Hollow Gaussian beams in strongly nonlocal nonlinear media*

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The propagation of hollow Gaussian beams in strongly nonlocal nonlinear media is studied in detail. Two analytical expressions are derived. For hollow Gaussian beams, the intensity distribution always evolves periodically. However the second-order moment beam width can keep invariant during propagation if the input power is equal to the critical power. The interaction of two hollow Gaussian beams and the vortical hollow Gaussian beams are also discussed. The vortical hollow Gaussian beams with an appropriate topological charge can keep their shapes invariant during propagation.

Keywords: strong nonlocality, hollow Gaussian beam, beam propagation

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1. Introduction

In past several years, the so-called dark hollow beams with zero central intensity have attracted much attention because of their wide applications in atom optics.[1−3] Experimentally, various techniques have been used to generate various dark hollow beams.[4−6] Theoretically, several models are introduced to describe dark hollow beams. A useful mathematical model, i.e. the hollow Gaussian beam,[7] is introduced to describe the propagation of dark hollow beams and analyse guiding, focusing, and trapping of ultracold atoms, even Bose–Einstein condensates. Subsequently Deng *et al.*[8] calculated the propagation factor of hollow Gaussian beams. Gao *et al.*[9] studied the nonparaxial propagation of hollow Gaussian beams. Wu *et al.*[10] described the vectorial structure of hollow Gaussian beams in far field. More recently, Liu *et al.*[11] demonstrated how to generate hollow Gaussian beams by phase-only filtering. So far the propagation properties of the hollow Gaussian beams have been investigated through several optical systems (or in some media) such as a paraxial misaligned optical system,[12] an apertured paraxial optical system,[13] fractional Fourier transform system,[14] a turbulent atmosphere,[15] a uniaxial crystal,[16] and so on.

On the other hand, an important class of material systems is the nonlocal media. Such nonlocality arises in several branches of physics, ranging from Bose–Einstein condensates to atomic vapour.[17−21] If the characteristic length of the material response function is much longer than the beam width, these media can be called strongly nonlocal nonlinear media (SNNM). In optics, strong nonlocality can be encountered in several materials, for example liquid crystals[19,20] and thermal media.[21] Recently, the beam propagation in SNNM has also been paid increasing attention due to its intriguing properties different from those in local media.[22−34] To the best of our knowledge, the propagation of a hollow Gaussian beam in SNNM remains unexploited. In this paper, the propagation of hollow Gaussian beams in SNNM is investigated in detail. Two analytical expressions describing the propagation of hollow Gaussian beams and the beam width are derived. The interaction of two hollow Gaussian beams and the vortical hollow Gaussian beams are also discussed.

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2. Analytical expressions of hollow Gaussian beams

The propagation of the (1+2)-dimensional optical beams in nonlocal nonlinear media is governed phenomenologically by the nonlocal nonlinear Schrödinger equation (NNLSE)

$$2\,\mathrm{i}\,kn_0\frac{\partial \Phi}{\partial z}+n_0\nabla_\perp^2\,\Phi+2k^2\triangle n\,\Phi=0, \tag{1}$$

where $\Phi=\Phi(\boldsymbol{r})$ is the complex amplitude of the light field, $\boldsymbol{r}$ is the two-dimensional transverse coordinate vectors, k is the wave number in the media without nonlinearity, n_0 is the linear refractive index of the media, $\nabla_\perp^2=(\partial_x^2+\partial_y^2)$ is the two-dimensional transverse Laplacian operator. The nonlinear perturbation of the refractive index $\triangle n$ can be generally expressed as

$$\triangle n=n_2\int_{-\infty}^{+\infty}R(\boldsymbol{r}-\boldsymbol{r}')|\,\Phi(\boldsymbol{r}',z)|^2\mathrm{d}^2\boldsymbol{r}', \tag{2}$$

where n_2 is the nonlinear index coefficient, and $R(\boldsymbol{r})$ is the normalized symmetrical real spatial response function of the media. If R is a delta function, equation (1) degenerates to the well-known nonlinear Schrödinger equation in the local nonlinear media.

For the strongly nonlocal case, the NNLSE can be deduced to the linear Snyder–Mitchell model which can be expressed in the form in cylindrical coordinates (r,θ,z),

$$2\,\mathrm{i}\,k\frac{\partial \Phi}{\partial z}+\frac{1}{r}\frac{\partial \Phi}{\partial r}+\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}-k^2\gamma^2P_0r^2\,\Phi=0, \tag{3}$$

where γ is a material constant associated with the response function R, and P_0 is the input power. Because the Snyder–Mitchell model is a linear model, the linear method can be employed to investigate the beam propagation in SNNM.[23,24] It has been demonstrated that the beam propagation in SNNM can be regarded as a self-induced fractional Fourier transform[23,35]

