Correlation and fractal properties of speckles in the extremely deep Fresnel diffraction region*

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Based on the Kirchhoff approximation and the theoretical analysis of the random light fields, the speckle intensity distributions in the extremely deep Fresnel diffraction region are simulated. The fractal property of the speckles as well as the relation between the speckle intensity distribution and the corresponding random surface is investigated. We design a microscope system to detect experimentally the speckles in the extremely deep Fresnel diffraction region, and the experimental results prove the conclusions drawn from our simulations.

Keywords: fractal, speckle, extremely deep Fresnel diffraction region

PACC: 4225H, 0555, 4230K

1. Introduction

Coherent light transmitted by a random diffuser or reflected from a rough surface produces a granular interference pattern, known as a speckle pattern.[1,2] They appear in many optical phenomena related to light propagations[2−6] and have wide applications in a variety of scientific and technical fields.[7−9] For the far field speckles, including the speckles in Fresnel and Fraunhofer diffraction regions, Van Cittert and Zernike (VCZ) theorem[10,11] describes the spatial coherence properties of radiation scattered from a planar incoherent source in terms of the two-dimensional (2D) Fourier transform (2DFT) of the source intensity distribution. The VCZ theorem states that the space correlation function of speckles in the far field is dependent only on the intensity distribution of the scattering volume, and independent of the physical properties of the random surface. Thus, the speckles in this region obey the Gaussian distribution and have no fractal properties, and so it is difficult to find any relations between the speckles and the rough surface parameters, i.e. the roughness w, the lateral correlation length ξ (or average scattering grain size, as it is called) and the fractal exponent α. Recently, some detailed studies of the speckles in the deep Fresnel diffraction region were performed,[12−14] and most of these papers in the literature focused on the theoretical analysis of the correlation function with a certain degree of approximation.

In the present paper, we first make a theoretical analysis of speckles in the region infinitely close to the random surface (also called the extremely deep Fresnel diffraction region) based on the Kirchhoff approximation and the Green's function theory, and then we produce the 2D speckles by numerical simulation. By comparison with the corresponding random surfaces, we find that the intensity space distribution and the fractal property of the speckles are both closely related to the random height distribution of the surfaces. The intensity probability density of the speckles is gained, which represents the characters that do not obey the negative exponential law, and it is related to the surface. Finally, we design a microscope system to detect the speckles in the extremely deep Fresnel diffraction region and give experimental evidence of the conclusions reached from our numerical simulations.

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2. Theoretical analysis and numerical simulation of speckles in extremely deep Fresnel diffraction region

The schematic diagram for generating 2D speckles in the extremely deep Fresnel diffraction region is shown in Fig. 1. The random surface placed on the object plane (x_1, y_1) is illuminated by a parallel laser beam with wavelength λ and amplitude U_0. $P_1(x_1, y_1)$ is an arbitrary point on the plane with a unit normal vector $\boldsymbol{n}$ and the random height distribution on this point is $h(x_1, y_1)$. The observation plane (x, y) is located in the extremely deep Fresnel diffraction region with a distance z from the object plane. Based on the Kirchhoff approximation,[15] the expression of the scattered light field at a point $P(x, y)$ in the observation plane can be written as

$$U(x,y) = \frac{1}{4\pi}\iint\left(G\frac{\partial U_0(x_1,y_1)}{\partial n} - U_0(x_1,y_1)\frac{\partial G}{\partial n}\right)\mathrm{d}S \quad (1)$$

with

$$U_0(x_1,y_1) = \exp[\mathrm{j}k(m-1)h(x_1,y_1)], \quad (2)$$

$$G = \frac{\exp[\mathrm{j}kr]}{r}, \quad (3)$$

where the integrating factor in Eq. (1) is around the object point $P_1(x_1, y_1)$ on the random surface, and G is Green's function, m is the refractive index of the sample, and $k = 2\pi/\lambda$ is the magnitude of the wave vector. The distance between the object point P and the observation point P_1 is $r = \{(x-x_1)^2+(y-y_1)^2+[z-h(x_1,y_1)]^2\}^{1/2}$.

