Two-qutrit maximally entangled states prepared via adiabatic passage in ion-trapped system*

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This paper considers a scheme for the preparation of two-qutrit entangled states via adiabatic passage in ion-trapped system. In the proposal, the two three-level V-type ions are initially cooled to the ground states and need not be separately addressed. Moreover, only the ionic states act as memory and the system evolves in the dark space during the whole procedure, which makes the system robust against the decoherence and the fluctuation of the laser pulse.

Keywords: two-qutrit, adiabatic passage, trapped ions

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Quantum entanglement is one of the most striking features of quantum mechanics and holds the key of applications in quantum information processing.[1−3] Usually, most of the previous schemes use two-dimensional systems (qubits). In recent years, there has been aroused much interest in high-dimensional entangled states which play a more important role in testing quantum nonlocality and quantum information.[4−6] For example, using entangled qutrits (three-dimensional systems), it can violate local realism more strongly[4] and obtain quantum cryptography more securely than entangled qubits.[5] Therefore, there are several groups devoting themselves to studying the preparation of the high-dimensional entangled states. In Refs. [7] and [8], high-dimensional entanglement for photon has been observed. And with cavity QED, several theoretical schemes have also been reported.[9−13]

On the other hand, ion-trapped systems are considered to be a promising candidate for the preparation of entangled states and the implementation of quantum information.[14−16] Based on trapped ions, Zheng had proposed a scheme to generate an entangled state.[17] However, their scheme is based on conditional dynamical evolution, which is sensitive to decoherence[18,19] and fluctuations in experimental parameters.[20] In this paper, we propose an alternate proposal to prepare the two-qutrit entangled states via adiabatic passage in ion-trap systems.

We begin with by describing the model of two three-level V-type ions being trapped in a linear potential. Each ion has a ground state $|g\rangle$ with the free energy $\omega_{\rm g}$ (where we have set $\hbar = 1$) and two excited states $|e\rangle$ and $|f\rangle$ with $\omega_{\rm e}$ and $\omega_{\rm f}$, respectively. We simultaneously drive the transitions $|g\rangle \rightarrow |e\rangle$ of the two ions with a laser of frequency $\omega_1 = \omega_{\rm e} - \omega_{\rm g} + \nu + \delta$ and $|g\rangle \rightarrow |f\rangle$ of the two ions with the other laser with frequency $\omega_2 = \omega_{\rm f} - \omega_{\rm g} - \nu - \delta$, where ν represents the frequency of the collective vibrational mode and δ is the detuning. Suppose that δ is much smaller than ν, and thus other vibrational modes can be neglected. In this case the Hamiltonian for the system is given by the following expression:

$$H = H_0 + V, \tag{1a}$$

where

$$H_0 = \nu a^{\dagger}a + \sum_{j=1}^{2}(\omega_{\rm g}|g\rangle_j\langle g| + \omega_{\rm e}|e\rangle_j\langle e| + \omega_{\rm f}|f\rangle_j\langle f|), \tag{1b}$$

$$V = \sum_{j=1}^{2}[\Omega_1|e\rangle_j\langle g|{\rm e}^{{\rm i}\,\eta(a^{\dagger}+a)-{\rm i}\,\omega_1 t} + \Omega_2|f\rangle_j\langle g|{\rm e}^{{\rm i}\,\eta(a^{\dagger}+a)-{\rm i}\,\omega_2 t}] + {\rm H.c.}, \tag{1c}$$

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where Ω_j $(j=1,2)$ and a $(a^\dagger)$ represent the Rabi frequency for the j-th laser and the creation (annihilation) operator for the collective vibrational mode respectively, η is the Lamb–Dicke parameter, and H.c. stands for Hermitian conjugate. We can easily obtain the interaction Hamiltonian H_I with respect to H_0, i.e.

$$H_{\rm I} = \sum_{j=1}^{2}[\Omega_1|e\rangle_j\langle g|{\rm e}^{{\rm i}\,\eta(a^\dagger {\rm e}^{{\rm i}\,\nu t}+a{\rm e}^{-\,{\rm i}\,\nu t})}{\rm e}^{-i(\nu+\delta)\,t} + \Omega_2|f\rangle_j\langle g|{\rm e}^{{\rm i}\,\eta(a^\dagger {\rm e}^{{\rm i}\,\nu t}+a{\rm e}^{-\,{\rm i}\,\nu t})}{\rm e}^{{\rm i}(\nu+\delta)\,t}] + {\rm H.c.} \quad (2)$$

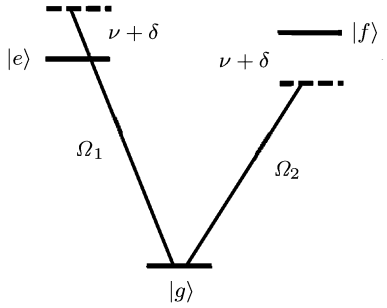


Fig. 1. The transition of two three-level V-type ions.