$$\begin{aligned}\mathscr{F}_\alpha\{\Phi(x_0,y_0)\}(x,y)\,=\,&\frac{\exp[\mathrm{i}(\alpha-\pi/2)]}{2\pi w_\mathrm{p}^2\sin\alpha}\exp\left[\frac{\mathrm{i}(x^2+y^2)}{2w_\mathrm{p}^2\tan\alpha}\right]\\&\times\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Phi(x_0,y_0)\exp\left[\frac{\mathrm{i}(x_0^2+y_0^2)}{2w_\mathrm{p}^2\tan\alpha}-\frac{\mathrm{i}(xx_0+yy_0)}{w_\mathrm{p}^2\sin\alpha}\right]\mathrm{d}x_0\mathrm{d}y_0,\end{aligned} \tag{4}$$

where $\mathscr{F}_\alpha\{\cdot\}$ represents the fractional Fourier transform, $\alpha=\sqrt{P_0}\gamma z$ is the transform order, $w_\mathrm{p}^2=1/(k\gamma\sqrt{P_0})$. In cylindrical coordinates, equation (4) can be rewritten as

$$\begin{aligned}\mathscr{F}_\alpha\{\Phi(r_0,\theta_0)\}(r,\theta)\,=\,&\frac{\exp[\mathrm{i}(\alpha-\pi/2)]}{2\pi w_\mathrm{p}^2\sin\alpha}\exp\left[\frac{\mathrm{i}\,r^2}{2w_\mathrm{p}^2\tan\alpha}\right]\\&\times\int_0^{2\pi}\int_0^{\infty}\Phi(r_0,\theta_0)\exp\left[\frac{\mathrm{i}\,r_0^2}{2w_\mathrm{p}^2\tan\alpha}-\frac{2\,\mathrm{i}\,rr_0\cos(\theta-\theta_0)}{w_\mathrm{p}^2\sin\alpha}\right]r_0\mathrm{d}r_0\mathrm{d}\theta_0.\end{aligned} \tag{5}$$

The optical field distribution of hollow Gaussian beams at the source plane is given by[7]

$$\Phi(r_0)=C_0\left(\frac{r_0^2}{2w_0^2}\right)^n\exp\left(-\frac{r_0^2}{2w_0^2}\right), \tag{6}$$

where $n=0,1,2,3,\ldots$ is the order of hollow Gaussian beams. If $n=0$, equation (6) reduces to the fundamental Gaussian beam. The radius of the bright ring in the irradiance distribution of a hollow Gaussian beam increases with n increasing. C_0 is a normalized constant associated with the order of n and ensures the input power to be P_0. w_0 is the waist width of Gaussian beam.

Substituting Eq. (6) into Eq. (5) and recalling the integral formulae[36]

$$\mathrm{J}_0(x)=\frac{1}{2\pi}\int_0^{2\pi}\exp(\mathrm{i}\,x\cos\theta)\mathrm{d}\theta \tag{7}$$

and

$$\int_0^{\infty}x^{n+\frac{1}{2}\nu}\mathrm{e}^{-\alpha x}\mathrm{J}_\nu\left(2\beta\sqrt{x}\right)\mathrm{d}x=n!\beta^\nu\mathrm{e}^{-\beta^2/\alpha}\alpha^{-n-\nu-1}\mathrm{L}_n^\nu\left(\frac{\beta^2}{\alpha}\right) \tag{8}$$

with $\mathrm{J}_\nu(x)$ being the Bessel function and $\mathrm{L}_n^\nu(x)$ being Laguerre polynomials, after tedious integral calculations, we obtain the propagation expression as

$$\Phi(r,\theta,z)\,=\,\frac{\mathrm{i}\,C_0n!(w'^2\tan\alpha)^n}{\cos\alpha(w'^4\tan^2\alpha+1)^{(n+1)/2}}\mathrm{L}_n\left[\frac{(w'^2\tan\alpha+\mathrm{i})r'^2}{w'^2\sin2\alpha(w'^4\tan^2\alpha+1)}\right]\exp\left[-\frac{(\tan^2\alpha+1)r'^2}{2(w'^4\tan^2\alpha+1)}\right]$$

$$\times \exp\left[\mathrm{i}\frac{\tan\alpha(w'^4-1)r'^2}{2w'^2(w'^4\tan^2\alpha+1)}-\mathrm{i}(n+1)\arctan\left(\frac{\cos\alpha}{w'^2\sin\alpha}\right)\right], \tag{9}$$

where $w'=w_{\mathrm{p}}/w_0$, $r'=r/w_0$.

