A scattering matrix approach to quantum pumping: beyond the small-AC-driving-amplitude limit*

Zhu Rui(朱 瑞)†

Department of Physics, South China University of Technology, Guangzhou 510641, *China*

(Received 26 February 2010; revised manuscript received 15 May 2010)

In the adiabatic and weak-modulation quantum pump, net electron flow is driven from one reservoir to another by absorbing or emitting an energy quantum $\hbar\omega$ from or to the reservoirs. This paper considers high-order dependence of the scattering matrix on the time. Non-sinusoidal behaviour of strong pumping is revealed. The relation between the pumped current and the ac driving amplitude varies from power of 2, 1 to 1/2 when stronger modulation is exerted. Open experimental observation can be interpreted by multi-energy-quantum-related processes.

Keywords: quantum pumping, scattering matrix approach, multi-energy-quantum-related processes

PACC: 7200, 7335C

1. Introduction

Generally speaking, the transport of matter from low potential to high potential excited by absorbing energy from the environment can be described as a pump process. The driving mechanics of classic pumps is straightforward and well understood.[1] The concept of a quantum pump is initiated several decades ago[2] with its mechanism involving coherent tunneling and quantum interference. Research on quantum pumping has attracted heated interest since its experimental realization in an open quantum dot.[3−35]

The current and noise properties in various quantum pump structures and devices were investigated such as the magnetic-barrier-modulated two-dimensional electron gas,[8] mesoscopic one-dimensional wire,[10,26] quantum-dot structures,[1,9,15,16,32,34,35] mesoscopic rings with Aharonov–Casher and Aharonov–Bohm effect,[11] magnetic tunnel junctions,[15] chains of tunnel-coupled metallic islands,[29] the nanoscale helical wire,[30] the Tomonaga–Luttinger liquid,[28] and garphene-based devices.[24,25] Theory also predicts that charge can be pumped by oscillating one parameter in particular quantum configurations.[27] A recent experiment[31] based on two parallel quantized charge pumps offers a way forward to the potential application of quantum pumping in quantum information processing, the generation of single photons in pairs and bunches, neural networking, and the development of a quantum standard for electrical current. Correspondingly, theoretical techniques have been put forward for the treatment of the quantum pumps.[4−7,22,26,29] One of the most prominent techniques is the scattering approach proposed by Brouwer[4] who presented a formula that relates the pumped current to the parametric derivatives of the scattering matrix of the system. Driven by adiabatic and weak modulation (the AC driving amplitude is small in comparison with the static potential), the pumped current was found to vary in a sinusoidal manner as a function of the phase difference between the two oscillating potentials. It increases linearly with the frequency in line with experimental finding.

Although the quantum pump has been extensively discussed in literatures, little attention was paid to experimentally observed deviation from the weak-pumping theory with only the first-order parametric derivative of the scattering matrix considered. We improved the scattering approach by expanding the scattering matrix to higher orders of the time and modulation amplitude, which enables us to go farther in investigation of the problem.

*Project supported by the National Natural Science Foundation of China (Grant No. 11004063), the Fundamental Research Funds for the Central Universities (Grant No. 2009ZM0299), the Natural Science Foundation of South China University of Technology (Grant No. x2lxE5090410), and the Graduate Course Construction Project of South China University of Technology (Grant No. yjzk2009001).

†Corresponding author. E-mail: rzhu@scut.edu.cn

© 2010 Chinese Physical Society and IOP Publishing Ltd

2. Theoretical formulation

The scattering-matrix equation $\hat{b}_\alpha = S_{\alpha\beta}\hat{a}_\beta$, where α and β are indexing leads for channel and spin introduced in the Landauer–Büttiker conductance,[36−38] characterizes the transport properties through a conductor at a certain bias.

In a more general situation with dynamic processes, e.g. in quantum pumping, a time-dependent scattering matrix can be introduced as follows:

$$\hat{b}_\alpha(t) = \int_{-\infty}^{\infty} S_{\alpha\beta}(t,t')\hat{a}_\beta(t')\mathrm{d}t', \quad t \geq t', \tag{1}$$

where α and β are generally indexing leads for channel and spin. An incident state $\hat{a}_\beta$ at time t' is scattered into the outgoing state $\hat{b}_\alpha$ at time t with the amplitude $S_{\alpha\beta}(t,t')$.