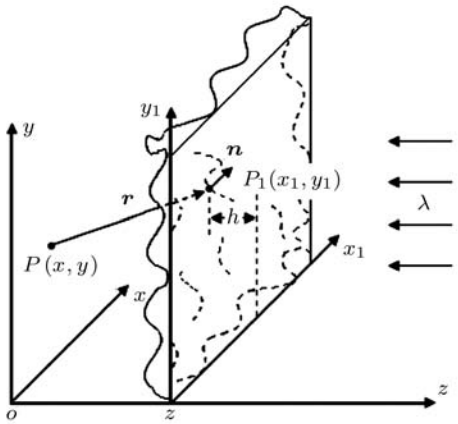


Fig. 1. Schematic diagram for generating speckles in the extremely deep Fresnel diffraction region.

The partial derivative is

$$\frac{\partial}{\partial n} = (\boldsymbol{n}\cdot\boldsymbol{\nabla}) = \frac{1}{\gamma}\left[-\frac{\partial h(x_1,y_1)}{\partial x_1}\frac{\partial}{\partial x_1} - \frac{\partial h(x_1,y_1)}{\partial y_1}\frac{\partial}{\partial y_1} + \frac{\partial}{\partial z}\right] \quad (4)$$

with

$$\gamma = \left\{1 + \left[\frac{\partial h(x_1,y_1)}{\partial x_1}\right]^2 + \left[\frac{\partial h(x_1,y_1)}{\partial y_1}\right]^2\right\}^{1/2}, \quad (5)$$

then equation (1) can be written as

$$\begin{aligned} U(x,y) &= \frac{1}{4\pi}\iint \mathrm{d}S\left(G\frac{\partial U_0(x_1,y_1)}{\partial n} - U_0(x_1,y_1)\frac{\partial G}{\partial n}\right) \\ &= \frac{1}{4\pi}\iint \mathrm{d}S\exp\{\mathrm{j}k[(m-1)h(x_1,y_1)+r]\} \\ &\quad\times\left[\frac{\mathrm{j}k(m-1)\gamma}{r} - \left(\mathrm{j}k - \frac{1}{r}\right)\frac{\chi}{r^2}\right] \end{aligned} \quad (6)$$

with

$$\begin{aligned} \chi = {} & h(x_1,y_1) - z - (x_1-x)\frac{\partial h(x_1,y_1)}{\partial x_1} \\ & - (y_1-y)\frac{\partial h(x_1,y_1)}{\partial y_1}. \end{aligned} \quad (7)$$

The autocorrelation function of speckle intensities $R_\mathrm{I}(x, y; x', y')$ is defined as

$$\begin{aligned} R_\mathrm{I}(x,y;x',y') &= \langle I(x,y)\cdot I(x',y')\rangle \\ &= \langle U(x,y)U^*(x,y)\cdot U(x',y')U^*(x',y')\rangle, \end{aligned} \quad (8)$$

where $\langle\ \rangle$ denotes the ensemble average, $I(x, y)$ and $I(x', y')$ are intensities of two arbitrary observation points.

Based on the Gaussian moment theorem and some approximate calculations, the analytical expression of normalized autocorrelation function of the speckle intensity in the deep Fresnel diffraction region is given as[14]

$$\begin{aligned} \kappa_\mathrm{I}(\rho) &= [R_\mathrm{I}(\rho) - \langle I\rangle^2]/\langle I\rangle^2 \\ &= \exp\{-2[2\pi(m-1)/\lambda]^2[w^2 - R_\mathrm{h}(\rho)]\}, \end{aligned} \quad (9)$$

where $\rho = |x'-x|$ or $\rho = |y'-y|$ is the correlation separation along the x or the y axis, and $R_\mathrm{h}(\rho)$ is the height autocorrelation function of the rough surface, and the roughness w satisfies $w^2 = R_\mathrm{h}(\rho = 0)$. The height autocorrelation function and the height–height correlation function of a random self-affine fractal surface, which can describe a variety of practical surfaces

ranging from growth fronts to ground interfaces,[16,17] are phenomenologically given by

$$R_{\rm h}(\rho) = \langle h(r)h(r+\rho)\rangle = w^2\exp[-(\rho/\xi)^{2\alpha}], \quad (10)$$

$$H_{\rm h}(\rho_1) = \langle[h(r_1+\rho_1)-h(r_1)]^2\rangle = 2w^2\{1-\exp[-(\rho_1/\xi)^{2\alpha}]\}, \quad (11)$$

where the fractal exponent α related to the fractal dimension $d_{\rm f}$ by $d_{\rm f} = d - \alpha$, with d being the embedded dimension,[18] when $\alpha = 1$, the random surfaces reduce to those with Gaussian correlation.