By using the following formula,

$$x^{2n}=\frac{n!}{2^n}\sum_{m=0}^{n}(-1)^m A_{nm}\mathrm{L}_m(2x^2), \tag{10}$$

where A_{nm} denotes a binomial coefficient, equation (6) can be also expressed in another form

$$\Phi(r_0)=\frac{n!}{2^n}\sum_{m=0}^{n}(-1)^m A_{nm}\mathrm{L}_m\left(2\frac{r_0^2}{w_0^2}\right)\exp\left(-\frac{r_0^2}{w_0^2}\right). \tag{11}$$

Substituting Eq. (11) into Eq. (5), after integration, we obtain the alternative propagation expression

$$\begin{aligned}\Phi(r,\theta,z)=&\frac{C_0 n!}{2^n}\sum_{m=0}^{n}\frac{(-1)^m A_{nm}}{\sqrt{\cos^2\alpha+w'^4\sin^2\alpha}}\mathrm{L}_m\left(\frac{r'^2}{w'^4\sin^2\alpha+\cos^2\alpha}\right)\exp\left[-\frac{r'^2}{2(w'^4\sin^2\alpha+\cos^2\alpha)}\right]\\&\times\exp\left[-\mathrm{i}\frac{\sin(2\alpha)(w'^4-1)}{4w'^2(w'^4\sin^2\alpha+\cos^2\alpha)}r'^2+\mathrm{i}(2m+1)\arctan\left(w'^2\frac{\sin\alpha}{\cos\alpha}\right)\right].\end{aligned} \tag{12}$$

Equations (9) and (12) are two analytical expressions describing the propagation of hollow Gaussian beams in SNNM.

3. Evolution of hollow Gaussian beams

3.1. Propagation of a single hollow Gaussian beam

According to Eq. (9), the intensity distribution of hollow Gaussian beams during propagation can be expressed as

$$\begin{aligned}I(r,\theta,z)=&\frac{(C_0 n!)^2(w'^2\tan\alpha)^{2n}}{\cos^2\alpha(w'^4\tan^2\alpha+1)^{n+1}}\\&\times\exp\left[-\frac{\tan^2\alpha+1}{w'^4\tan^2\alpha+1}r'^2\right]\\&\times\left|\mathrm{L}_n\left[\frac{(w'^2\tan\alpha+\mathrm{i})r'^2}{w'^2\sin 2\alpha(w'^4\tan^2\alpha+1)}\right]\right|^2.\end{aligned} \tag{13}$$

Then, one obtains the evolution of the on-axis intensity

$$I(0,z)=\frac{(C_0 n!)^2(w'^2\tan\alpha)^{2n}}{\cos^2\alpha(w'^4\tan^2\alpha+1)^{n+1}}. \tag{14}$$

The beam propagation in SNNM is always periodical.[27] For hollow Gaussian beams, it can be seen from Eqs. (13) and (14) that z always appears in $\tan(\alpha)$, therefore the beam evolution varies periodically. In fact, the propagation of hollow Gaussian beams in SNNM is the self-induced fractional Fourier transform, as a result, the propagation should be periodical. It is known that the soliton power of the Gaussian and higher Gaussian beams, i.e. the critical power, is $P_{\mathrm{gc}}=1/(\gamma^2 z_{\mathrm{R}}^2)$,[22,25,26] where z_{R} is Rayleigh distance of Gaussian beam. Then α can be expressed as

$$\alpha=\sqrt{m}\frac{z}{z_{\mathrm{R}}}, \tag{15}$$

where $m=P_0/P_{\mathrm{gc}}$. Hence one can obtain the evolution period of hollow Gaussian beams,

$$\Delta z=\frac{\pi z_{\mathrm{R}}}{\sqrt{m}}. \tag{16}$$

As shown in Eq. (16) that the shorter the period is, the higher the input power is, because a higher input power provides a stronger self-focusing.

Because of the complexity of the intensity distribution of hollow Gaussian beams during propagation, we take the second-order moment beam width to describe the beam width. For the axisymmetric beams, the second-order moment beam width is defined as

$$w^2(z)=\frac{2\int_0^{+\infty}\int_0^{2\pi}|\Phi(r,\theta,z)|^2 r^3\mathrm{d}r\mathrm{d}\theta}{\int_0^{+\infty}\int_0^{2\pi}|\Phi(r,\theta,z)|^2 r\mathrm{d}r\mathrm{d}\theta}. \tag{17}$$

By following the method in Ref. [37], we obtain the beam width of hollow Gaussian beams

$$\begin{aligned}w^2(z)=&w_0^2\left[(2n+1)\cos^2\left(\sqrt{m}\frac{z}{z_{\mathrm{R}}}\right)\right.\\&\left.+\frac{1}{m}\sin^2\left(\sqrt{m}\frac{z}{z_{\mathrm{R}}}\right)\right].\end{aligned} \tag{18}$$