For simplification, the interaction Hamiltonian is expanded to the first order in the Lamb–Dicke limit $(\eta \ll 1)$ as

$$H_{\rm I} = \sum_{j=1}^{2}\{\Omega_1|e\rangle_j\langle g|[1+{\rm i}\,\eta(a^\dagger {\rm e}^{{\rm i}\,\nu t}+a{\rm e}^{-\,{\rm i}\,\nu t})]{\rm e}^{-\,{\rm i}(\nu+\delta)\,t} + \Omega_2|f\rangle_j\langle g|[1+{\rm i}\,\eta(a^\dagger {\rm e}^{{\rm i}\,\nu t}+a{\rm e}^{-\,{\rm i}\,\nu t})]{\rm e}^{{\rm i}(\delta+\nu)\,t}\} + {\rm H.c.} \quad (3)$$

We further consider the case of the weak-field regime $(\delta \gg \eta\Omega_j)$, the fast-oscillating terms

$$\Omega_1\sum_{j=1}^{2}|e\rangle_j\langle g|({\rm e}^{-\,{\rm i}(\delta+\nu)\,t}+{\rm i}\,\eta a{\rm e}^{-\,{\rm i}(\delta+2\nu)\,t}) + \Omega_2\sum_{j=1}^{2}|f\rangle_j\langle g|({\rm e}^{{\rm i}(\delta+\nu)\,t}+{\rm i}\,\eta a^\dagger {\rm e}^{{\rm i}(\delta+2\nu)\,t})+{\rm H.c.}$$

can be eliminated, leading the interaction Hamiltonian to

$$H_{\rm I} = {\rm i}\,\eta\sum_{j=1}^{2}[(\Omega_1|e\rangle_j\langle g|-\Omega_2|g\rangle_j\langle f|)a^\dagger {\rm e}^{-\,{\rm i}\,\delta\,t} - (\Omega_1|g\rangle_j\langle e|-\Omega_2|f\rangle_j\langle g|)a{\rm e}^{{\rm i}\,\delta\,t}]. \quad (4)$$

In addition, in such a weak field, there is no energy exchange between ions and the vibrational mode, and the effective Hamiltonian for the system is

$$\begin{aligned} H_{\rm eff} = {}& \frac{\eta^2}{\delta}\Bigg\{\sum_{j=1}^{2}[(\Omega_1^2|e\rangle_j\langle e|+\Omega_2^2|g\rangle_j\langle g|)a^\dagger a-(\Omega_1^2|g\rangle_j\langle g|+\Omega_2^2|f\rangle_j\langle f|)aa^\dagger] \\ &+\sum_{\substack{j,k=1\\ j\neq k}}^{2}(\Omega_1^2|e\rangle_j\langle g|\times|g\rangle_k\langle e|+\Omega_2^2|g\rangle_j\langle f|\times|f\rangle_k\langle g|)a^\dagger a \\ &-\sum_{\substack{j,k=1\\ j\neq k}}^{2}(\Omega_1\Omega_2|e\rangle_j\langle g|\times|f\rangle_k\langle g|+\Omega_1\Omega_2|g\rangle_j\langle f|\times|g\rangle_k\langle e|)a^\dagger a \\ &-\sum_{\substack{j,k=1\\ j\neq k}}^{2}(\Omega_1^2|g\rangle_j\langle e|\times|e\rangle_k\langle g|+\Omega_2^2|f\rangle_j\langle g|\times|g\rangle_k\langle f|)aa^\dagger \\ &+\sum_{\substack{j,k=1\\ j\neq k}}^{2}(\Omega_1\Omega_2|g\rangle_j\langle e|\times|g\rangle_k\langle f|+\Omega_1\Omega_2|f\rangle_j\langle g|\times|e\rangle_k\langle g|)aa^\dagger\Bigg\}. \end{aligned} \quad (5)$$

In the case of the initial vibrational mode in the vacuum state, the effective Hamiltonian described in Eq. (5) reduces to

$$\begin{aligned} H_{\rm eff} = {}& -\frac{\eta^2}{\delta}\Bigg\{\sum_{j=1}^{2}(\Omega_1^2|g\rangle_j\langle g|+\Omega_2^2|f\rangle_j\langle f|)+\sum_{\substack{j,k=1\\ j\neq k}}^{2}(\Omega_1^2|g\rangle_j\langle e|\times|e\rangle_k\langle g|+\Omega_2^2|f\rangle_j\langle g|\times|g\rangle_k\langle f|) \\ &-\sum_{\substack{j,k=1\\ j\neq k}}^{2}(\Omega_1\Omega_2|g\rangle_j\langle e|\times|g\rangle_k\langle f|+\Omega_1\Omega_2|f\rangle_j\langle g|\times|e\rangle_k\langle g|)\Bigg\}. \end{aligned} \quad (6)$$