The time-dependent scattering-matrix picture described by Eq. (1) is exactly equivalent to the time-dependent Schrödinger equation with the elements of the scattering matrix amplitudes of the wave function. In usual cases, the time-dependent Schrödinger equation cannot be solved exactly, similarly to the time-dependent scattering matrix. In the static or adiabatic cases, it is advantageous to use an analog of the Wigner transform for the matrix $S_{\alpha\beta}(t,t')$,[39]

$$S_{\alpha\beta}(E,t) = \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\,E(t-t')} S_{\alpha\beta}(t,t')\mathrm{d}t'. \tag{2}$$

An on-time scattering process $S_{\alpha\beta}(t,t')$ with $t = t'$ is sufficient to describe the bias-driven conductance. Namely, from Eq. (2), we can use $S_{\alpha\beta}(E)$ with E labeling the energy channel to fully capture the transport physics.

When the scattering time $t - t'$ is small (i.e., the dynamic characteristic frequency is much smaller than the inverse Wigner time delay), the dynamics can be approximated to the instant-scattering picture. Physically this means that the scattering matrix changes only a little while an electron is scattered by the mesoscopic sample under dynamic modulation, in which we use the term "adiabatic". In adiabatic dynamics, we can use low-order Fourier components of $S_{\alpha\beta}(E,t)$ to characterize transport physics. Therefore, small $t - t'$ is transformed into variation of the particle energy by side-band broadening around the Fermi level.

We consider the response of a mesoscopic phase-coherent sample to two slowly oscillating (with a frequency ω) external real parameters $X_j(t)$ (gate potential, magnetic flux, etc.),[7]

$$X_j(t) = X_{0,j} + X_{\omega,j}\mathrm{e}^{\mathrm{i}(\omega t-\varphi_j)} + X_{\omega,j}\mathrm{e}^{-\mathrm{i}(\omega t-\varphi_j)}, \quad j = 1,2. \tag{3}$$

$X_{0,j}$ and $X_{\omega,j}$ measure the static magnitude and AC driving amplitude of the two parameters, respectively. The phase difference between the two drivers is defined as $\phi = \varphi_1 - \varphi_2$. The mesoscopic conductor is connected to two reservoirs at zero bias. The scattering matrix $\hat{s}$ being a function of parameters $X_j(t)$ depends on time.

We assume that the scattering time $t - t'$ is small. Up to corrections of order $\hbar\omega/\gamma$ (γ measures the escape rate), the matrix $S_{\alpha\beta}(E,t)$ is equal to the "instantaneous" scattering matrix $S_X(E)$, which is obtained by "freezing" all parameters X_j to their values at time t. Below, we use the instant scattering matrix $\hat{s}(t)$ in place of $S_{\alpha\beta}(E,t)$ to describe the physics for simplicity. The kinetic properties (charge current, heat current, etc.) depend on the values of the scattering matrix within the energy interval of the order of max $(k_BT, \hbar\omega)$ near the Fermi energy. In the low-frequency ($\omega \to 0$) and low-temperature ($T \to 0$) limit we assume the scattering matrix to be energy independent. To investigate the deviation from the small amplitude $X_{\omega,j}$ limit, we expand the scattering matrix $\hat{s}(t)$ into Taylor series of $X_j(t)$ to second order at $X_{0,j}$ with the terms linear and quadratic of $X_{\omega,j}$ presented in the expansion,

$$\begin{aligned}\hat{s}(t) \approx\ & \hat{s}_0(X_{0,j}) + \hat{s}_{-\omega}\mathrm{e}^{\mathrm{i}\,\omega t} + \hat{s}_{+\omega}\mathrm{e}^{-\mathrm{i}\,\omega t} + \hat{s}_2 \\ & + \hat{s}_{-2\omega}\mathrm{e}^{2\,\mathrm{i}\,\omega t} + \hat{s}_{+2\omega}\mathrm{e}^{-2\,\mathrm{i}\,\omega t}\end{aligned} \tag{4}$$