According to Eq. (18), the power with which the beam width of hollow Gaussian beams keeps invariant during propagation, i.e. the critical power, is obtained

$$P_c = \frac{1}{2n+1} P_{\rm gc}. \tag{19}$$

Figure 1 gives the intensity evolution of hollow Gaussian beams in SNNM. It shows that the intensity evolution varies with a period of $\pi z_{\rm R}/\sqrt{m}$. From 0 to $\pi z_{\rm R}/2\sqrt{m}$, the beam energy converges to the centre gradually, and the maximal intensity emerges in the central area at $\pi z_{\rm R}/2\sqrt{m}$. From $\pi z_{\rm R}/2\sqrt{m}$ to $\pi z_{\rm R}/\sqrt{m}$, the evolution of the intensity distribution is the inverse process of that from 0 to $\pi z_{\rm R}/2\sqrt{m}$. At the plane of $\pi z_{\rm R}/\sqrt{m}$, the intensity distribution comes back to the original distribution. This is identical with the period of the fractional Fourier transform.

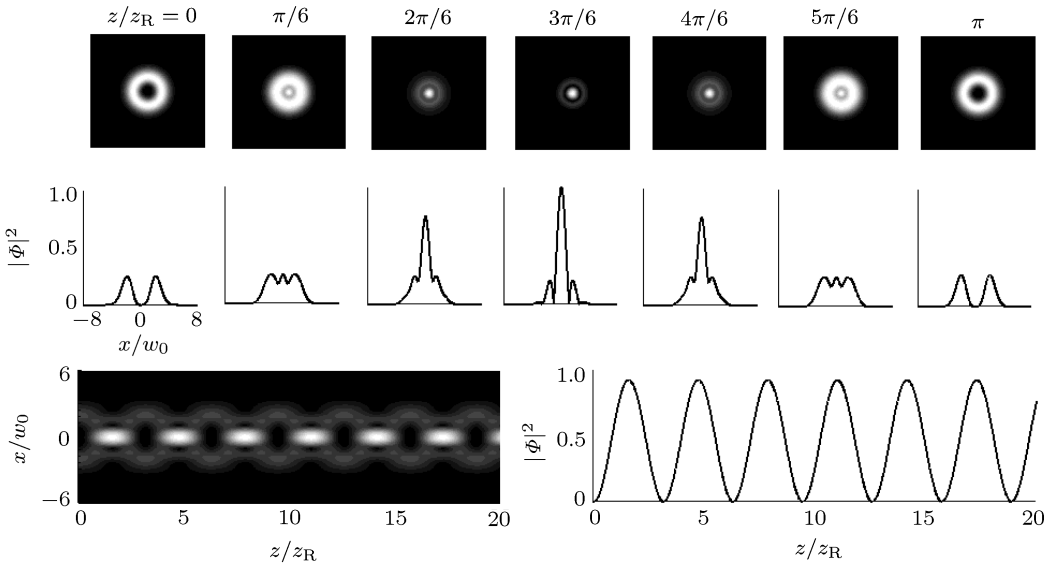


Fig. 1. Evolutions of the intensity of the hollow Gaussian beam. Row 1: the transversal intensity distributions at different distances given in the top of the figure; row 2: the transversal intensity distributions in x direction at the distances corresponding to row 1; row 3: the evolution of the intensity of the hollow Gaussian beam (left) and the evolution of the on-axis intensity of the hollow Gaussian beam (right). Parameters: $n = 2$, $P_0 = P_{\rm gc}$.

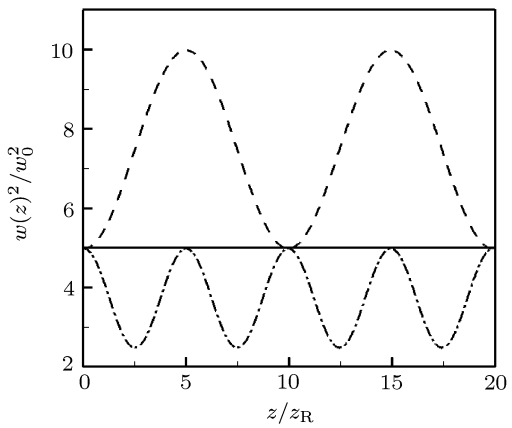


Fig. 2. Evolutions of the beam width of the hollow Gaussian beam for $n = 2$. Solid curve: $P_0 = 0.2P_{\rm gc}$; dashed curve: $P_0 = 0.1P_{\rm gc}$; dash-dotted curve: $P_0 = 0.4P_{\rm gc}$.

Figure 2 shows the evolution of the beam width with different input powers. When the input power is equal to the critical power ($P_{\rm c} = 0.2P_{\rm gc}$ for $n = 2$), the beam width keeps invariant during propagation. When the input power is not equal to the critical power, the beam width varies periodically during propagation. The shorter the period is, the higher the input power is, which is in agreement with the evolution of the intensity. The beam width variation of hollow Gaussian beams is similar to that of Gaussian beams. The difference is that the shape of hollow Gaussian beams does vary during propagation, however the shape of Gaussian beams remains invariant.