We now begin our scheme for the generation of two-qutrit maximally entangled states. The two ions are initially cooled to the ground state $|gg\rangle_{12}$ and the vibrational mode is in the vacuum state. Under the action of the effective Hamiltonian described in Eq. (6), the initial ionic state will evolve in the subspace $\{|gg\rangle_{12}, \frac{1}{\sqrt{2}}(|ef\rangle + |fe\rangle)_{12}\}$, where the effective Hamiltonian is rewritten as

$$H_{\rm eff} = -\frac{\eta^2}{\delta}\begin{pmatrix} 2\Omega_1^2 & -\sqrt{2}\Omega_1\Omega_2 \\ -\sqrt{2}\Omega_1\Omega_2 & \Omega_2^2 \end{pmatrix}, \tag{7}$$

whose dark state is given by

$$|D\rangle = \frac{\Omega_2}{\sqrt{2\Omega_1^2+\Omega_2^2}}|gg\rangle_{12} + \frac{\Omega_1}{\sqrt{2\Omega_1^2+\Omega_2^2}}(|ef\rangle + |fe\rangle)_{12}. \tag{8}$$

Based on the dark state, a two-qutrit maximally entangled state can be prepared from the ground state $|gg\rangle_{12}$ via adiabatic passage. The detailed procedure is described as follows. We suppose that the first Rabi frequency Ω_1 is initially turned off, and the second Rabi frequency Ω_2 is turned on. In this condition, the system is initially in the dark state

$$|D(0)\rangle = |gg\rangle_{12}. \tag{9}$$

If we adiabatically increase the first Rabi frequency Ω_1 and decrease the second one Ω_2 until $\Omega_1 = \Omega_2$, the two ions will remain their state in the dark state (8) and will eventually evolve to

$$|\psi_{\rm id}\rangle = \frac{1}{\sqrt{3}}(|gg\rangle + |ef\rangle + |fe\rangle)_{12}, \tag{10}$$

which describes a maximal two-qutrit entangled state.

Next, we will give a brief discussion on the feasibility for the generation of the two-qutrit entangled states. In our scheme, the states $|e\rangle$, $|g\rangle$, and $|f\rangle$ can be represented by the hyperfine levels $|F=2,\ m=2\rangle$, $|F=2,\ m=1\rangle$, and $|F=2,\ m=0\rangle$ of $^2S_{1/2}$ of $^9Be^+$, respectively.[21,22] The collective vibrational mode and the ionic state can be prepared in the vacuum state and the ground state with the fidelity approximated to unit.[23] We then consider the effect of the fluctuation of the laser pulses. The fidelity $F = |\langle\psi_{\rm id}/\psi_{\rm F}\rangle|^2$ ($|\psi_{\rm F}\rangle$ is the generated entangled state) versus different ratios of the first Rabi frequency Ω_1 and the second one Ω_2 is illustrated in Fig. 2. The result shows that the prepared entangled state is insensitive to the fluctuation of the laser pulse as long as Ω_2 is slightly larger than Ω_1. For example, if the final values of Ω_1 and Ω_2 satisfying the relation $\Omega_2 = 1.01\Omega_1$, the fidelity is $F = 0.9998$. Even when $\Omega_2 = 1.1\Omega_1$, the fidelity still reaches 0.9981.

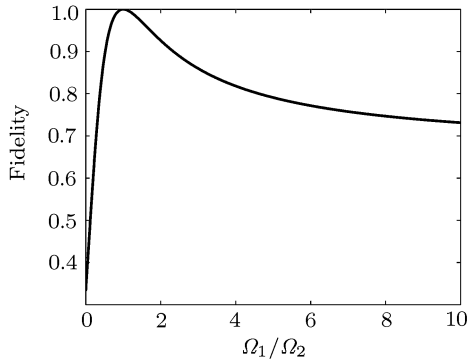


Fig. 2. Fidelity versus the ratio Ω_1/Ω_2.

In conclusion, we have proposed a scheme for the preparation of two-qutrit maximally entangled states via adiabatic passage in ion-trapped system. The scheme is robust against decoherence and fluctuations of the experimental parameters due to adiabatic passage. The two ions are only driven globally, making the system insensitive to the imperfection of separately addressing the ions. Therefore, the present scheme may be realizable based on current technologies.

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