with

$$\begin{cases} \hat{s}_{\pm\omega} = \displaystyle\sum_{j=1,2} X_{\omega,j}\mathrm{e}^{\pm\,\mathrm{i}\,\varphi_j}\partial\hat{s}/\partial X_j, \\ \hat{s}_2 = \displaystyle\sum_{j=1,2} X_{\omega,j}^2\partial^2\hat{s}/\partial X_j^2, \\ \hat{s}_{\pm2\omega} = \dfrac{1}{2}\displaystyle\sum_{j=1,2} X_{\omega,j}^2\mathrm{e}^{\pm2\,\mathrm{i}\,\varphi_j}\partial^2\hat{s}/\partial X_j^2. \end{cases} \tag{5}$$

It can be seen from the equations that higher orders of the Fourier spectra enter into the scattering matrix. As a result, both the nearest and next nearest sidebands are taken into account, which implies that a scattered electron can absorb or emit an energy quantum of $\hbar\omega$ or $2\hbar\omega$ before it leaves the scattering region. In principle, third or higher orders in the Taylor series can be obtained accordingly. However, the higher-order parametric derivatives of the scattering matrix diminish dramatically and approximate to zero. Numerical calculation demonstrates that even in relatively large amplitude modulation, their contribution is negligible.

The pumped current depends on the values of the scattering matrix within the energy interval of the order of $\max(k_BT, 2\hbar\omega)$ near the Fermi energy. In the low-temperature limit ($T \to 0$), an energy interval of $2\hbar\omega$ is opened during the scattering process.

The mesoscopic scatterer is coupled to two reservoirs with the same temperatures T and electrochemical potentials μ. Electrons with the energy E entering the scatterer are described by the Fermi distribution function $f_0(E)$, which approximates to a step function at a low temperature. Due to the interaction with an oscillating scatterer, an electron can absorb or emit energy quanta that changes the distribution function. A single transverse channel in one of the leads is considered. Applying the hypothesis of an instant scattering, the scattering matrix connecting the incoming and outgoing states can be written as

$$\hat{b}_\alpha(t) = \sum_\beta s_{\alpha\beta}(t)\hat{a}_\beta(t). \tag{6}$$

Here $s_{\alpha\beta}$ is an element of the scattering matrix $\hat{s}$; the time-dependent operator is $\hat{a}_\alpha(t) = \int \mathrm{d}E\hat{a}_\alpha(E)\mathrm{e}^{-\mathrm{i}Et/\hbar}$, and the energy-dependent operator $\hat{a}_\alpha(E)$ annihilates particles with total energy E incident from the α lead into the scatter and obey the following anticommutation relations

$$[\hat{a}^\dagger_\alpha(E), \hat{a}_\beta(E')] = \delta_{\alpha\beta}\delta(E - E'). \tag{7}$$

Note that the above expressions correspond to single-(transverse) channel leads and spinless electrons. For the case of many-channel leads each lead index (α, β, etc.) includes a transverse channel index and any repeating lead index implies implicitly a summation over all the transverse channels in the lead. Similarly an electron spin can be taken into account.

Using Eqs. (4) and (6) and after a Fourier transformation we obtain

$$\begin{aligned}\hat{b}_\alpha(E) = \sum_\beta [&\hat{s}_{0,\alpha\beta}\hat{a}_\beta(E) + \hat{s}_{2,\alpha\beta}\hat{a}_\beta(E) \\ &+ \hat{s}_{-\omega,\alpha\beta}\hat{a}_\beta(E+\hbar\omega) + \hat{s}_{+\omega,\alpha\beta}\hat{a}_\beta(E-\hbar\omega) \\ &+ \hat{s}_{-2\omega,\alpha\beta}\hat{a}_\beta(E+2\hbar\omega) \\ &+ \hat{s}_{+2\omega,\alpha\beta}\hat{a}_\beta(E-2\hbar\omega)].\end{aligned} \tag{8}$$