Although the result shown in Eq. (9) is approximate, moreover, the exact results cannot be strictly derived, from the approximate results we can still see that the speckles in the extremely deep Fresnel diffraction region are closely linked to the random surface. When the observation plane is in the extremely deep Fresnel diffraction region, the area in the object plane contributing to the light field at an observation point becomes more finite and much smaller than the area of illuminating beam. This condition is no longer applicable for the VCZ theorem, so the speckle is no longer obey the Gaussian distribution.

Then, the speckles in the extremely deep Fresnel diffraction region produced by the scattering of random self-affine fractal surface can be simulated based on Eq. (6). First, three random surface samples with different parameters are produced by grinding glass sufficiently with different grit sizes of corundum, and these samples are labeled as I, II and III, respectively. Then the random height fluctuations of these samples are measured by atomic force microscope (AFM), and the statistical parameters of these samples can be extracted based on Eq. (11),[17] and the results are shown in Table 1. The measured range of the AFM is 100 μm $\times$ 100 μm with the imaging lattice 256×256, so the lattice step, $100/255 = 0.3922$ μm, is much close to the wavelength of the illuminating light 0.6328 μm (corresponding to that of the He–Ne laser). We increased the lattice to 2041×2041 by the interpolation method, and then the lattice step is reduced to $100/2040 = 0.049$ μm. After the light field $U(x,y)$ is calculated based on Eq. (6), we can gain the corresponding intensity distribution by employing the formula $I(x,y) = U(x,y)U^*(x,y)$. In practical calculations, the refractive index of the sample is $m = 1.532$, the distance z is the maximum height difference of the random surface $z = h_{\rm MAX} - h_{\rm MIN}$, then $z_{\rm I} = 8.32828$ μm, $z_{\rm II} = 5.99659$ μm and $z_{\rm III} = 3.45001$ μm.

Table 1. Statistical parameters of the samples I, II and III.

	I	II	III
w/μm	0.921	0.674	0.487
ζ/μm	8.121	5.473	4.149
α	0.572	0.632	0.648

Figures 2(a), 2(b) and 2(c) show the partial height distributions of these three samples I, II and III measured by AFM, and figures 2(d), 2(e) and 2(f) display the corresponding speckle intensity distributions in the extremely deep Fresnel diffraction region, simulated in the paper, respectively. Comparing the speckle intensity distributions with their corresponding random surfaces, we find there are some structural similarities between them, i.e., the local fluctuations in speckle intensity are directly induced by the fractal characteristics of the random surfaces. Each peak in random surfaces will produce a bigger speckle granule, in which there are many little speckle spots produced by the microstructures of the surfaces. This means that there are obvious resemblances between the speckle intensities and the surface heights. These resemblances originate from the local smoothness of the surface, of which the peaks and the valleys act as convex and concave micro-lenses, respectively. The convex parts converge the light and the concave parts diverge the light, and then the bright spots with larger intensities and the dark areas are formed behind them.[14] That is to say, speckles in the extremely deep Fresnel diffraction region no longer satisfy the VCZ theorem, and they are dependent only on the physical properties of the random surface, but independent of the intensity distributions of scattering volume. Moreover, the detection distances decrease gradually as well as the resemblances to their scattering random surfaces are increasing (see the evolution from Figs. 2(d) to 2(f)). Therefore, the scattering distances determine the intensity distributions, and as the distance turns shorter, the intensities distributions are more similar to those of the scattering random surfaces.

Based on Eq. (11) we define the intensity–intensity correlation function of the speckles as

$$S_{\rm I}(\rho) = \langle[I(r+\rho)-I(r)]^2\rangle, \quad (12)$$

which is known from the fractal theory,[19] for the generic fractal structure, $S_{\rm I}(\rho)$ is given by the following power law

$$S_{\rm I}(\rho) \propto \rho^{2\alpha}. \quad (13)$$

The solid square dots in Figs. 3(a) and 3(b) denote the intensity-intensity correlation functions of the speckles in the extremely deep Fresnel diffraction region as shown in Figs. 2(d)–2(f) respectively. We can see that the dots are asymptotic to a line with slope 2α while ρ is small in logarithm coordinate. Based on the linear fitting of the dots, shown in Fig. 3 with real lines, the fractal exponents of the speckles are extracted, and they are 0.865, 0.883 and 0.905, respectively. These values are bigger than the fractal exponents of their corresponding samples shown in Table 1, and the reason is that the scattering effect weakens the fractal characteristics of the speckles.

