ring is composed of 7 QDs ($N = 7$), the output spin current defined by $\Delta T(E) = T_\uparrow(E) - T_\downarrow(E)$ is positive (i.e. the spin current with spin-up polarization) for the case of $\alpha = 0.3$; while the $\Delta T(E)$ will be negative (the spin-down current) for the case of $\alpha = 0.5$. However the results will be contrary to those when the ring is composed of 6 QDs ($N = 6$). On the other hand, when we keep the magnetic flux constant for $\alpha = 0.3$, in some other regions (where in the energy window: $2.44t$ to $2.60t$, etc.), the output spin current is positive (the spin-up current) for the case of $N = 6$; while the $\Delta T(E)$ is negative (the spin-down current) for the case of $N = 7$. Figure 3(a) with $\alpha = 0.3$ and figure 3(b) with $\alpha = 0.5$ are plots of the relative transmission probabilities between two spin components versus the energy where the solid curve is for $N = 11$, and the dash curve is for $N = 12$. From these, we can see how the net spin transition is modulated by the number of the QDs in the quantum ring and the magnetic flux. For example, when the energy of the transmitted electron is between $2.68t$ and $2.91t$ for the case of $\alpha = 0.3$, there is no spin current output when the ring consists of 12 QDs, while the output spin current will be spin-up polarization when the ring has 11 QDs. This QD number dependence is the consequence of quantum phase interference. On the other hand, to explore the relation between the output spin-current and the magnetic flux in the AB ring with different values of QD number N, we can fix the ring to contain 11 QDs. From Figs. 3(a) and 3(b) we can see that when the

energy of transmitted electron is defined as being in a range from $2.45t$ to $2.67t$ with $\alpha = 0.3$, there is no spin current output, while the spin-down current has observed under the condition of $\alpha = 0.5$. Further, both the total and the net probabilities are periodic functions of the magnetic flux in the quantum unit Φ_0, indicating that the AB ring oscillates. A similar result has been found in Ref. [28].

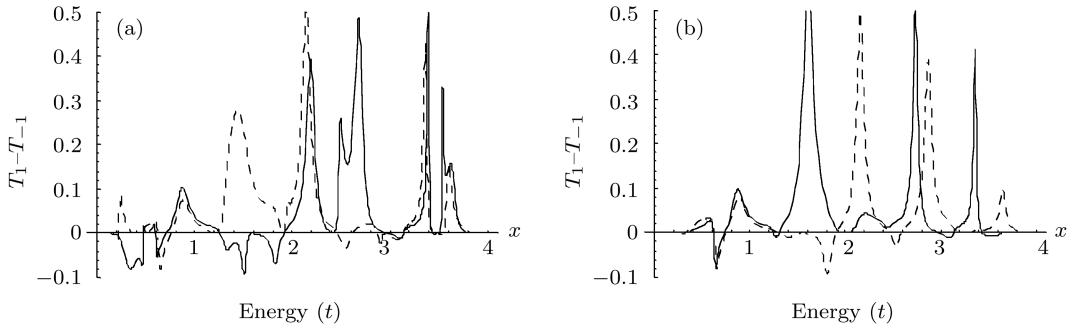


Fig. 2. Output relative transmission probabilities between two spin components as a function of energy for $N = 6$ and $N = 7$.

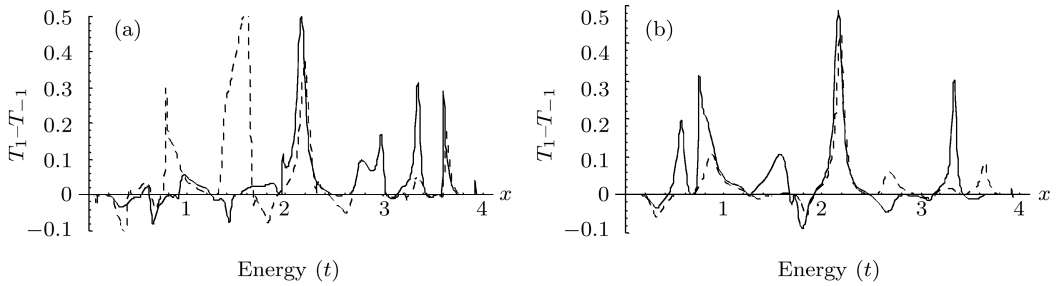


Fig. 3. Output relative transmission probabilities between two spin components as functions of energy for $N = 11$ and $N = 12$.