The distribution function for electrons leaving the scatterer through the lead α is $f^{(\mathrm{out})}_\alpha(E) = \langle \hat{b}^\dagger_\alpha(E)\hat{b}_\alpha(E)\rangle$, where $\langle\cdots\rangle$ means quantum-mechanical averaging. Substituting Eq. (8) into the above equation we find

$$\begin{aligned}f^{(\mathrm{out})}_\alpha(E) = \sum_\beta [&|\hat{s}_{0,\alpha\beta} + \hat{s}_{2,\alpha\beta}|^2 f_0(E) \\ &+ |\hat{s}_{-\omega,\alpha\beta}|^2 f_0(E+\hbar\omega) \\ &\times |\hat{s}_{+\omega,\alpha\beta}|^2 f_0(E-\hbar\omega) \\ &+ |\hat{s}_{-2\omega,\alpha\beta}|^2 f_0(E+2\hbar\omega) \\ &+ |\hat{s}_{+2\omega,\alpha\beta}|^2 f_0(E-2\hbar\omega)].\end{aligned} \tag{9}$$

The distribution function for outgoing carriers is a nonequilibrium distribution function generated by the nonstationary scatterer. The Fourier amplitudes of the scattering matrix $|\hat{s}_{-\omega,\alpha\beta}|^2$ ($|\hat{s}_{+\omega,\alpha\beta}|^2$) is the probability for an electron entering the scatterer through the lead β and leaving the scatterer through the lead α to emit (to absorb) an energy quantum $\hbar\omega$ and $|\hat{s}_{-2\omega,\alpha\beta}|^2$ ($|\hat{s}_{+2\omega,\alpha\beta}|^2$) is that of the energy quantum $2\hbar\omega$ process. The $|\hat{s}_{0,\alpha\beta} + \hat{s}_{2,\alpha\beta}|^2$ is the probability for the same scattering without the change of an energy with the second-order term $\hat{s}_{2,\alpha\beta}$ much smaller than the zero-order term $\hat{s}_{0,\alpha\beta}$ in weak-modulation limit ($X_{\omega,j} \ll X_{0,j}$) and can be omitted therein.

Using the distribution functions $f_0(E)$ for incoming electrons and $f^{\mathrm{out}}_\alpha(E)$ for outgoing electrons, we find that the pumped current measured at lead α is

$$I_{\mathrm{p}} = \frac{e}{2\pi\hbar}\int_0^\infty \langle \hat{b}^\dagger_\alpha(E)\hat{b}_\alpha(E)\rangle - \langle \hat{a}^\dagger_\alpha(E)\hat{a}_\alpha(E)\rangle \mathrm{d}E. \tag{10}$$

Substituting Eqs. (9) and (5) into Eq. (10) we obtain

$$\begin{aligned}I_{\mathrm{p}} = &\frac{e\omega}{2\pi}\sum_{\beta,j_1,j_2} X_{\omega,j_1}X_{\omega,j_2}\frac{\partial s_{\alpha\beta}}{\partial X_{j_1}}\frac{\partial s^*_{\alpha\beta}}{\partial X_{j_2}} 2\,\mathrm{i}\sin(\varphi_{j_1} - \varphi_{j_2}) \\ &+ \frac{e\omega}{2\pi}\sum_{\beta,j_1,j_2} X^2_{\omega,j_1}X^2_{\omega,j_2} \\ &\times \frac{\partial^2 s_{\alpha\beta}}{\partial X^2_{j_1}}\frac{\partial^2 s^*_{\alpha\beta}}{\partial X^2_{j_2}}\,\mathrm{i}\sin[2(\varphi_{j_1} - \varphi_{j_2})].\end{aligned} \tag{11}$$

Quantum pumping properties beyond former theory based on first-order parametric derivative of the scattering matrix are demonstrated in Eq. (11). By taking higher orders of the Fourier spectrum of the scattering matrix into consideration, double $\hbar\omega$ energy quantum (or a $2\hbar\omega$ energy quantum) emission (absorption) processes coact with single $\hbar\omega$ quantum processes. In the weak-modulation limit, the second term on the right-hand side of Eq. (11) is small, which implies that double $\hbar\omega$ quantum processes are weak and therefore not observable. As the AC driving amplitude is enlarged, this term increases markedly and contribution from double $\hbar\omega$ quantum processes takes effect. As a result, the dependence of the pumped current on the phase difference between two driving oscillations deviates from sinusoidal and changes from $\sin\phi$ to $\sin 2\phi$, which is observed in experiment.[3] Moreover, the relation between the pumped current and the AC driving

amplitude $X_{\omega,j}$ is reshaped. It is also seen that the linear dependence of the pumped current on the oscillation frequency holds for multi-quanta-related processes. In the next section, numerical results of the pumped current in a two-oscillating-potential-barrier modulated nanowire are presented and compared with experiment.