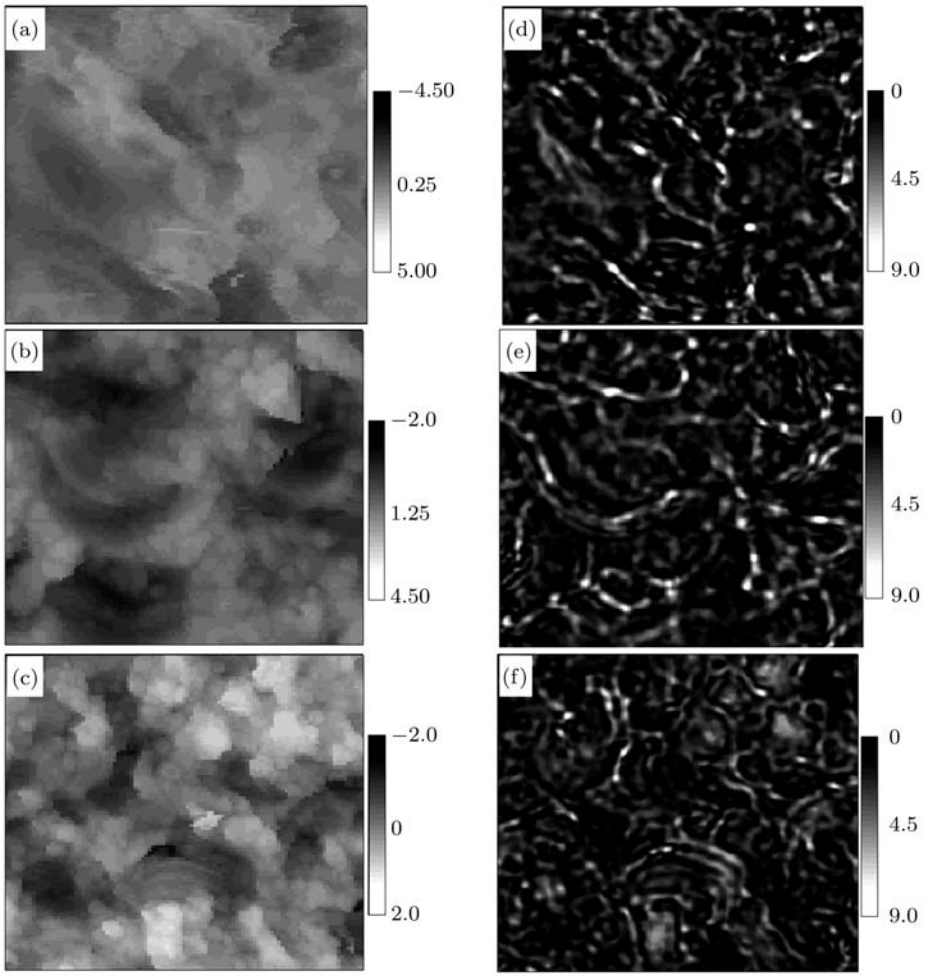


Fig. 2. Grey scale maps of height distributions of the random surface samples ((a), (b), (c)), and corresponding speckle intensity distributions in the extremely deep Fresnel diffraction region ((d), (e), (f)). The ranges of all the six maps are 40 μm × 40 μm.

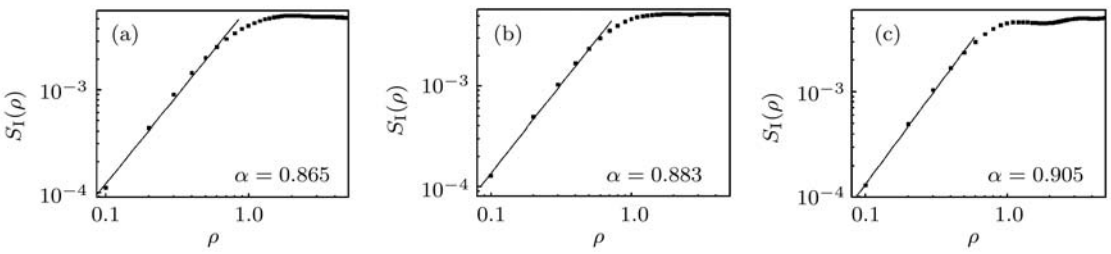


Fig. 3. Intensity–intensity correlation functions (■) of the speckles shown in Figs. 2(d)–2(f) and results of linear fitting (solid lines).