3.2. Interaction of two hollow Gaussian beams

By utilizing the linear rule and the shift rule of the fractional Fourier transform, one can further investigate the interaction of two or more beams. The linear rule and the shift rule are given by[35]

$$\mathscr{F}_\alpha\{Ag(\xi)+Bh(\xi)\}(x)=A\mathscr{F}_\alpha\{g(\xi)\}(x)+B\mathscr{F}_\alpha\{h(\xi)\}(x), \tag{20}$$

and

$$\mathscr{F}_\alpha\{g(\xi+a)\}(x)=\exp\left[\mathrm{i}\,a\sin\alpha\left(x+\frac{a\cos\alpha}{2}\right)\right]\mathscr{F}_\alpha\{g(\xi)\}(x+a\cos\alpha), \tag{21}$$

where A, B, a are constants. In the following the interaction of two beams is illustrated as an example.

As shown in the first figure in row 1 in Fig. 3, we suppose that the centres of two beams at the source plane are located at $(\pm d, 0)$, i.e.

$$\Phi_\pm(r_0,\theta_0)=C_0\left[\frac{(r_0\cos\theta_0\pm d)^2+(r_0\sin\theta_0)^2}{2w_0^2}\right]^n\exp\left[-\frac{(r_0\cos\theta_0\pm d)^2+(r_0\sin\theta_0)^2}{2w_0^2}\right]. \tag{22}$$

Combining Eqs. (9), (20), (21) and (22), the combined filed of the two beams, i.e. $\Phi_A=\Phi_+ + \Phi_-$, can be obtained as

$$\begin{aligned}\Phi_A(r,\theta,z)=&\frac{\mathrm{i}\,C_0 n!}{w'^2\sin\alpha}\left(\frac{w'^2\tan\alpha}{w'^2\tan\alpha+\mathrm{i}}\right)^{n+1}\exp\left[-\frac{(\mathrm{i}\,w'^2\cos\alpha+\sin\alpha)r'^2\sin^2\theta}{2w'^2(w'^2\sin\alpha+\mathrm{i}\cos\alpha)}+\frac{\mathrm{i}\,d'^2\sin 2\alpha}{4w'^2}\right]\\
&\times\left\{\exp\left[-\frac{(\mathrm{i}\,w'^2\cos\alpha+\sin\alpha)(r'\cos\theta-d'\cos\alpha)^2}{2w'^2(w'^2\sin\alpha+\mathrm{i}\cos\alpha)}-\frac{\mathrm{i}\,d'r'\cos\theta\sin\alpha}{2w'^2}\right]\right.\\
&\times\mathrm{L}_n\left[\frac{(r'\cos\theta-d'\cos\alpha)^2+r'^2\sin^2\theta}{w'^2(2w'^2\sin^2\alpha+\mathrm{i}\sin 2\alpha)}\right]\\
&+\exp\left[-\frac{(\mathrm{i}\,w'^2\cos\alpha+\sin\alpha)(r'\cos\theta+d'\cos\alpha)^2}{2w'^2(w'^2\sin\alpha+\mathrm{i}\cos\alpha)}+\frac{\mathrm{i}\,d'r'\cos\theta\sin\alpha}{2w'^2}\right]\\
&\times\left.\mathrm{L}_n\left[\frac{(r'\cos\theta+d\cos\alpha)'^2+r'^2\sin^2\theta}{w'^2(2w'^2\sin^2\alpha+\mathrm{i}\sin 2\alpha)}\right]\right\}.\end{aligned} \tag{23}$$

As shown in Eq. (23) and Fig. 3, the propagation is periodical which is similar to that of a single beam. Moreover, the combined intensity distribution becomes more complicated, and there occur the interference fringes in the combined intensity pattern because of the interference of the two beams. The pattern is no longer axisymmetric.