3. Numerical results and interpretations

We consider a nanowire modulated by two gate potential barriers with equal width $L = 20$ Å (1 Å=0.1 nm) separated by a $2L = 40$ Å width well (see Fig. 1). The electrochemical potential of the two reservoirs μ is set to be 60 meV according to the resonant level within the double-barrier structure. The two oscillating parameters in Eq. (3) correspond to the two AC driven potential gates $X_{1,2}(t) \to U_{1,2}(t)$ while all the other notations correspond accordingly. We set the static magnitude of the two gate potentials $U_{0,1} = U_{0,2} = U_0 = 100$ meV and the AC driving amplitude of the modulations equal $U_{\omega,1} = U_{\omega,2} = U_\omega$.

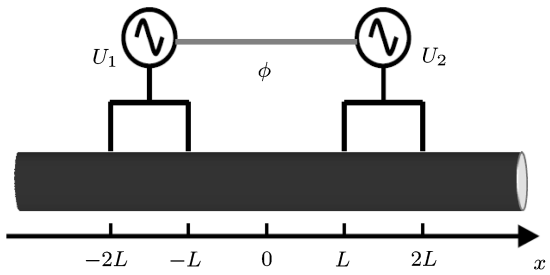


Fig. 1. Schematic diagram of the quantum pump: a nanowire modulated by two AC driven potential barriers.

In Fig. 2, the dependence of the pumped current on the phase difference between the two AC oscillations is presented. In weak-modulation regime (namely U_ω are small), sinusoidal behaviour dominates. Here, three relatively large U_ω are selected to reveal the deviation from the sinusoidal dependence. (The magnitude of the pumped current mounts up in power-law relation as a function of U_ω as shown in Fig. 3. The sinusoidal curve for small U_ω would be flat and invisible in the same coordinate range.) It can be seen from the figure that the $I_{\rm p}$–ϕ relation varies from sinusoidal ($\sin\phi$) to double-sinusoidal ($\sin 2\phi$) as the AC oscillation amplitude is increased. The interpretation follows from Eq. (11). The single $\hbar\omega$ quantum emission (absorption) processes feature a sinusoidal behaviour while the $2\hbar\omega$ quantum emission (absorption) processes feature a double-sinusoidal behaviour when the Fourier index is doubled. As U_ω increases, double $\hbar\omega$ quantum processes gradually parallel and outweigh the single $\hbar\omega$ quantum ones. It is also demonstrated that when the single $\hbar\omega$ quantum processes have the effect of $\sin\phi$ dependence, the double $\hbar\omega$ quantum processes induce a $-\sin 2\phi$ contribution with a sign flip, which can be understood from the sign change of the derivative of the scattering matrix. The effect of three- and higher $\hbar\omega$ quantum processes is small even for large U_ω comparable to U_0. The experimental observations[3] as a deviation from the weak-modulation limit are revealed by our theory.

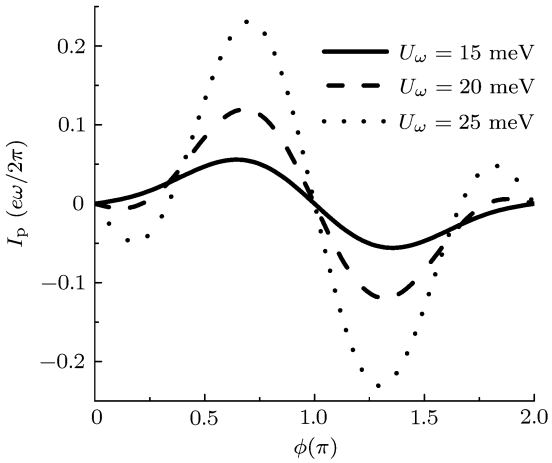


Fig. 2. Pumped current as a function of the phase difference between the two modulations for different AC driving amplitudes.