As is well known, for the circular Gaussian statistic speckles, the intensity probability density obeys a negative exponential law[2,20]

$$p_{\rm I}(I) = \begin{cases} \dfrac{1}{\langle I\rangle}\exp\left(-\dfrac{I}{\langle I\rangle}\right), & I \geq 0, \\ 0, & \text{otherwise}, \end{cases} \tag{14}$$

but the speckles in the extremely deep Fresnel diffraction region no longer have a Gaussian probability distribution as we have analysed above, so the intensity probability density no longer obeys Eq. (14). In Fig. 4, solid square line (a), solid triangle line (b), and solid star line (c) denote the intensity probability densities of the speckles produced by samples I, II and III shown in Figs. 2(d)–2(f), respectively. We can see that the ranges of intensity fluctuations are different from each other. Solid square line (a) shows that the maximum intensity of speckles is 10 times more than the average $\langle I\rangle$, while the multiples are about 8 and 7 in solid triangle line (b) and solid star line (c), respectively. Otherwise, these probability density curves decrease in the minimum $I/\langle I\rangle$ area and become smaller and smaller from solid square line to solid star line (c), they do not reach their maxima as the theoretical curve does. We can use the scattering theory to explain the properties. In the extremely deep Fresnel diffraction region, light field cannot be adequately scattered for the smaller scattering distance z. The smaller the scattering distance z is, the more inadequate the scattered light field will be, so the constructive interference and the destructive interference are inadequate. Then we can say that the intensity distribution properties are related to the scattering distance z.

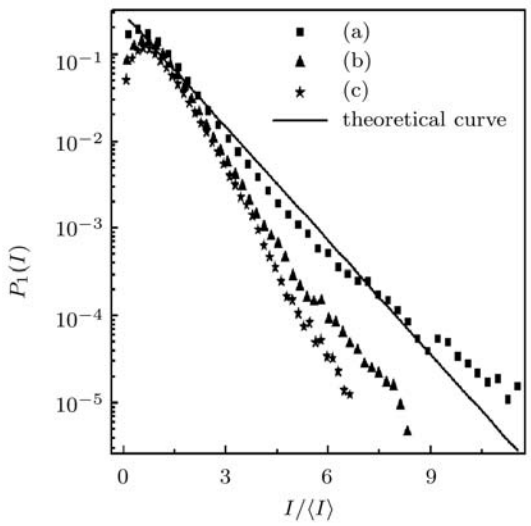


Fig. 4. Solid dots (a)–(c) denote the intensity probability densities of the speckles shown in Figs. 2(d)–2(f) respectively, and theoretical results (solid line).

With the foregoing analysis, we conclude that the intensity distribution properties of speckles in the extremely deep Fresnel diffraction region are determined by the statistics of microstructures of the random surface rather than the intensity distribution of the illumination, and are related to scattering distance.

3. Experimental analysis of the speckles in extremely deep Fresnel diffraction region

We detect the speckles in the extremely deep Fresnel diffraction region by using a microscope system depicted in Fig. 5. A linearly polarized light beam from He–Ne laser illuminates perpendicularly the random surface samples fixed on a nanometer mobile platform (NMP). To detect the speckles in the region very close to the random surface, we use a 40× microscope objective (MO) and a lens (L, f = 50 mm) to enlarge the light field near the random surface, and the image is recorded by a charge-coupled device (CCD) camera (Cascade-1K) with 1004 × 1002 pixels and the pixel size 8 μm × 8 μm.

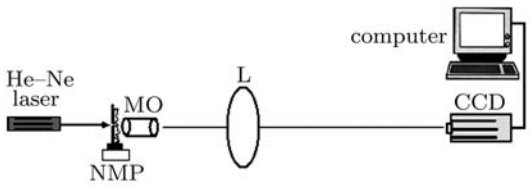


Fig. 5. Experimental setup for the detection of the speckles in extremely Fresnel diffraction region.

We illuminate the random surface by white light at first and slide the surface back and forth with special care by using an NMP until we can use the CCD to distinctly record the image of the random surface. For the bad coherence of the white light, we can observe the speckles only when the observation plane is very close to the random surface. Then the speckles in the extremely deep Fresnel diffraction region should be detected when we replace white light with He–Ne laser. The mobile size of the speckles recorded by CCD is 5208 μm when the random surface is laterally slid 30 μm by NMP, so the magnification of the system is 173.6, and the range of the object plane is 46.1 μm × 46.3 μm.