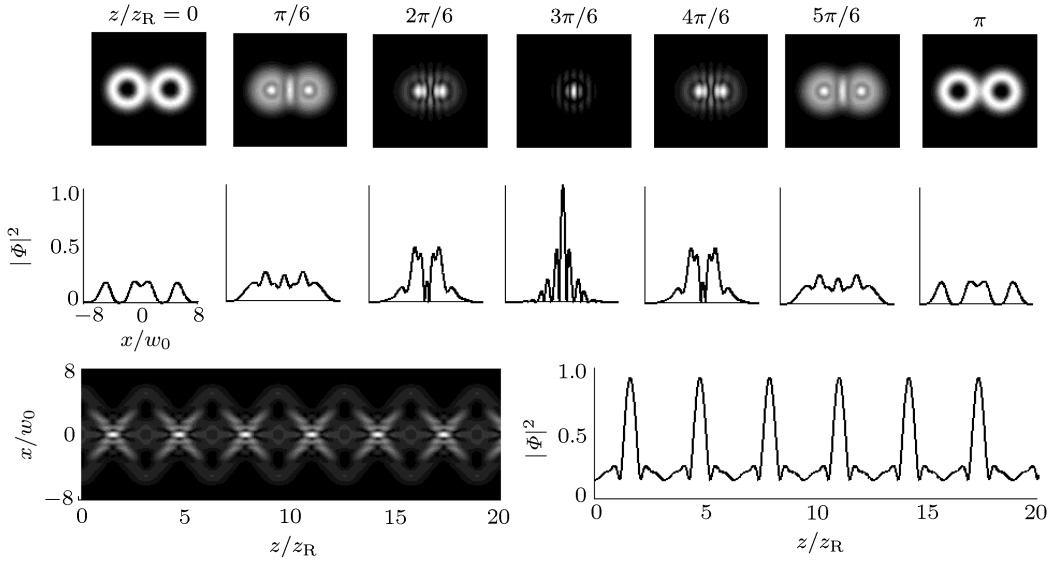


Fig. 3. Evolutions of the intensity of two interactional hollow Gaussian beams. Row 1: the transversal intensity distributions at different distances; row 2: the transversal intensity distributions in x direction at the distances corresponding to row 1; row 3: the evolution of the intensity of two interactional hollow Gaussian beam (left) and the evolution of the on-axis intensity of two interactional hollow Gaussian beam (right). Parameters: $n=2, d=\pm 3.0w_0$, $P_0=0.5P_\mathrm{gc}$.

4. Vortical hollow Gaussian beams

Nowadays the vortical beams have also attracted much attention. If a phase factor is added to the hollow Gaussian beams an optical vortex will be induced. One can obtain the vortical hollow Gaussian beams

$$\Phi(r_0) = C_0 \left(\frac{r_0^2}{2w_0^2}\right)^n \exp\left(-\frac{r_0^2}{2w_0^2}\right) \exp(2\,\mathrm{i}\, n\theta_0), \tag{24}$$

where n is the topological charge of the vortex.

Substituting Eq. (24) into Eq. (5) and recalling the integral formulae[36]

$$\int_0^{2\pi} \exp[\mathrm{i}\,\beta\cos(\theta-\theta_0)]\exp(-\mathrm{i}\,n\theta_0)\mathrm{d}\theta_0 = \mathrm{i}^n 2\pi \mathrm{J}_n(\beta)\exp(-\mathrm{i}\,m\theta), \tag{25}$$

$$\int_0^{\infty} x^{v+1}\mathrm{e}^{-\alpha x^2}\mathrm{J}_v(\beta x)\mathrm{d}x = \frac{\beta^v}{(2\alpha)^{v+1}}\exp\left(-\frac{\beta^2}{4\alpha}\right), \tag{26}$$

after complex integral calculations, we obtain the propagation expression of vortical hollow Gaussian beams in SNNM

$$\Phi(r,\theta,z) = \left[\frac{1+\mathrm{i}\,w'^2\tan\alpha}{\cos\alpha(w'^4\tan^2\alpha+1)}\right]^{2n+1} r'^{2n}\exp\left[-\mathrm{i}\,\frac{r'^2}{w'^2\tan\alpha} - \frac{r'^2}{w'^2\sin\alpha\cos\alpha(\mathrm{i}+w'^2\tan\alpha)} - \mathrm{i}\,2n\theta\right], \tag{27}$$

which is identical with the $(0, 2n)$ mode Laguerre Gaussian beams.[25]

According to Eq. (27), the intensity distribution is

$$I(r,\theta,z) = \frac{r'^{2n}}{(w'^4\sin^2\alpha+\cos^2\alpha)^{n+1/2}}\exp\left(-\frac{r'2}{w'^4\sin^2\alpha+\cos^2\alpha}\right). \tag{28}$$

According to Eq. (17), the beam width is

$$w(z)^2 = w_0^2(w'^4\sin^2\alpha+\cos^2\alpha). \tag{29}$$

Thus the critical input power of vortical hollow Gaussian beams is

$$P_{\mathrm{c}} = P_{\mathrm{gc}}. \tag{30}$$

It is shown in Eq. (28) that the pattern of vortical hollow Gaussian beams keeps invariant and the on-axis intensity always equals zero during propagation, which is much different from the general hollow Gaussian beams mentioned in the previous section.