Experiment[3] also discovered that for weak pumping the dependence of the pumped current on the pumping strength obeys a power of 2 relation, as expected from the simple loop-area argument;[4–6] for strong pumping, power of 1 and 1/2 relation is observed beyond former theory. We presented in Fig. 3 the numerical results based on our theory of the $I_{\rm p}$–U_ω relation at a fixed ϕ. To demonstrate its power-law dependence, natural logarithm of the variables is applied. From Eq. (11), it can be seen that for large AC driving amplitude U_ω, contribution of double $\hbar\omega$ quantum processes (formulated in the second term on the right-hand side of the equation) causes the $I_{\rm p}$–U_ω relation to deviate from its weak-modulation limit, the latter of which is $I_{\rm p} \propto U_\omega^2$. For different phase difference between the two AC drivers, the deviation is different. At $\phi = \pi$ the pumped current is invariably zero regardless of the order of approximation determined by time-reversal symmetry. At $\phi = \pi/2$, $\sin 2\phi$

is exact zero, and no difference is incurred by introducing higher order effect. If we shift the value of ϕ to 0.49π, the abating effect of the double $\hbar\omega$ quantum processes has the order of U_ω^4 with the small second-order parametric derivative of the scattering matrix smoothing that effect a bit. Consequently, a power of $2 \to 1 \to 1/2$ relation is obtained and visualized by

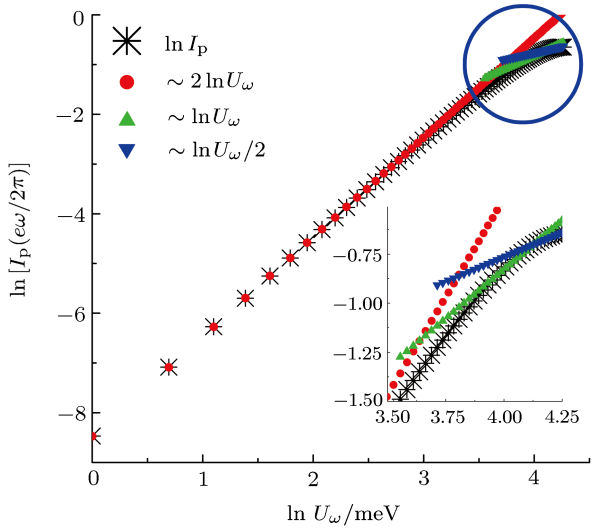


Fig. 3. Pumped current as a function of the AC driving amplitude U_ω along with fits to $I_p \propto U_\omega^2$ (red solid circle) below 35 meV, $I_p \propto U_\omega$ (green upward triangle) below 41 meV, and $I_p \propto U_\omega^{1/2}$ above 41 meV (blue downward triangle). To demonstrate its power-law dependence, natural logarithm of the variables is applied. The phase difference between the two AC driver $\phi = 0.49\pi$. Inset is the zoom-in of the circled region.

our curve fit, which is analogous to experimental findings. For different values of ϕ, sharper abating and augmental effect occurs with analogous mechanisms. It is possible that the experiment[3] was done at the phase difference close to $\pi/2$ while trying to approach maximal pumped current in the adiabatic and weak-pumping limit.

4. Conclusions

Based on the AC scattering approach, we further expand the time-dependent scattering matrix to higher orders of the modulation amplitude and the time. It is demonstrated in our theory that $2\hbar\omega$ quantum emission (absorption) processes coact with those of single $\hbar\omega$ quantum when we go beyond the small-frequency and weak-modulation limit. Nonsinusoidal dependence on the phase difference between two oscillating modulations is incurred by higher order Fourier components. The pumped current versus modulation amplitude relation has a power law of $2 \to 1 \to 1/2$ passage with the increase of the oscillating amplitude. Numerical results for a two-AC-gate modulated nanowire interpret experimental findings at large AC driving amplitudes.

Acknowledgements

The author would like to express sincere appreciation to professor Wenji Deng, Dr. Brian M. Walsh, professor Jamal Berakdar, professor Michael Moskalets, and professor Liliana Arrachea for valuable enlightenment on the topics discussed with them.