Speckles in the extremely deep Fresnel diffraction region, produced by illuminating the random surface samples I, II and III are detected by the CCD, and

they are shown in Figs. 6(a), 6(b) and 6(c) respectively. According to the statement previously mentioned, although the detection distances cannot be determined accurately, we are convinced that the observation planes are very close to the random surfaces, and the distances between them are smaller and smaller from Figs. 6(a) to 6(c). From this figure, we can see that the structures of the speckles are different from those in the ordinary far field, and they fully reflect the structural features of the random surfaces. Therefore, the intensity distribution properties of the speckles depend on the microstructures of the random surface samples. The intensity–intensity correlation functions are shown in Fig. 7 and the intensity probability densities are shown in Fig. 8. The results shown in Fig. 7 prove that the speckles in the extremely deep Fresnel diffraction region have fractal properties and the fractal exponent α relies on the fractal property of the random surface. Moreover, the experimental results shown in Fig. 7 are different from the simulation results shown in Fig. 3, this is probably because the detection distances cannot be accurately determined in experiment. From Fig. 8 we can see that the intensity probability density of these speckles is related to the random surface and does not obey the negative exponential law. These curves are different from those in Fig. 4, which is maybe because of the effect of the noise produced by CCD.

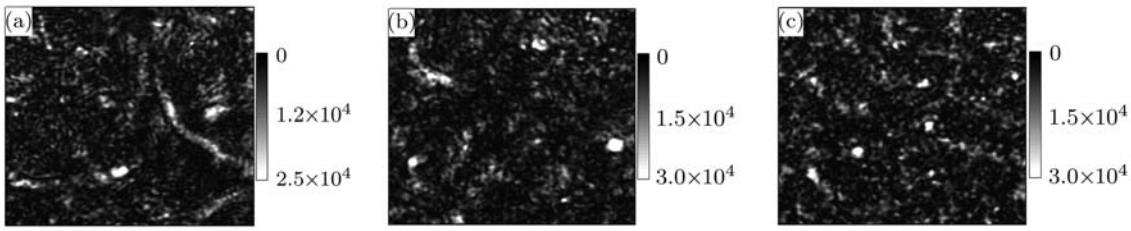


Fig. 6. Speckles in extremely deep Fresnel diffraction region, detected by CCD.

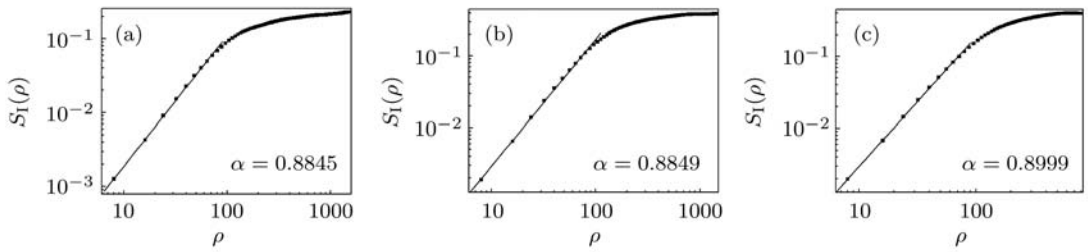


Fig. 7. Curves of intensity–intensity correlation functions (■) of the speckle fields shown in Figs. 6(a)–6(c) and results of linear fitting (solid lines).

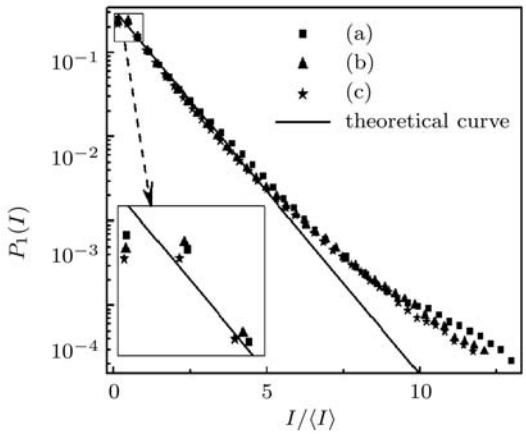


Fig. 8. Intensity probability densities (solid dots (a), (b), (c)) of the speckle fields shown in Figs. 6(a)–6(c) respectively, and theoretical results (solid line).

4. Summary

In this paper, we make theoretical analysis on the correlation of speckles in the extremely deep Fresnel diffraction region by using the Kirchhoff approximation and we simulate the speckle intensity distributions. Further, we compare the speckle intensity distributions with the corresponding random surface samples and investigate the connection between them as well as the fractal properties of the speckles. Finally, we give experimental evidence of the conclusion from our simulations by a microscope system.

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