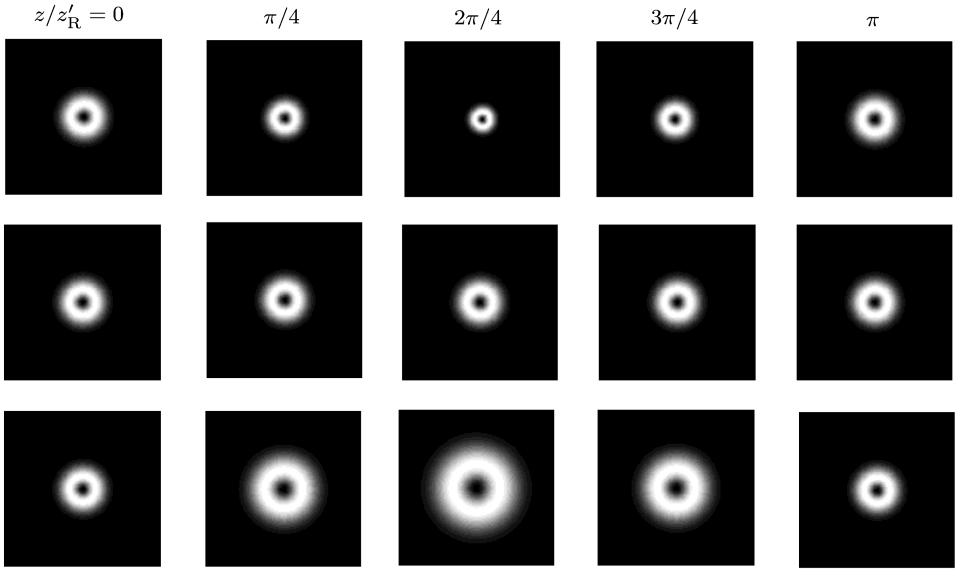


Fig. 4. Evolutions of the intensity distributions of the vortical hollow Gaussian beam for $n = 1$ with different input powers at different distances. $z'_{\mathrm{R}} = z_{\mathrm{R}}/\sqrt{m}$. Row 1: $P_0 = 3.0P_{\mathrm{gc}}$; row 2: $P_0 = 1.0P_{\mathrm{gc}}$; row 3: $P_0 = 0.3P_{\mathrm{gc}}$.

As shown in Fig. 4, the vortical hollow Gaussian beam propagates as a soliton when the input power is equal to the critical power (row 2), otherwise a breather comes (row 1 and row 3). The vortex of optics beams always relates to the singularity of phase. Figure 5 gives the phase comparison between the vortical hollow Gaussian beams and the hollow Gaussian beams during propagation. The vortical hollow Gaussian beam owns a staircase-like phase distribution, and it is a nonlinear function of the polar angle. The intensity peak with the larger energy moves along the phase gradient, therefore the beam has an angular momentum. As a result the spin staircase-like phase in the ring preserves the beam shape for a hollow Gaussian beam with an appropriate parameter n.[28,29,38]

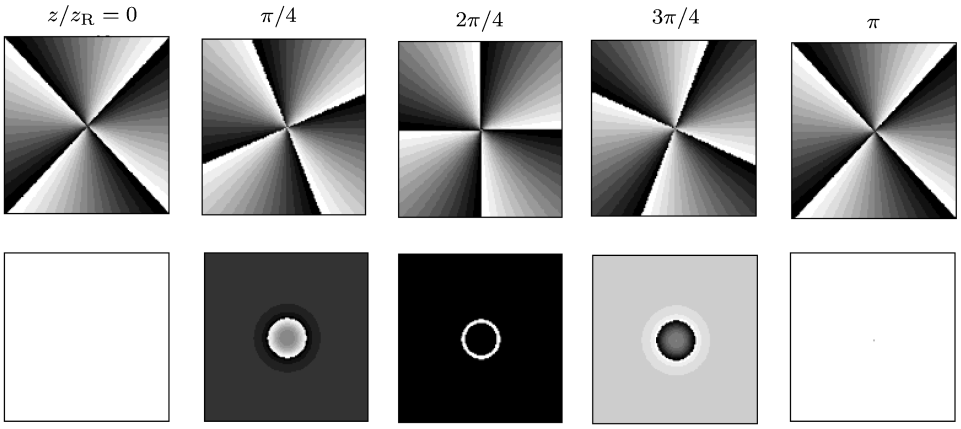


Fig. 5. Comparisons of the phase between the vortical hollow Gaussian beam (top row) and the hollow Gaussian beam (bottom row) at different distances. Parameters: $n=1, P_0=P_{\rm gc}$.

5. Conclusion

In conclusion, the propagation of hollow Gaussian beams in SNNM is investigated in detail. Two analytical expressions are derived. The results show that the intensity distribution of hollow Gaussian beams always varies with a period of $\Delta z=\pi z_{\rm R}/\sqrt{m}$. And the evolution in the first half period is the inverse process of the latter half period. The period is dependent on the input power. The shorter the period is, the higher the input power is, because a higher input power provides a stronger nonlinearity. The beam width can keep invariant if the input power is equal to the critical power, although the intensity distribution varies during propagation. The interaction of two hollow Gaussian beams in SNNM is more complicated. There occur the interference fringes in the combined intensity pattern. When the vortical hollow Gaussian beams with an appropriate topological charge reduce to solitons or breathers, their critical input power is equal to the critical input power of Gaussian beams independent of the order of beams. The difference of the phase distributions between the vortical hollow Gaussian beams and the general hollow Gaussian beams results in the difference of their propagation properties.