References

[1] Kouwenhoven L P, Johnson A T, van der Vaart N C, Harmans C J P M and Foxon C T 1991 *Phys. Rev. Lett.* **67** 1626
[2] Thouless D J 1983 *Phys. Rev.* B **27** 6083
[3] Switkes M, Marcus C M, Campman K and Gossard A C 1999 *Science* **283** 1905
[4] Brouwer P W 1998 *Phys. Rev.* B **58** R10135
[5] Büttiker M, Thomas H and Prêtre A 1994 *Z. Phys.* B **94** 133
[6] Büttiker M, Prêtre A and Thomas H 1993 *Phys. Rev. Lett.* **70** 4114
[7] Moskalets M and Büttiker M 2002 *Phys. Rev.* B **66** 035306
[8] Benjamin R and Benjamin C 2004 *Phys. Rev.* B **69** 085318
[9] Park H C and Ahn K H 2008 *Phys. Rev. Lett.* **101** 116804
[10] Devillard P, Gasparian V and Martin T 2008 *Phys. Rev.* B **78** 085130
[11] Citro R and Romeo F 2006 *Phys. Rev.* B **73** 233304
[12] Moskalets M and Büttiker M 2005 *Phys. Rev.* B **72** 035324
[13] Moskalets M and Büttiker M 2007 *Phys. Rev.* B **75** 035315
[14] Romeoa F and Citro R 2006 *Euro. Phys. J.* B **50** 483
[15] Splettstoesser J, Governale M and König J 2008 *Phys. Rev.* B **77** 195320
[16] Strass M, Hänggi P and Kohler S 2005 *Phys. Rev. Lett.* **95** 130601
[17] Avron J E, Elgart A, Graf G M and Sadun L 2001 *Phys. Rev. Lett.* **87** 236601
[18] Wang B G, Wang J and Guo H 2002 *Phys. Rev.* B **65** 073306
[19] Wang B G and Wang J 2002 *Phys. Rev.* B **66** 125310
[20] Wang B G, Wang J and Guo H 2003 *Phys. Rev.* B **68** 155326
[21] Arrachea L 2005 *Phys. Rev.* B **72** 125349
[22] Tserkovnyak Y, Brataas A, Bauer G E W and Halperin B I 2005 *Rev. Mod. Phys.* **77** 1375

[23] Ralph D C and Stiles M D 2008 *J. Magn. Magn. Mater.* **320** 1190
[24] Zhu R and Chen H 2009 *Appl. Phys. Lett.* **95** 122111
[25] Prada E, San-Jose P and Schomerus H 2009 Arxiv:0907.1568v1 (unpublished)
[26] Agarwal A and Sen D 2007 *J. Phys.: Condens. Matter* **19** 046205
[27] Torres L E F F 2005 *Phys. Rev.* B **72** 245339
[28] Agarwal A and Sen D 2007 *Phys. Rev.* B **76** 235316
[29] Winkler N, Governale M and König J 2009 *Phys. Rev.* B **79** 235309
[30] Qi X L and Zhang S C 2009 *Phys. Rev.* B **79** 235442
[31] Wright S J, Blumenthal M D, Pepper M, Anderson D, Jones G A C, Nicoll C A and Ritchie D A 2009 *Phys. Rev.* B **80** 113303
[32] Romeo F and Citro R 2009 *Phys. Rev.* B **80** 165311
[33] Moskalets M and Büttiker M 2002 *Phys. Rev.* B **66** 205320
[34] Li L, Kaestner B, Blumenthal M D, Giblin S, Janssen T J B M, Pepper M, Anderson D, Jones G, Ritchie D A and Gao J 2008 *Acta Phys. Sin.* **57** 1878 (in Chinese)
[35] Wu F and Wang T H 2003 *Acta Phys. Sin.* **52** 696 (in Chinese)
[36] Landauer R 1957 *IBM J. Res. Develop.* **1** 223
[37] Landauer R 1970 *Phil. Mag.* **21** 863
[38] Büttiker M, Imry Y, Landauer R and Pinhas S 1985 *Phys. Rev.* B **31** 6207
[39] Polianski M L, Vavilov M G and Brouwer P W 2002 *Phys. Rev.* B **65** 245314