It is also interesting to see the total persistent current density j and the spin persistent current Δj in the ring, which are respectively determined by

$$j = \frac{J}{\frac{2eV_c(\sigma)}{\hbar}} = \mathrm{Im}[b^*_{2,\uparrow} b_{1,\uparrow}] + \mathrm{Im}[b^*_{2,\downarrow} b_{1,\downarrow}],$$
$$\Delta j = \frac{\Delta J}{\frac{2eV_c(\sigma)}{\hbar}} = \mathrm{Im}[b^*_{2,\uparrow} b_{1,\uparrow}] - \mathrm{Im}[b^*_{2,\downarrow} b_{1,\downarrow}]. \quad (22)$$

Figure 4 shows the numerical results of total persistent current density j versus magnetic flux α with energy of the transmitted electron $0.5t$ for $N = 11$ (Fig. 4(a)) and $N = 12$ (Fig. 4(b)). Obviously, the total persistent current is a periodic function of magnetic flux in quantum unit $\Phi_0 = hc/e$ and this character is due to the fact that the total transmission probability $T_{\mathrm{tot}}(E)$ defined by $T_\uparrow(E) + T_\downarrow(E)$ is a periodic function of magnetic flux which indicates the AB ring oscillates. It can also see that the curve is discontinuous at the half-integer number of magnetic flux, which is in agreement with the observation in a polyacetylene AB ring[35]. Figures 5(a) ($N = 11$) and 5(b) ($N = 12$) show the plots of the spin persistent current Δj as functions of the magnetic flux. Similar to the total persistent current shown in Fig. 4, the spin persistent current density is also a periodic function of the magnetic flux in the quantum unit, and the periods for both the total and the spin currents are independent of the size of the AB ring; but it is important to point out that the value and the flux window of the spin persistent current are dependent on AB ring remarkably (Fig. 5). When the Δj in the ring is positive,

it means the spin persistent current is spin-up polarization; otherwise it is the spin-down polarization. The widths of the flux window for the different polarizations are strongly dependent on the size of the ring (the number of the QDs in the ring), this enables one to be able to control the spin persistent current by modulating the AB ring.

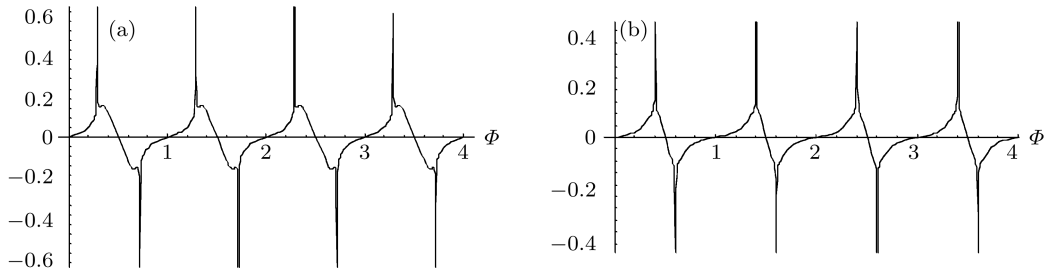


Fig. 4. Total persistent current density j in AB ring versus magnetic flux α.

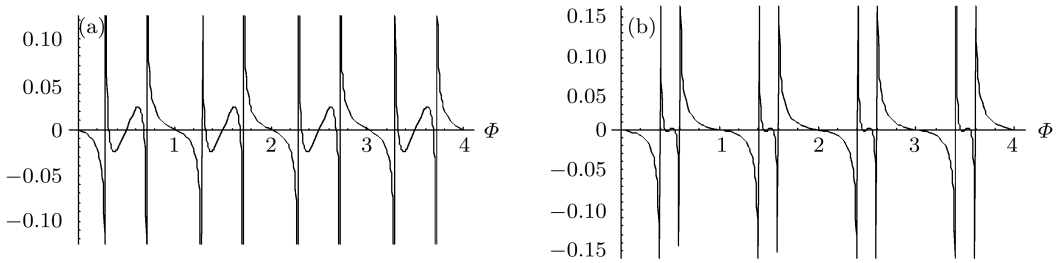


Fig. 5. Spin persistent current Δj in the AB ring as a function of the magnetic flux α.

4. Summary and conclusion

We have shown that output spin current is a periodic function of magnetic flux and modulated by the number of QDs in the ring due to the quantum phase interference. Moreover, our new observation is that the persistent current in the AB ring is also spin-polarization dependent resulting from the coupling with the QD array in which there exists the spin current.

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