References

[1] Kuga T, Torii Y, Shiokawa N, Hirano T, Shimizu Y and Sasada H 1997 *Phys. Rev. Lett.* **78** 4713

[2] Ovchinnikov Y B, Manek I and Grimm R 1997 *Phys. Rev. Lett.* **79** 2225

[3] Song Y, Milam D and Hill W T 1999 *Opt. Lett.* **24** 1805

[4] Ito H, Sakaki K, Jhe W and Ohtsu M 1997 *Phys. Rev.* A **56** 712

[5] Liu Z, Zhao H, Liu J, Lin J, Ahmad M A and Liu S 2007 *Opt. Lett.* **32** 2076

[6] Anand S 2009 *Opt. Commun.* **282** 1335

[7] Cai Y J, Lu X H and Lin Q 2003 *Opt. Lett.* **28** 1084

[8] Deng D M 2005 *Phys. Lett.* A **341** 352

[9] Gao Z H and Lü B D 2006 *Phys. Lett.* A **23** 106

[10] Wu G H, Lou Q H and Zhou J 2008 *Opt. Express* **16** 6417

[11] Liu Z J, Dai J M, Sun X G and Liu S T 2008 *Opt. Express* **16** 19926

[12] Cai Y J and Zhang L 2006 *Opt. Commun.* **265** 607

[13] Cai Y J and He S L 2006 *J. Opt. Soc. Am.* A **23** 1410

[14] Zheng C W 2006 *Phys. Lett.* A **355** 156

[15] Cai Y J and He S L 2006 *Opt. Express* **14** 1353

[16] Deng D G, Yu H, Xu S Q, Shao J D and Fan Z X 2008 *Opt. Commun.* **281** 202

[17] Skupin S, Saffman M and Królikowski W 2007 *Phys. Rev. Lett.* **98** 263902

[18] Pedri P and Santos L 2005 *Phys. Rev. Lett.* **95** 200404

[19] Peccianti M, Conti C and Assanto G 2005 *Opt. Lett.* **30** 415

[20] Hu W, Zhang T, Guo Q, Li X and Lan S 2006 *Appl. Phys. Lett.* **89** 071111

[21] Rotschild C, Cohen O, Manela O and Segev M 2005 *Phys. Rev. Lett.* **95** 213904

[22] Guo Q, Luo B, Yi F H, Chi S and Xie Y Q 2004 *Phys. Rev.* E **69** 016602

[23] Lu D Q, Hu W, Zheng Y J, Liang Y B, Cao L G, Lan S and Guo Q 2008 *Phys. Rev.* A **78** 043815

[24] Lu D Q and Hu W 2009 *Phys. Rev.* A **80** 083818

[25] Deng D M and Guo Q 2008 *J. Opt.* A: *Pure Appl. Opt.* **10** 035101

[26] Deng D M, Guo Q and Hu W 2009 *Phys. Rev.* A **79** 023803

[27] Kaminer I, Rotschild C, Manela O and Segev M 2007 *Opt. Lett.* **32** 3209

[28] Desyatnikov A S, Sukhorukov A A and Kivshar Y S 2005 *Phys. Rev. Lett.* **95** 203904

[29] Lopez-Aguayo S, Desyatnikov A S and Kivshar Y S 2006 *Opt. Lett.* **31** 1100

[30] Ding N and Guo Q 2009 *Chin. Phys.* B **18** 4298

[31] Shen M, Xi N, Kong Q, Ge L J, Shi J L and Wang Q 2009 *Chin. Phys.* B **18** 2822

[32] Ge L J, Wang Q, Shen M, Shi J L, Kong Q and Hou P 2009 *Chin. Phys.* B **18** 616

[33] Chen L X, Lu D Q, Hu W, Yang Z J, Cao W W, Zheng R and Guo Q 2010 *Acta Phys. Sin.* **59** 2537 (in Chinese)

[34] Zheng Y J, Xuan W T, Lu D Q, Ouyang S G, Hu W and Guo Q 2010 *Acta Phys. Sin.* **59** 1075 (in Chinese)

[35] Song F J and Jutamulia S 2001 *Advanced Optical Information Processing* (Beijing: Peking University Press) pp. 96–103 (in Chinese)

[36] Gradshteyn I S and Ryzhik I M 2000 *Table of Integrals, Series, and Products* (6th edn.) (Singapore: Academic Press) pp. 486, 698, 701, 902

[37] Bélanger P A 1991 *Opt. Lett.* **16** 196

[38] Desyatnikov A S and Kivshar Y S 2002 *Phys. Rev. Lett.* **88** 053901